

Computation of the structure of a magnetized strange quark star

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Abstract We have calculated some properties of spin polarized strange quark matter (SQM) in a strong magnetic field at zero temperature using the MIT bag model. We showed that the equation of state of spin polarized SQM is stiffer than that for unpolarized cases. We have also computed the structural properties of a spin polarized strange quark star (SQS) and found that the presence of a magnetic field leads to a more stable SQS when compared to the structural properties of an unpolarized SQS.

Key words: dense matter — equation of state — magnetic field

1 INTRODUCTION

Strange quark stars (SQSs) consist mainly of self-bound strange quark matter (SQM). The surface density of an SQS is equal to the density of the SQM at zero pressure ($\sim 10^{15}$ g cm³), which is fourteen orders of magnitude greater than the surface density of a normal neutron star. The central density of these stars is about five times greater than that of their surface density (Haensel et al. 2007; Glendenning 2000; Weber 1999; Camenzind 2007). The existence of SQSs made of SQM was first proposed by Itoh (1970) before Quantum Chromodynamics (QCD) had even been fully developed. Later Bodmer (1971) discussed the fate of an astronomical object collapsing to such a state of matter. In the 1970s, after the formulation of QCD, perturbative calculations of the equation of state of SQM were developed, but the area of validity for these calculations was restricted to very high densities (Collins & Perry 1975). The existence of SQSs was also discussed by Witten (1984), who conjectured that a first order QCD phase transition in the early universe could concentrate most of the quark excess in dense quark nuggets. He suggested that the true state of matter was SQM. Witten proposed that SQM composed of light quarks is more stable than nuclei, therefore SQM can be considered as the ground state of matter. An SQS would be the bulk SQM phase consisting of almost equal numbers of up, down and strange quarks, plus a small number of electrons to ensure charge neutrality. A typical electron fraction is less than 10^{-3} and it decreases from the surface to the center of an SQS (Haensel et al. 2007; Glendenning 2000; Weber 1999; Camenzind 2007). SQM would have a lower charge-to-baryon ratio compared to the nuclear matter and can show itself in the form of an SQS (Witten 1984; Alcock et al. 1986; Haensel et al. 1986; Kettner et al. 1995).

The collapse of a massive star may lead to the formation of an SQS. An SQS may also be formed from a neutron star and is denser than the neutron star (Bhattacharyya et al. 2006). If sufficient additional matter is added to an SQS, it will collapse into a black hole. Neutron stars with masses

of $1.5 - 1.8M_{\odot}$ with rapid spins are theoretically the best candidates for conversion to an SQS. An extrapolation based on this indicates that up to two quark-novae occur in the observable universe each day. In addition, recent Chandra observations indicate that objects RX J185635–3754 and 3C58 may contain SQSs (Prakash et al. 2003).

It is known that compact objects such as neutron stars, pulsars, magnetars and strange quark stars are under the influence of a strong magnetic field, which is typically about $10^{15} - 10^{19}$ G (Kouveliotou et al. 1999, 1998; Haensel et al. 2007; Glendenning 2000; Weber 1999; Camenzind 2007). Therefore, in astrophysics, it is of special interest to study the effect of a strong magnetic field on SQM properties, which can be found in the core of neutron stars and also in SQSs. We note that in the presence of a magnetic field, the conversion of neutron stars to bare quark stars cannot take place unless the value of the magnetic field exceeds 10^{20} G (Ghosh & Chakrabarty 2001).

Recent investigations also show that the object SWIFT J1749.4–2807 may be an SQS (Yu & Xu 2010). We have also computed the structural properties of a neutron star with a quark core at finite temperature (Yazdizadeh & Bordbar 2011).

In this article, we focus on an SQS that is purely composed of spin polarized SQM and investigate the effects of a strong magnetic field on the different properties of such a star. In Section 2, we study spin polarized SQM in the absence and presence of a strong magnetic field. In Section 3, by numerically solving the Tolman-Oppenheimer-Volkoff equation, we obtain the structural properties of a spin polarized SQS. Moreover, we discuss the stability of spin polarized SQSs.

2 ENERGY CALCULATION FOR SPIN POLARIZED SQM

Several authors have investigated the properties of SQS using different methods (Alverdyan 2010; Li et al. 2011). We consider spin polarized SQM composed of u , d and s quarks with spin up (+) and down (–). We denote the number density of quark i with spin up by $\rho_i^{(+)}$ and spin down by $\rho_i^{(-)}$. We introduce the polarization parameter ξ_i by

$$\xi_i = \frac{\rho_i^{(+)} - \rho_i^{(-)}}{\rho_i}, \quad (1)$$

where $0 \leq \xi_i \leq 1$ and $\rho_i = \rho_i^{(+)} + \rho_i^{(-)}$. Under the conditions of beta-equilibrium and charge neutrality, we get the following relation for the number density,

$$\rho = \rho_u = \rho_d = \rho_s, \quad (2)$$

where ρ is the total baryonic density of the system.

Now, we calculate the energy density of spin polarized SQM. To calculate the total energy of spin polarized SQM, we use the MIT bag model, in which the total energy is the sum of the kinetic energy of the quarks plus a bag constant (B_{bag}) (Chodos et al. 1974). The bag constant B_{bag} can be interpreted as the difference between the energy densities of the noninteracting and interacting quarks. Dynamically, it acts as a pressure that keeps the quark gas at a constant density and potential. In MIT bag models, different values are considered for the bag constant such as 55 and $90 \frac{\text{MeV}}{\text{fm}^3}$. We calculate the energy density of SQM in the absence and presence of a magnetic field in the following two subsections.

2.1 Energy Density of Spin Polarized SQM in the Absence of a Magnetic Field

The total energy of spin polarized SQM in the absence of a magnetic field ($B = 0$) is given by

$$\varepsilon_{\text{tot}}^{(B=0)} = \varepsilon_u + \varepsilon_d + \varepsilon_s + B_{\text{bag}}, \quad (3)$$

where ε_i is the kinetic energy per volume of quark i ,

$$\varepsilon_i = \sum_{p=\pm} \sum_{k^{(p)}} \sqrt{m_s^2 c^4 + \hbar^2 k^{(p)2} c^2}. \quad (4)$$

We ignore the masses of quarks u and d , while we assume $m_s = 150 \text{ MeV}$ for quark s . After performing some algebra, supposing that $\xi_s = \xi_u = \xi_d = \xi$, we obtain the following relation for the total energy of spin polarized SQM,

$$\begin{aligned} \varepsilon_{\text{tot}}^{(B=0)} &= \frac{3}{16\pi^2 \hbar^3} \sum_{p=\pm} \left[\frac{\hbar}{c^2} k_F^{(p)} E_F^{(p)} \left(2\hbar^2 k_F^{(p)2} c^2 + m_s^2 c^4 \right) - m_s^4 c^5 \ln \left(\frac{\hbar k_F^{(p)} + E_F^{(p)}/c}{m_s c} \right) \right] \\ &+ \frac{3 \hbar c \pi^{2/3}}{4} \rho^{4/3} \left[(1 + \xi)^{4/3} + (1 - \xi)^{4/3} \right] + B_{\text{bag}}, \end{aligned} \quad (5)$$

where

$$k_F^{\pm} = (\pi^2 \rho)^{1/3} (1 \pm \xi)^{1/3}, \quad (6)$$

and

$$E_F^{\pm} = \left(\hbar^2 k_F^{(\pm)2} c^2 + m_s^2 c^4 \right)^{1/2}. \quad (7)$$

In Figure 1, we have plotted the total energy density of spin polarized SQM as a function of the density for different values of the polarization (ξ) in the absence of a magnetic field. Figure 1 shows that the energy is an increasing function of the density, however the rate of increase of energy versus density increases with increasing polarization. For each density, we see that the energy of spin polarized SQM increases with increasing polarization, especially at high densities.

For spin polarized SQM, we can also calculate the equation of state (EoS) using the following relation,

$$P(\rho) = \rho \frac{\partial \varepsilon_{\text{tot}}}{\partial \rho} - \varepsilon_{\text{tot}}, \quad (8)$$

where P is the pressure and ε_{tot} is the energy density which, in the absence of a magnetic field, is obtained from Equation (5).

In Figure 2, we have shown the pressure of spin polarized SQM as a function of the density for various values of the polarization parameter in the absence of a magnetic field. We see that for a given density, the pressure increases with increasing polarization. This shows that the EoS of spin polarized SQM is stiffer than that of the unpolarized case. From Figure 2, it can be seen that by increasing polarization, the density corresponding to zero pressure takes lower values.

2.2 Energy Density of Spin Polarized SQM in the Presence of a Magnetic Field

In this section, we consider spin polarized SQM which is under the influence of a strong magnetic field (\mathbf{B}). For this system, the contribution of magnetic energy is $E_M = -\mathbf{M} \cdot \mathbf{B}$. If we assume the magnetic field is along the z direction, the contribution of the magnetic energy of the spin polarized SQM is given by

$$E_M = - \sum_{i=u,d,s} M_z^{(i)} B, \quad (9)$$

where $M_z^{(i)}$ is the magnetization of the system corresponding to particle i which is given by

$$M_z^{(i)} = N_i \mu_i \xi_i. \quad (10)$$

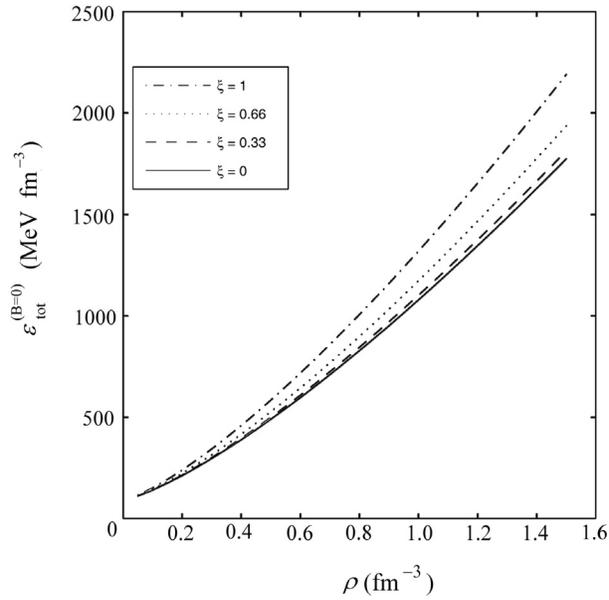


Fig. 1 Total energy density of spin polarized SQM as a function of density (ρ) at different values of the polarization parameter (ξ) in the absence of a magnetic field.

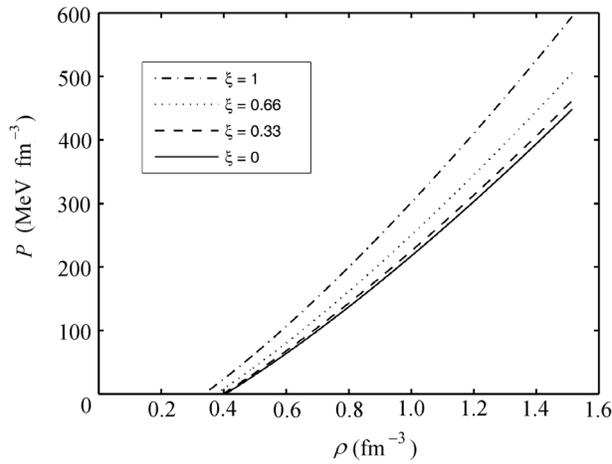


Fig. 2 Same as Fig. 1, but for the equation of state of spin polarized SQM.

In the above equation, N_i and μ_i are the number and magnetic moment of particle i , respectively. By some simplification, the contribution of the magnetic energy density of spin polarized SQM, $\epsilon_M = \frac{E_M}{V}$, can be obtained as follows

$$\epsilon_M = - \sum_{i=u,d,s} \rho_i \mu_i \xi_i B. \quad (11)$$

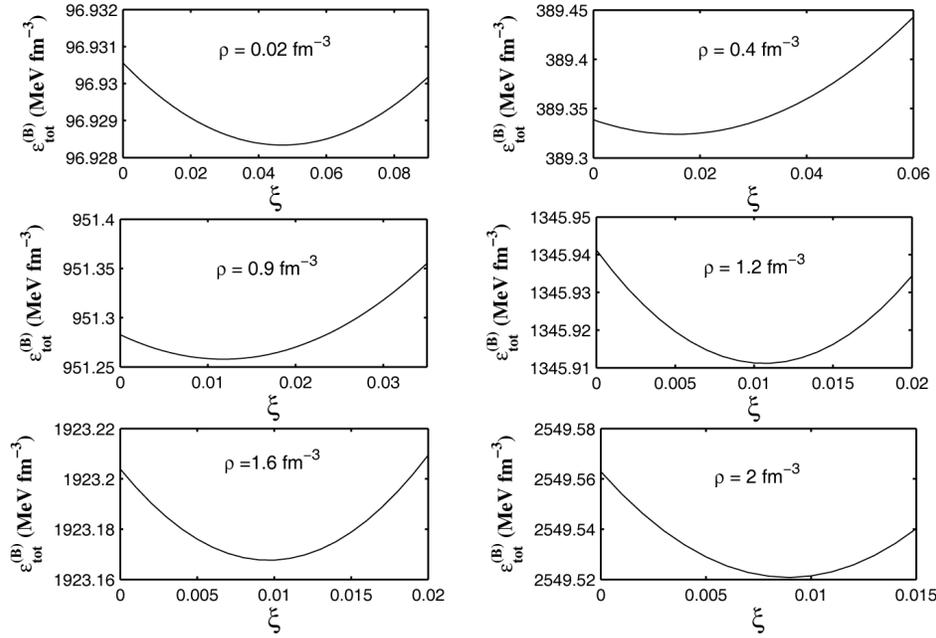


Fig. 3 Total energy density of polarized SQM as a function of the polarization parameter (ξ) for $B = 5 \times 10^{18}$ G at different densities (ρ).

Consequently, the total energy density of spin polarized SQM in the presence of a magnetic field can be written as

$$\varepsilon_{\text{tot}}^{(B)} = \varepsilon_{\text{tot}}^{(B=0)} + \varepsilon_M. \quad (12)$$

In Figure 3, we have shown the total energy density of spin polarized SQM as a function of the polarization parameter (ξ), for $B = 5 \times 10^{18}$ G at various densities. From Figure 3, we have seen that the energy curve shows a minimum for each relevant density. This behavior indicates that for each density there is a metastable state. We have also seen that as the density increases, this metastable state is shifted to lower values of the polarization parameter. Therefore, we can conclude that the metastable state disappears at high densities. We have also found that at high densities, the system becomes nearly identical to the unpolarized system. These results agree with those of reference (Pal et al. 2009).

In Figure 4, we have plotted the total energy density of spin polarized SQM versus the number density in the presence of a magnetic field. We have seen that the total energy increases by increasing the density. We have found that the energy density of spin polarized SQM in the presence of a magnetic field is nearly identical to that of the unpolarized case, which has been clarified in panel (b) of Figure 4. As we will see in the next paragraph, this is due to the fact that the polarization parameter in the presence of a magnetic field is very small, especially at high densities.

In Figure 5, we have presented the polarization parameter corresponding to the minimum point of energy density as a function of the number density at $B = 5 \times 10^{18}$ G. We see that the polarization parameter decreases by increasing the number density. From Figure 5, it can be seen that for $\rho < 0.2 \text{ fm}^{-3}$, the rate of decrease of polarization versus density is substantially higher than that for $\rho > 0.2 \text{ fm}^{-3}$.

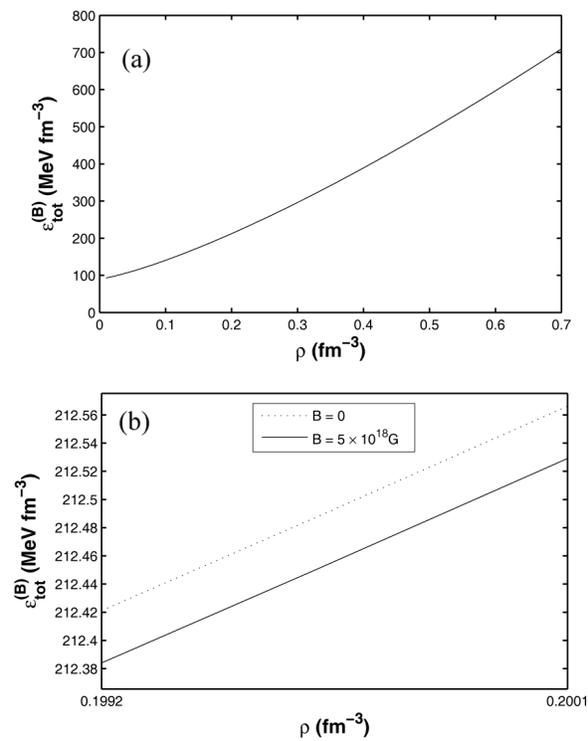


Fig. 4 (a) Total energy density of spin polarized SQM vs. density (ρ) at $B = 5 \times 10^{18}$ G. (b) Comparison between the total energy for two cases of $B = 5 \times 10^{18}$ G and $B = 0$.

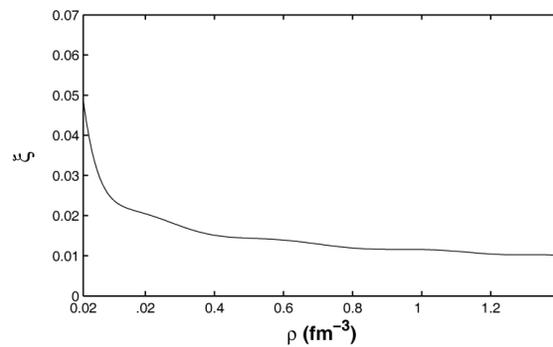


Fig. 5 Polarization parameter (ξ) corresponding to the minimum points of energy density vs. density (ρ) at $B = 5 \times 10^{18}$ G.

In Figure 6, we have shown the polarization parameter versus the magnetic field for different values of the number density. For each density, we can see that polarization increases by increasing the magnetic field. This figure also shows that the rate of increase of polarization versus magnetic field increases with increasing density.

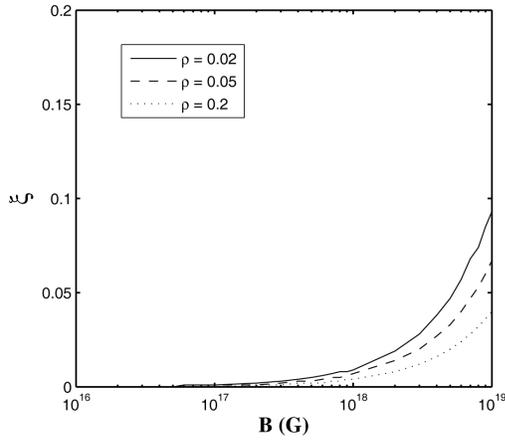


Fig. 6 Polarization parameter (ξ) corresponding to the minimum points of energy density vs. magnetic field (B) for different values of density (ρ).

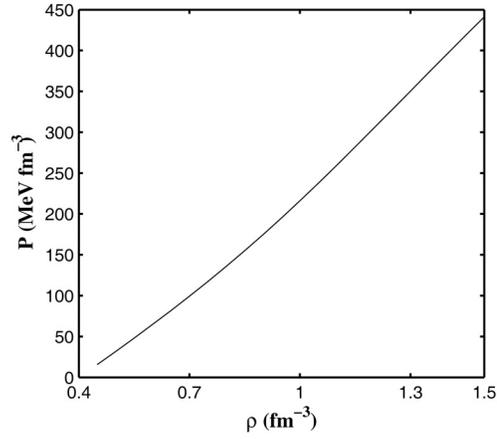


Fig. 7 Pressure (P) vs. density (ρ) for spin polarized SQM at $B = 5 \times 10^{18}$ G.

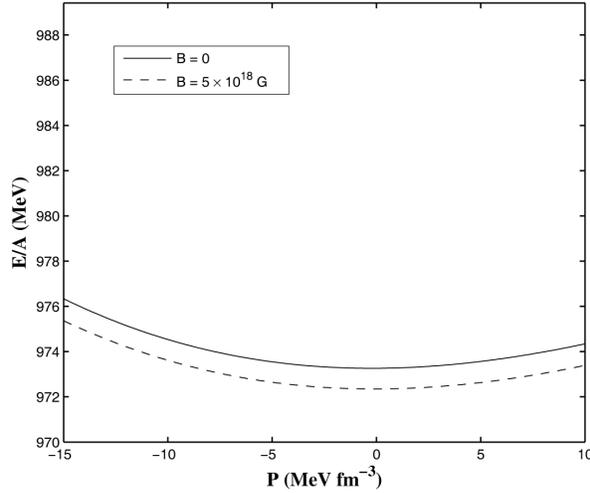


Fig. 8 Energy per baryon vs. pressure (P) for spin polarized SQM at $B = 0$ (solid line) and $B = 5 \times 10^{18}$ G (dashed line).

We have also calculated the EoS of spin polarized SQM in the presence of a magnetic field, where the contribution of magnetic pressure ($\frac{B^2}{8\pi}$) should be added to Equation (8) in which the total energy density is obtained from Equation (12).

In Figure 7, we have plotted the EoS of spin polarized SQM where the magnetic field is switched on. We found that this EoS is nearly identical to that of the unpolarized case. This is due to the fact that polarization at the energy minimum is very low, especially at high densities.

In Figure 8, we have plotted the energy per baryon (E/A) for spin polarized SQM as a function of pressure at $B = 5 \times 10^{18}$ G. Our results for the case of SQM in the absence of a magnetic field ($B = 0$) are also given for comparison. We have seen that the zero point of pressure in the presence of a magnetic field has a lower E/A compared to the case of SQM in the absence of a magnetic field ($B = 0$). This indicates that in the presence of a magnetic field, spin polarized SQM is more stable than that in the absence of a magnetic field.

3 STRUCTURE OF A SPIN POLARIZED SQS

The gravitational mass (M) and radius (R) of compact stars are of special interest in astrophysics. In this section, we calculate the structural properties of a spin polarized SQS and compare the results of this calculation with those of the unpolarized case. Using the EoS of spin polarized SQM, we can obtain M and R by numerically integrating the general relativistic equations of hydrostatic equilibrium, the Tolman-Oppenheimer-Volkoff (TOV) equations, which are as follows (Shapiro & Teukolsky 1983),

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \varepsilon(r), \\ \frac{dP}{dr} &= -\frac{Gm(r)\varepsilon(r)}{r^2} \left(1 + \frac{P(r)}{\varepsilon(r)c^2}\right) \left(1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right) \left(1 - \frac{2Gm(r)}{c^2 r}\right)^{-1}, \end{aligned} \quad (13)$$

where $\varepsilon(r)$ is the energy density, G is the gravitational constant, and

$$m(r) = \int_0^r 4\pi r'^2 \varepsilon(r') dr' \quad (14)$$

has the interpretation of the mass inside radius r . By selecting a central energy density ε_c , under the boundary conditions $P(0) = P_c$ and $m(0) = 0$, we integrate the TOV equation outwards to a radius $r = R$, at which P vanishes. This yields the radius R and mass $M = m(R)$ (Shapiro & Teukolsky 1983).

Our results for the structure of a spin polarized SQS in the absence and presence of a magnetic field are given in the two following subsections.

3.1 Structure of a Spin Polarized SQS in the Absence of a Magnetic Field

In Figures 9 and 10, we have plotted the gravitational mass and radius of a spin polarized SQS in the absence of a magnetic field vs. the central energy density (ε_c) for different values of the polarization parameter (ξ). From these figures, we see that for each central density, the mass and radius of an SQS decrease by increasing the polarization parameter. This is due to the fact that by increasing the polarization parameter, the pressure of spin polarized SQM increases, which leads to a stiffer equation of state for this system (Fig. 2). Figures 9 and 10 show that for a given polarization parameter, the gravitational mass and radius of an SQS increase by increasing the central density. From Figure 9, it can be seen that the gravitational mass of an SQS reaches a limiting value called the maximum mass.

In Figure 11, we have plotted our results for the gravitational mass of a spin polarized SQS as a function of radius (mass-radius relation) in the absence of a magnetic field. For this system, we see that the gravitational mass increases by increasing the radius. It is seen that the rate of increase of mass versus radius increases with increasing the polarization.

In Table 1, the maximum mass (M_{\max}) and the corresponding radius (R) of a spin polarized SQS are given for different values of the polarization parameter (ξ) in the absence of a magnetic field. We can see that both maximum mass and the corresponding radius decrease with increasing ξ . This shows that increasing polarization leads to a more stable SQS.

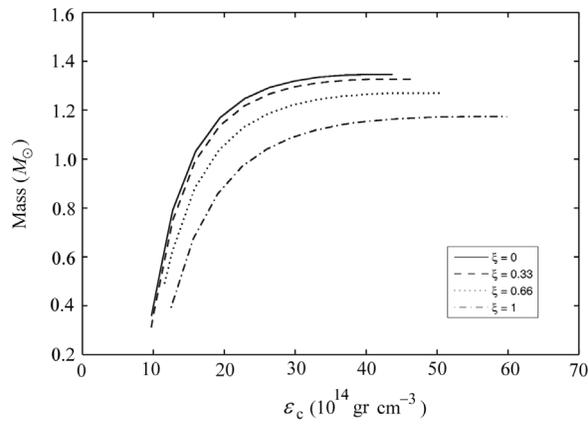


Fig. 9 Gravitational mass of a spin polarized SQS vs. central density (ϵ_c) for different values of the polarization parameter (ξ) in the absence of a magnetic field.

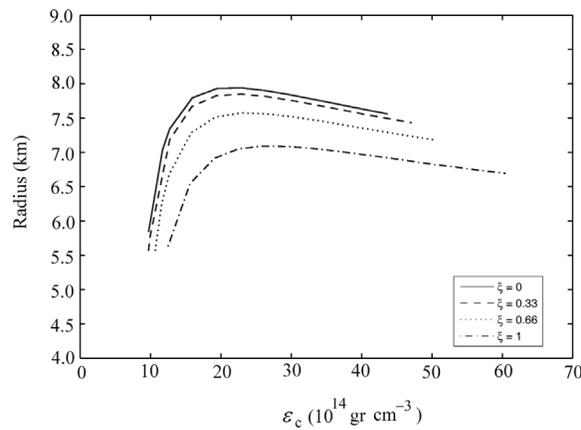


Fig. 10 Same as Figure 9, but for radius of a spin polarized SQS.

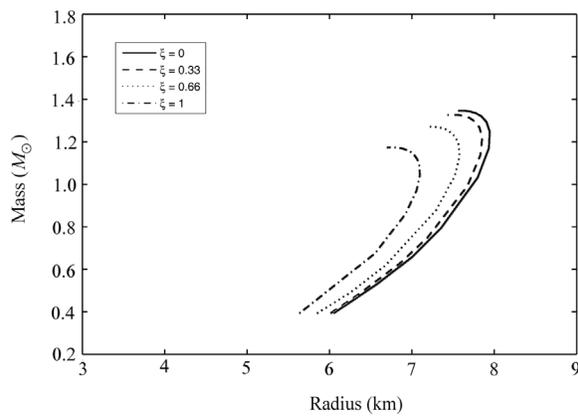


Fig. 11 Mass-radius relation for a spin polarized SQS in the absence of a magnetic field at different values of the polarization parameter (ξ).

Table 1 Maximum gravitational mass (M_{\max}) and the corresponding radius (R) of a spin polarized SQS for different values of the polarization parameter.

Star	M_{\max} (M_{\odot})	R (km)
Unpolarized SQS ($\xi = 0$)	1.35	7.6
Polarized SQS ($\xi = 0.33$)	1.33	7.5
Polarized SQS ($\xi = 0.66$)	1.27	7.2
Polarized SQS ($\xi = 1$)	1.17	6.7

Table 2 Maximum gravitational mass (M_{\max}) and the corresponding radius (R) of an SQS for $B = 0$ and 5×10^{18} G.

Star	M_{\max} (M_{\odot})	R (km)
Unpolarized SQS ($B = 0$)	1.35	7.6
Polarized SQS ($B = 5 \times 10^{18}$ G)	1.31	7.4

3.2 Structure of a Spin Polarized SQS in the Presence of a Magnetic Field

In this section, we present our calculations for the structure of an SQS in the presence of a magnetic field. It should be noted that a strong magnetic field changes the spherical symmetry of the system. However, for magnetic fields less than 10^{19} G, this effect is negligible (Felipe & Martínez 2009; Perez Martinez et al. 2010), therefore, we can solve the TOV equations using a spherical metric, which leads to Equation (13). Our results for the gravitational mass and radius of a spin polarized SQS in the presence of magnetic field versus the central energy density (ε_c) are shown in Figures 12 and 13, respectively. In these figures, our results for an unpolarized SQS ($B = 0$) are also given for comparison.

Figures 12 and 13 show that for all values of central density, mass and radius of an SQS decrease when the magnetic field is switched on. From Figure 12, we see that as the central density increases, the gravitational mass of an SQS increases and finally reaches a limiting value (maximum mass). In Table 2, we have given the maximum mass and the corresponding radius of an SQS for two cases: $B = 0$ (unpolarized SQS) and $B = 5 \times 10^{18}$ G. It is shown that the presence of a magnetic field

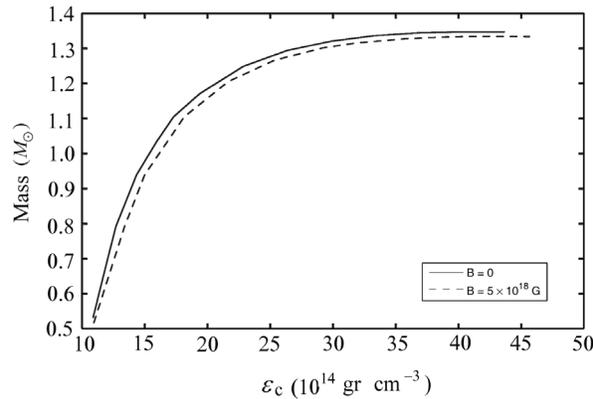


Fig. 12 Gravitational mass vs. central density (ε_c) for a spin polarized SQS at $B = 0$ and $B = 5 \times 10^{18}$ G.

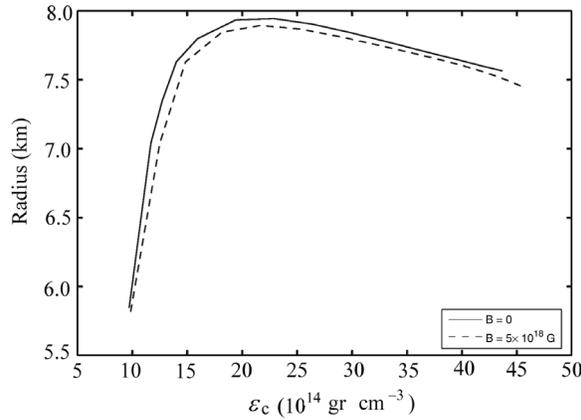


Fig. 13 Same as Figure 12, but for radius of a spin polarized SQS.

leads to lower values for both the maximum mass and the corresponding radius of an SQS showing a more stable SQS compared to an unpolarized SQS.

4 SUMMARY AND CONCLUSIONS

We have studied spin polarized SQM both for cases in the absence and presence of a magnetic field. We calculated some of the bulk properties of this system such as energy, equation of state (EoS) and polarization. We have shown that the energy of spin polarized SQM in the absence of a magnetic field increases with increasing polarization. Calculations of the energy in the presence of a magnetic field demonstrated that for each density, there is a minimum point for the energy of SQM showing a metastable state. We have seen that the EoS of spin polarized SQM becomes stiffer as the polarization increases. We have also seen that spin polarized SQM in the presence of a magnetic field is more stable than unpolarized SQM. The structural properties of a spin polarized SQS were also calculated in the absence and presence of a magnetic field. We have seen that for each central density, the mass and radius of a spin polarized SQS decrease with increasing polarization. We have also seen that both maximum mass and the corresponding radius of this system decrease by increasing polarization.

Our calculations indicated that in the presence of a magnetic field, the maximum mass and the corresponding radius of a polarized SQS acquire lower values than those of an unpolarized SQS. Therefore, we can conclude that the presence of a magnetic field leads to a more stable SQS compared to an unpolarized SQS.

Our results for the maximum mass and radius of an SQS (Tables 1 and 2) are consistent with those observed for the object SAX J1808.4–3658 (Li et al. 1999). We can conclude that this object is a good candidate for an SQS.

One of the other astrophysical implications of our results is the calculation of the surface redshift (z_s) of an SQS. This parameter is of special interest in astrophysics and can be obtained from the mass and radius of the star using the following relation (Camenzind 2007),

$$z_s = \left(1 - \frac{2GM}{Rc^2}\right)^{-\frac{1}{2}} - 1. \quad (15)$$

Our results corresponding to the maximum mass and radius of an SQS lead to $z_s = 0.45 \text{ m s}^{-1}$ in the absence of a magnetic field and $z_s = 0.44 \text{ m s}^{-1}$ for a magnetic field $B = 5 \times 10^{18} \text{ G}$. This indicates that the presence of a magnetic field approximately leads to lower values for the surface redshift.

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