

## String cosmological models in the Brans-Dicke theory for five-dimensional space-time

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Received 2011 February 3; accepted 2011 June 1

**Abstract** Five-dimensional space-time string cosmological models generated by a cloud of strings with particles attached to them are studied in the Brans-Dicke theory. We obtain two types of interesting models by taking up the cases of geometric strings (or Nambu strings) and  $p$ -strings (Takabayasi strings), and study their different physical and dynamical properties. The roles of the scalar field in getting different phases, such as the inflationary phase and the string-dominated phase, are discussed. An interesting feature obtained here is that in one of the models there is a “bounce” at a particular instant of its evolution.

**Key words:** cosmology: theory — string universes — Brans-Dicke theory

### 1 INTRODUCTION

Cosmologists have taken considerable interest in the study of cosmic strings since they are believed to play an important role in the description of the universe in the early stages of its evolution (Kibble 1976) and give rise to density perturbations leading to the formation of galaxies (Zeldovich 1980). The existence of a large scale network of strings in the early universe does not contradict present day observations of the universe. These strings possess stress energy and are coupled with a gravitational field. The very early universe underwent phase transitions which gave rise to topologically stable structures of particular interest (Stachel 1980), linear structures called geometric strings. Letelier (1983) presented a model of string dust in which incoherent matter particles are attached to geometric strings along their extension. The presence of strings results in anisotropy in the space-time, though strings are not observable in the present epoch. Unlike domain walls and monopoles, strings cause no harm (to the cosmological models), but rather can lead to very interesting astrophysical consequences (Kibble 1976). The string gas cosmology will lead to a dynamical evolution of the early universe very different from what is obtained in standard and inflationary cosmology and can already be seen by combining the basic ingredients from string theory discussed so far. As the radius of a cloud of strings decreases from an initially large value which maintains thermal equilibrium, the temperature first rises as in standard cosmology since the occupied string states (the momentum modes) get heavier. However, as the temperature approaches the Hagedorn temperature, the energy begins to flow into the oscillatory modes and the increase in temperature levels off. As the radius decreases below the string scale, the temperature begins to decrease as the energy begins to flow into the winding modes whose energy decreases as the radius decreases. The temperature singularity

of early universe cosmology should be resolved in string gas cosmology. The equations that govern the background of string cosmology are not known. The Einstein equations are not correct in this case since they do not obey the T-duality symmetry of string theory. Many early studies of string gas cosmology were based on using dilation gravity equations. However, these equations are not satisfactory either. First we expect that these theory correction terms to the low energy effective action of string theory become dominant in the Hagedorn phase. Once the dilation becomes large, it is unproductive to focus on fundamental string states rather than brane states. In other words, using dilation gravity as a background for string gas cosmology does not correctly reflect the S-duality symmetry of string theory. Recently, a background for string gas cosmology including a rolling tachyon was proposed by Brandenberger et al. (2007) which allows a background in the Hagedorn phase with constant scale factor and dilation.

The transition between the quasi-static Hagedorn phase and the radiation phase at the time is a consequence of the annihilation of string winding modes into string loops. This process corresponds to the production of radiation, hence string gas cosmology may provide a natural mechanism for explaining why there are three large spatial dimensions.

The possibility of geometrically unifying the fundamental interactions of the universe motivates the study of higher dimensional cosmological models. It was suggested by Marciano (1984); Randjbar-Daemi et al. (1984) that the experimental detection of the time variation of fundamental constants could provide strong evidence for the existence of extra dimensions.

The extra dimensions, being too small, are unobservable at present. Cosmological models in which the space coordinates expand while the fifth dimension contracts or remains constant were studied by some authors, namely D'Adda et al. (1982); Appelquist & Chodos (1983); Rahaman et al. (2002); Singh et al. (2004); Mohanty et al. (2006, 2007a,b); Mohanty & Mahanta (2007); Rathore & Mandawat (2009).

Since the universe was much smaller in the early stages of its evolution than it is today, the present four-dimensional stage of the universe could have been preceded by a higher dimensional stage. The extra dimensions become unobservable due to dynamical contraction (Appelquist et al. 1987; Chatterjee 1992 and Chodos & Detweiler 1980), which leads to the present four-dimensional space-time of the universe. Guth (1981) and Alvarez & Gavela (1983) observed that during the contraction process extra dimensions produce large amounts of entropy, which provides an alternative solution to the flatness and horizon problems than the usual inflationary scenario. Studies in theories with more than four dimensions in which the extra dimensions are compactified to the small size of a Planck length during the evolution of the universe will be interesting. These studies in higher dimensions will lead to the discovery of an anomalous free super string theory, which is an approach to the unification of the fundamental forces of nature.

The study of the cosmological models of the Brans-Dicke theory (Brans & Dicke 1961), which develops Mach's principle in a relativistic framework by assuming the interaction of inertial masses of fundamental particles with some cosmic scalar field, coupled with the large scale distribution of matter in motion, has gained momentum. Models involving inflation (Mathiazhagan & Johri 1984), extended inflation (La & Steinhardt 1989 and Steinhardt & Accetta 1990), hyper-extended inflation and extended chaotic inflation (Linde 1990) are based on the Brans-Dicke theory of gravitation and general scalar tensor theories. String cosmological models in Brans-Dicke and other alternative theories of gravitation were obtained by Gundlach & Ortiz (1990); Barros & Romero (1995); Banerjee et al. (1996); Sen (2000); Barros et al. (2001); Bhattacharjee & Baruah (2001); Rahaman et al. (2003); Reddy (2005a,b); Reddy et al. (2006); Rathore & Mandawat (2009), whereas five dimensional string cosmological models in alternative theories of gravitation were presented by Reddy & Naidu (2007); Mohanty & Mahanta (2007); Mohanty et al. (2007b); Yoshimura (1984); Krori et al. (1994), etc. In this paper we investigate string cosmological models in the Brans-Dicke theory of gravitation for five-dimensional space-time, and obtain some interesting solutions. Further studies of these models will be very stimulating.

## 2 FIELD EQUATIONS AND THEIR SOLUTIONS

For this problem we consider the metric

$$ds^2 = -dt^2 + A^2 dx_1^2 + B^2(dx_1^2 + dx_3^2) + C^2 dx_4^2, \quad (1)$$

where  $A, B$  and  $C$  are arbitrary functions of time. The scalar tensor field equations in the Brans-Dicke theory are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi'^k\right) - \phi^{-1}(\phi_{ij} - g_{ij}\square\phi), \quad (2)$$

where

$$\square\phi = \phi'_{;k}{}^k = 8\pi\phi^{-1}T(3 + 2\omega)^{-1}. \quad (3)$$

Here  $T_{ij}$  is the stress energy tensor of matter,  $\phi$  the scalar field and  $\omega$  the dimensionless coupling constant. As a consequence of the field Equations (2) and (3) we get the equation

$$T_{,j}^{ij} = 0. \quad (4)$$

For cosmic strings the energy momentum tensor is given by

$$T_j^i = \rho u^i u_j - \lambda x^i x_j, \quad (5)$$

where  $\rho$  is the rest energy density of the cloud of strings with particles attached to them,  $\lambda$  the string tension density,  $u^i$  the cloud's four velocity and  $x^i$  the direction of the string satisfying the relations

$$u^i u_j = x^i x_j = -1, \quad u^i x_i = 0. \quad (6)$$

Relation (5) gives, for the co-moving coordinate system,

$$T_0^0 = -\rho, \quad T_1^1 = T_2^2 = T_3^3 = 0, \quad T_4^4 = -\lambda, \quad T_j^i = 0, \quad (7)$$

for  $i \neq j$ , and

$$T = (\rho + \lambda). \quad (8)$$

Here, the rest energy density of the particles  $\rho_p$  is given by

$$\rho_p = \rho - \lambda, \quad (9)$$

where  $\rho, \lambda$  and  $\rho_p$  are taken to be functions of time  $t$ . In this problem  $\phi$  is only considered to be a function of time.

Thus for the above metric the Brans-Dicke field equations are obtained as

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}^2}{B^2} = 8\pi\phi^{-1}\rho + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2\frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right], \quad (10)$$

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}^2}{B^2} = -\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{A}}{A}\left(\frac{\dot{\phi}}{\phi}\right) + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right], \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = -\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right], \quad (12)$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi\phi^{-1}\lambda - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{C}\dot{\phi}}{C\phi} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right], \quad (13)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\phi(\rho + \lambda)}{3 + 2\omega}. \quad (14)$$

We have the conservation equation

$$\dot{\rho} + \rho \left( \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \right) + (\rho - \lambda) \frac{\dot{C}}{C} = 0. \quad (15)$$

A “dot” denotes differentiation with respect to  $t$ . Here we have five independent equations in six unknowns  $A, B, C, \phi, \rho$  and  $\lambda$ . Therefore we need more relations to find the determinate solutions of these equations. Thus we try to get the solutions by taking up two cases, namely

- (i)  $\rho = (1 + \omega)\lambda$ , which is the case of  $p$ -strings or Takabayasi strings, where  $\omega (> 0)$  is a constant.
- (ii)  $\rho = \lambda$ , which is the case of geometric strings or Nambu strings.

**Case-I** In this case

$$\rho = (1 + \omega)\lambda. \quad (16)$$

Therefore, we obtain (from the field equations),

$$A = (a_1 t - a_0)^{a_2}, \quad (17)$$

$$B = b_0 (a_1 t - a_0)^{a_2 b_1}, \quad (18)$$

$$C = c_0 (a_1 t - a_0)^{c_1}, \quad (19)$$

$$\phi = (a_1 t - a_0)^{-1}, \quad (20)$$

$$\begin{aligned} \rho = \frac{1}{8\pi} (a_1 t - a_0)^{-3} & (2a_1^2 a_2^2 b_1 + a_2^2 a_1^2 b_1^2 + a_1^2 a_2 + a_1^2 a_2 b_1 \\ & - 2a_1^2 a_2 b_1 c_1 - a_1^2 a_2 c_1 - a_1^2 c_1 - \frac{\omega}{2} a_1^2 - 4a_1^2), \end{aligned} \quad (21)$$

$$\begin{aligned} \lambda = \frac{1}{8\pi(1 + \omega)} (a_1 t - a_0)^{-3} & (2a_1^2 a_2^2 b_1 + a_2^2 a_1^2 b_1^2 + a_1^2 a_2 + a_1^2 a_2 b_1 \\ & - 2a_1^2 a_2 b_1 c_1 - a_1^2 a_2 c_1 - a_1^2 c_1 - \frac{\omega}{2} a_1^2 - 4a_1^2), \end{aligned} \quad (22)$$

where  $a_0, a_1, a_2, b_0, b_1, c_0$  and  $c_1$  are arbitrary constants satisfying the relations

$$a_2^2 b_1^2 + a_2 b_1 c_1 + a_2 c_1 + a_2 b_1 + 2a_2 + 2 - 2a_2 b_1 c_1 - a_2^2 b_1 - a_2^2 = 0, \quad (23)$$

$$\begin{aligned} 3a_1^2 b_1 + 2a_1^2 b_1 \omega + 4a_2^2 b_1 + 3a_2^2 b_1^2 + 5a_2 b_1 + 2a_2 \omega + 4a_2 + a_2^2 \\ - 2c_1 \omega - 2a_2 b_1 c_1 - a_2^2 c_1 - 9c_1 - 4\omega - 12 = 0, \end{aligned} \quad (24)$$

$$2a_2 b_1 + a_2 - 3 + (2a_2 b_1 + a_2 - c_1 - 3)\omega = 0, \quad (25)$$

$$\begin{aligned} 2a_2 b_1 c_1 + a_2 c_1 + 3a_2 b_1 + a_2^2 + \omega + 2 - a_2 - c_1 \\ + \omega \left( a_2^2 b_1^2 + 2a_2^2 b_1 + 4a_2 b_1 + a_2^2 + \frac{\omega}{2} - 2c_1 - 2 \right) = 0, \end{aligned} \quad (26)$$

$$2a_2 b_1 + a_2 - c_1 - 2 = 0, \quad (27)$$

$$a_1 a_2 c_0 = d_0, \quad (28)$$

and where  $d_0$  is an integration constant,

$$\begin{aligned} b_0^2 c_0^{\frac{\omega}{1+\omega}} (2a_1^2 a_2^2 b_1 + a_2^2 a_1^2 b_1^2 + a_1^2 a_2 + a_1^2 a_2 b_1 \\ - 2a_1^2 a_2 b_1 c_1 - a_1^2 a_2 c_1 - a_1^2 c_1 - \frac{\omega}{2} a_1^2 - 4a_1^2) = 1. \end{aligned} \quad (29)$$

For this model we have

$$\rho_p = \frac{\omega}{8\pi(\omega+1)}(a_1t - a_0)^{-3}(2a_1^2a_2^2b_1 + a_2^2a_1^2b_1^2 + a_1^2a_2 + a_1^2a_2b_1 - 2a_1^2a_2b_1c_1 - a_1^2a_2c_1 - a_1^2c_1 - \frac{\omega}{2}a_1^2 - 4_1^2). \quad (30)$$

$$\sigma = \frac{1}{3\sqrt{2}}(a_1t - a_0)^{-1} \left[ (2a_1a_2(1 - b_0b_1) + a_1c_0c_1)^2 + b_0^4(a_1a_2(b_0b_1 - 1) + a_1c_0c_1)^2 + b_0^4(a_1a_2(b_0b_1 - 1) + a_1c_0c_1)^2 + c_0^4(2a_1c_0c_1 + a_1a_2(2b_0b_1 + 1))^2 \right]. \quad (31)$$

Volume,

$$V = b_0^2c_0(a_1t - a_0)^{a_2+2a_2b_1-c_1}, \quad (32)$$

expansion factor,

$$\theta = (a_1t - a_0)^{-1} [a_1a_2(2b_0b_1 + 1) + a_1c_0c_1], \quad (33)$$

and deceleration parameter,

$$q = -3(2a_2b_1 + a_2 - c_1 - a_1) [a_1a_2(2b_0b_1 + 1) - a_1c_0c_1]^{-1} - 1. \quad (34)$$

**Case-II.** In this case

$$\rho = \lambda. \quad (35)$$

Therefore we obtain (from the field equations),

$$A = \exp(\ell_1t + \ell_0), \quad (36)$$

$$B = m_0 \exp(\ell_1m_1t + \ell_0m_1), \quad (37)$$

$$C = \exp(n_0 - 2n_1t), \quad (38)$$

$$\varphi = \exp(s_1t - s_0), \quad (39)$$

$$\rho = \frac{1}{8\pi} \left( 2\ell_1^2m_1 + \ell_1^2m_1^2 - 3n_1\ell_1 - 5\ell_1m_1n_1 + 2n_1s_1 - \frac{\omega}{2}s_1^2 - 2s_1^2 \right) \exp(s_1t - s_0), \quad (40)$$

$$\lambda = \frac{1}{8\pi} \left( 2\ell_1^2m_1 + \ell_1^2m_1^2 - 3n_1\ell_1 - 5\ell_1m_1n_1 + 2n_1s_1 + \frac{\omega}{2}s_1^2 - 2s_1^2 \right) \exp(s_1t - s_0), \quad (41)$$

where  $\ell_0, \ell_1, m_0, m_1, n_0, n_1, s_0$  and  $s_1$  are arbitrary constants satisfying the relations

$$3\ell_1^2m_1^2 + 4n_1^2 - 4\ell_1m_1n_1 - \frac{\omega}{2}n_1^2 - 2\ell_1s_1 + s_1^2 - \ell_1m_1s_1 = 0, \quad (42)$$

$$\ell_1^2m_1^2 + \ell_1^2 + 4n_1^2 + \ell_1^2m_1 + 2n_1s_1 + \frac{\omega}{2}s_1^2 - 2\ell_1m_1n_1 - 2\ell_1m_1s_1 - 2\ell_1n_1 - \ell_1s_1 - s_1^2 = 0, \quad (43)$$

$$5\ell_1m_1n_1 + 2\ell_1m_1\omega s_1 + 2\ell_1\omega s_1 + 4\ell_1\omega s_1 + 2\omega s_1^2 + 3\ell_1n_1 + 4\ell_1s_1 + 4s_1^2 - 14n_1s_1 - 4n_1\omega s_1 - 4\ell_1^2m_1 - 4\ell_1^2m_1^2 - \ell_1^2 = 0, \quad (44)$$

$$2\ell_1^2m_1^2 + \omega s_1^2 + 3s_1^2 + \ell_1^2 + 3n_1\ell_1 + 5\ell_1m_1n_1 - \ell_1s_1 - \ell_1m_1s_1 = 0, \quad (45)$$

$$s_1 + \ell_1 + 2\ell_1m_1 = 0, \quad (46)$$

$$2\ell_0m_1 + \ell_0 + n_0 - s_0 = z_0, \quad (47)$$

where  $z_0$  is an integration constant,

$$2\ell_0 m_1 + \ell_0 - s_0 = 0, \quad (48)$$

$$m_0^2 \left( 2\ell_1^2 m_1 + \ell_1^2 m_1^2 - 3\ell_1 n_1 - 5\ell_1 m_1 n_1 + 2n_1 s_1 - \frac{\omega}{2} s_1^2 - 2s_1^2 \right) \exp(2\ell_0 m_1 + \ell_0 - s_0) = 1. \quad (49)$$

In this model

$$\theta = \ell_1 + 2\ell_1 m_1 + 2n_1, \quad (50)$$

$$q = -4, \quad (51)$$

$$V = m_0^2 \exp[(\ell_1 + \ell_1 m_1 - 2n_1)t + \ell_0 + \ell_0 m_1 + z_0], \quad (52)$$

$$\sigma = \frac{1}{3\sqrt{2}} \left[ (3\ell_1 - n_1)^2 + \frac{1}{2}(3\ell_1 + 5n_1)^2 + 25n_1^2 \right]^{\frac{1}{2}}. \quad (53)$$

### 3 DISCUSSION AND CONCLUSIONS

In Case-I it is seen that with the increase in time the fifth dimension of the Brans-Dicke model contracts and becomes unobservable after some time whereas the three space dimensions expand in the normal way for  $a_2 > 0, b_1 > 0$ . Thus for this model to be physically realistic, we must have  $a_2 > 0, a_2 b_1 > 0$ . It has no initial singularity, but there is a ‘‘bounce’’ at  $t = \frac{a_0}{a_1}$ . The expansion rate in this model decreases as  $t$  increases and becomes almost nil as  $t \rightarrow \infty$ . Here the shear  $\sigma$  is found to be a decreasing function of time. Since  $\lim_{t \rightarrow \infty} \left( \frac{\sigma^2}{\theta^2} \right) \neq 0$  this model is not isotropic for large values of  $t$ . Here the deceleration parameter  $q$  is found to be negative, which shows that the model inflates. From the expression of the volume of this model it is seen that this model follows the power law of inflation for  $a_2(1 + 2b_1) - c_1 > 0$ .

Here the rest energy density  $\rho$ , string tension density  $\lambda$  and particle density  $\rho_p$  are found to be decreasing functions of time. These physical parameters tend to zero if

$$2a_2^2 + a_2^2 b_1^2 + a_2 b_1 + a_2 = 2_2 b_1 c_1 + a_2 c_1 + c_1 + 4 + \frac{\omega}{2},$$

and in this case we see that our model degenerates into a five-dimensional vacuum model in the Brans-Dicke theory. On the other hand these density parameters tend to infinity at  $t = \frac{a_0}{a_1}$ . It is seen that there exists a linear relationship between  $\lambda$  and  $\rho_p$ , namely  $\frac{\rho_p}{\lambda} = \omega$ . Here it may be noted that this model becomes a string-dominated universe if  $w \sim 0$ .

The scalar field  $\phi$  is found to be a decreasing function of time  $t$ . At the beginning of the universe the scalar field has a significant role in establishing a string dominated era. At large cosmic time, when the effect of the scalar field is negligible, it is seen that particles dominate over the strings to fill up the volume of the universe.

The model universe in this case represents a shearing, non-rotating, realistic string cosmological model of Bianchi type-I, having inflationary character in the Brans-Dicke theory for five-dimensional space-time. It has a point type singularity at  $t = \frac{a_0}{a_1}$ . For  $b_0 = b_1 = c_0 = 1$  and  $c_1 = a_2$  the spatial sections of the space-time will be four flat spaces. The findings of this model may be of great help in understanding the early phases of the evolution of the universe.

The model in Case-II has no initial singularity. It is expanding at a constant rate. This model is also found to be shearing, but at a rate independent of time. Since here  $\frac{\sigma^2}{\theta^2} \neq 0$ , this universe remains anisotropic throughout the evolution. Again, as the deceleration parameter is found to be negative, our model is an inflationary one. In this model the metric coefficient of the fifth dimension is found to be constant at the initial epoch, then the fifth dimension contracts as time passes and becomes unobservable at infinite time. Here the three space dimensions expand in the normal way for  $\ell_1 > 0, m_1 > 0$ .

At the instant of the beginning of this model universe the scalar field is found to be a constant quantity, it then increases gradually with time until it becomes unity at  $t = \frac{s_0}{s_1}$ , and after that it again gradually increases. In view of the expression of the volume of this model universe, it is seen that it follows the exponential law of inflation for  $\ell_1(m_1 + 1) > 2n_1$ . Here the rest energy density  $\rho$ , as well as the string tension density  $\lambda$  are found to decrease with time until they become zero as  $t \rightarrow \infty$ . Thus this model represents a string cosmological model later dominated by matter particles which can therefore be considered as a realistic universe.

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