

# Transient accelerating scalar models with exponential potentials \*

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Received 2012 October 12; accepted 2012 December 12

**Abstract** We study a known class of scalar dark energy models in which the potential has an exponential term and the current accelerating era is transient. We find that, although a decelerating era will return in the future, when extrapolating the model back to earlier stages ( $z \gtrsim 4$ ), scalar dark energy becomes dominant over matter. So these models do not have the desired tracking behavior, and the predicted transient period of acceleration cannot be adopted into the standard scenario of the Big Bang cosmology. When couplings between the scalar field and matter are introduced, the models still have the same problem; only the time when deceleration returns will be varied. To achieve re-deceleration, one has to turn to alternative models that are consistent with the standard Big Bang scenario.

**Key words:** cosmology: cosmological parameters — cosmology: theory

## 1 INTRODUCTION

Cosmological observations have indicated that the universe is now in an accelerating expansion (Riess et al. 1998; Amanullah et al. 2010; Hanany et al. 2000; Colless et al. 2001; Boughn & Crittenden 2004; Contaldi et al. 2003). Within the framework of general relativity, the cause of acceleration can be attributed to the existence of some dark energy, which makes up  $\sim 70\%$  of the total cosmic energy in the universe. There are a number of possible candidates for dark energy driving the accelerating expansion. The simplest one is the cosmological constant  $\Lambda$ , which, however, has difficulties with fine-tuning and the coincidence problem (Carroll 2001). To overcome these difficulties within the framework of the theory of general relativity, dynamic dark energy models have been proposed. Among them, one type is based on some scalar fields, such as quintessence (Ratra & Peebles 1988), phantom (Caldwell 2002), k-essence (Armendáriz-Picón et al. 1999), tachyon field (Padmanabhan 2002), quintom (Hu 2005) and Chaplygin gas (Kamenshchik et al. 2001), while other types are dynamic vector models based upon the Yang-Mills fields (Zhang 2002; Xia & Zhang 2007, Bamba et al. 2008).

So far, there has been no observational evidence indicating whether the current cosmic acceleration is eternal or transient. Recently, the analysis of combined data of SNe Ia+BAO+CMB made by Shafieloo (2009) seems to indicate that the acceleration is slowing down. Most dark energy models

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\* Supported by the National Natural Science Foundation of China.

predict a scenario where the acceleration will be eternal. As is known, in such an eternally accelerating universe, there is an event horizon, and causality exists in a limited spacetime. From the perspective of string theory, an asymptotically large space at infinity is required for the existence of a conventional S-matrix, whose elements are connected to physical observables. Therefore, the S-matrix is ill-defined in an eternally accelerating universe (Fischler et al. 2001). If one wants to save string theory as a theory of physics, one has to either look for alternatives to the conventional S-matrix, or to construct cosmological models, in which the current acceleration is transient and the decelerating expansion will eventually return.

Several dynamic dark energy models have been proposed to achieve the possibility of cosmic expansion that is currently accelerating, then will be decelerating in the future (Townsend & Wohlfarth 2003; Sahni & Shtanov 2003; Russo 2004; Carvalho et al. 2006; Alcaniz 2010). One model was explored in the context of  $(4+n)$ -dimensional gravity compactified on an  $n$ -dimensional time-varying compact manifold (Townsend & Wohlfarth 2003). A class of braneworld models was also shown to admit a transient acceleration in certain regions of model parameters (Sahni & Shtanov 2003). The transient acceleration was also examined in a scalar cosmological model in  $d$ -dimensions with exponential potentials, and a general solution was obtained by a flat Robertson-Walker metric (Russo 2004). An extension was made to the case with a generalized exponential potential of the scalar field (Carvalho et al. 2006; Alcaniz 2010). Scalar fields with exponential potentials can occur generically in certain theories of particle physics, such as the compactified Kaluza-Klein theory with extra dimensions (Wetterich 1985), supersymmetry theories (Cremmer et al. 1983), and higher order gravity (Barrow & Cotsakis 1988), and have been extensively studied in a more general context related to for dark energy or inflation (Ferreira & Joyce 1997; Ratra & Peebles 1988).

It is interesting to explore the possibility of transient acceleration in these general cosmological models. In particular, we want to check if these models can be extended to earlier epochs and are consistent with the standard Big Bang scenario as a realistic dark energy model should be. So far in these scalar models of transient acceleration, either the cosmic matter component was not taken into consideration, or dark energy was assumed to be independent of matter. In the time-dependent  $\Lambda(t)$  model (Costa & Alcaniz 2010) and the fluid dark energy model (Fabris et al. 2010), interactions are introduced between dark energy and matter. Although so far there has been no observational indication of coupling between dark energy and matter, dark energy as a dynamic scalar field could have interactions with other components, such as matter. We will also study the models with coupling between scalar dark energy and matter, to examine the impacts of the coupling upon the dynamic evolution behavior.

## 2 THE COUPLING MODEL

We consider a spatially flat ( $k = 0$ ) Robertson-Walker spacetime with a metric  $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$ . The Lagrangian of the scalar field that drives the acceleration is given by  $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$ . The energy density and pressure are respectively  $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$  and  $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ , and the equation of state  $w = p_\phi/\rho_\phi$ . The dynamic expansion of the universe is determined by the Friedmann equations:

$$H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_m + \rho_r), \quad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_\phi + 3p_\phi + \rho_m + \rho_r + 3p_r), \quad (2)$$

where  $H = \dot{a}/a$  is the expansion rate and its present value  $H_0$  is the Hubble constant;  $\rho_m$  and  $\rho_r$  are the energy density of matter and radiation, respectively. The equations describing the evolution of the three components are given by

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = Q, \quad (3)$$

$$\dot{\rho}_m + 3H\rho_m = -Q, \tag{4}$$

$$\dot{\rho}_r + 4H\rho_r = 0, \tag{5}$$

where the coupling  $Q$  is a generic coupling term and represents the rate of energy exchange between dark energy and matter. The non-coupling model is  $Q = 0$ . When  $Q > 0$ , matter transforms energy into dark energy. When  $Q < 0$ , dark energy transforms energy into matter. We do not include the coupling between dark energy and radiation, which will not affect the following conclusion. Equation (2) can be derived from the set of Equations (1), (3), (4) and (5). It has been shown that, in the absence of the matter component in the non-coupling model (Russo 2004; Carvalho et al. 2006), for the current accelerating stage of expansion to be transient, i.e. for the expansion to again decelerate, the scalar field potential  $V(\phi)$  can take the following form

$$V(\phi) = \rho_{\phi 0} \left[ 1 - \frac{\lambda}{6} (1 + \alpha\sqrt{\sigma}\phi)^2 \right] \exp \left[ -\lambda\sqrt{\sigma} \left( \phi + \frac{\alpha\sqrt{\sigma}}{2} \phi^2 \right) \right], \tag{6}$$

where  $\rho_{\phi 0}$  is a constant energy density,  $\sigma = 8\pi G/\lambda$ , and  $\alpha$  and  $\lambda$  are two dimensionless, positive parameters of the model. In the context of this paper, they can take values around  $\alpha, \lambda \sim 1$ . In the limit  $\alpha \rightarrow 0$  the potential in Equation (6) reduces to an exponential potential,  $V(\phi) = V_0 \exp[-\sqrt{8\pi G\lambda}\phi]$ , a case that was examined in Russo (2004).

For convenience of computation, it is simpler to rewrite the set of equations, Equations (1), (3) and (4), in the following form,

$$h^2 = \frac{U(y) + x}{1 - \frac{1}{2} \left( \frac{dy}{dN} \right)^2}, \tag{7}$$

$$\frac{d^2y}{dN^2} - \frac{3}{2} \left( \frac{dy}{dN} \right)^3 + 3 \frac{dy}{dN} = \left( \frac{\Gamma}{\frac{dy}{dN}} + 1.5 \frac{dy}{dN} x - U'(y) \right) h^{-2}, \tag{8}$$

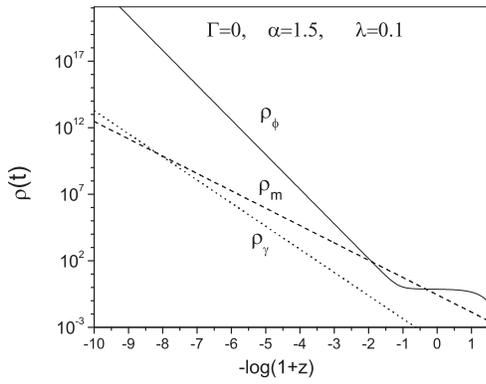
$$\frac{dx}{dN} = -\Gamma - 3x, \tag{9}$$

where  $h \equiv H/H_0$ ,  $N \equiv \ln a(t) = -\ln(1+z)$ ,  $y \equiv \sqrt{\frac{8\pi G}{3}}\phi$ ,  $x \equiv \rho_m/\rho_c$ ,  $\Gamma \equiv Q/H\rho_c$  and  $U(y) \equiv V(\phi)/\rho_c$ . All of these quantities are dimensionless. Specifically, we take the model with  $\Omega_\phi = 0.73$  and  $\Omega_m = 0.27$  at  $z = 0$ . The corresponding initial condition at  $z = 0$  is  $x_i = 0.27$ , and  $\phi_i = 0$  and  $\dot{\phi}_i = \sqrt{\lambda\rho_{\phi 0}/3}$ . The dynamical equations are then solved in the presence of the matter component and radiation. The solution yields a transient acceleration at about  $z = 0$  followed by a deceleration for the following three typical cases. Note that the parameter range of  $\lambda$  and  $\alpha$  for a transient acceleration in our model differs from those in Russo (2004), which assumed the absence of matter.

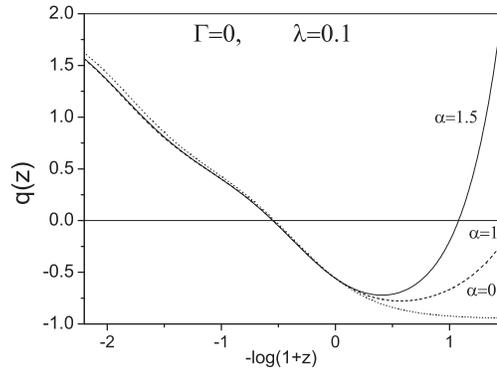
For the non-coupling case  $\Gamma = 0$ , Figure 1 shows the evolution of energy densities,  $\rho_\phi(t)$ ,  $\rho_m(t)$  and  $\rho_r(t)$ , and Figure 2 shows the evolution of the deceleration parameter  $q(z) = -\ddot{a}/a\dot{a}^2$  for various values of the parameter  $\alpha$ ; larger values of  $\alpha > 0$  yield a shorter period of transient acceleration and an earlier return of deceleration. The special case with  $\alpha = 0$  corresponds to the exponential potential (Russo 2004), in which the current acceleration is eternal and deceleration will not return.

Figure 3 shows  $\rho(t)$ , and Figure 4 shows  $q(z)$  for the coupling model where the scalar field transforms energy into matter, in which the rate is taken to be proportional to the matter density,  $Q \propto -H\rho_m$ , i.e.  $\Gamma \propto -x$ . The dependence upon  $\Gamma$  is demonstrated in Figure 4, and larger values of  $\Gamma$  yield an earlier return of deceleration.

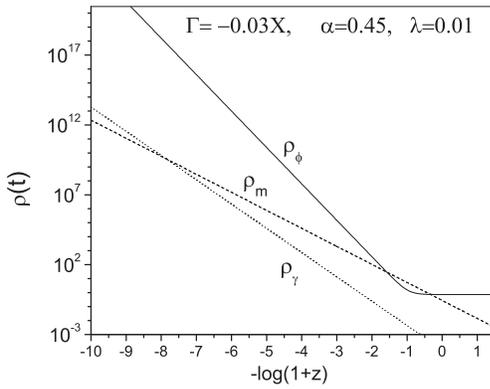
Figures 5 and 6 show the results for the coupling model in which matter transfers energy into the scalar field with  $\Gamma \propto x$ . The dependence upon the parameter  $\lambda$  is demonstrated in Figure 6, and larger values of  $\lambda$  yield a shorter period of transient acceleration.



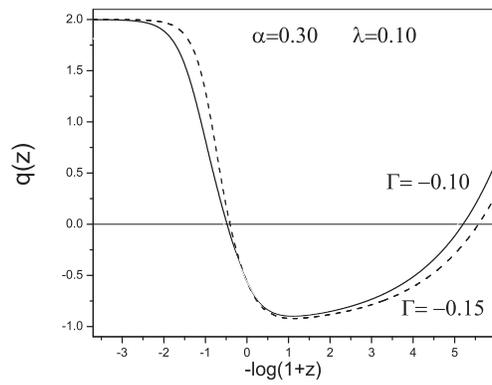
**Fig. 1**  $\rho(t)$  in the non-coupling model with  $\Gamma = 0$ . Notice that  $\rho_\phi$  dominates  $\rho_m$  and  $\rho_r$  for  $z \gtrsim 5$ , which is not compatible with Big Bang cosmology.



**Fig. 2** A transient acceleration is shown by the deceleration parameter  $q(z)$ , which is negative within  $a \sim (0.6, 2.5)$  for  $\alpha = 0.15$ . A greater  $\alpha$  yields a shorter duration of acceleration and an earlier return of deceleration.

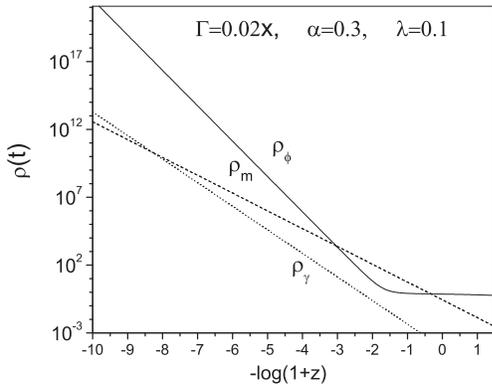


**Fig. 3** The model  $\Gamma < 0$ .  $\rho_\phi$  dominates  $\rho_m$  and  $\rho_r$  for  $z \gtrsim 4$ .

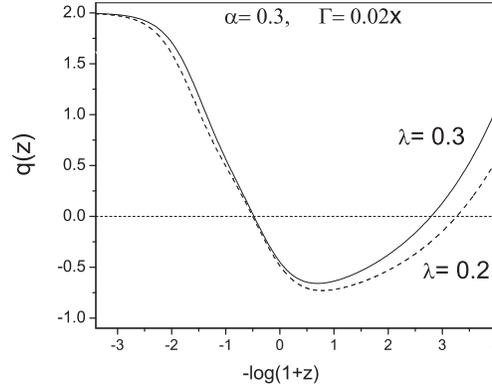


**Fig. 4**  $q(z)$  for various values of  $\Gamma < 0$ .  $q(z)$  is negative within  $a \sim (0.5, 5)$ .

In these plots we extrapolate the model to an earlier era. Figures 1, 3 and 5 reveal that, when the models are extrapolated back to the early stages,  $\rho_\phi(t)$  will be dominant over  $\rho_m(t)$  and  $\rho_r(t)$ , i.e. the  $\phi$  field is dominant over the matter, and the  $\phi$  field has a property that  $w = 1$  with  $\rho_\phi(t) = p_\phi(t) \propto a^{-6}(t)$ , a feature that has been discussed by Ferreira & Joyce (1997). The model predicts a scalar field dominated era for a range of redshift  $z > z_a$ , where  $z_a$  is some value  $\in (4 \sim 20)$ , depending upon the sign and value of  $\Gamma$  and upon the model parameters. This kind of dynamic behavior for the early era deviates drastically from the scenario described by standard cosmology. Therefore, the class of models with exponential potentials for a transient acceleration can be pertinent only for the rather recent era during the matter dominated stage. They cannot account for the early expansion of the universe with  $z > z_a$ . The standard Big Bang cosmology has a scenario where the matter component should be dominant in the past up to the radiation-matter equality at a redshift  $z \sim 3450$  (Spergel et al. 2003), and it also shows the acceleration era starts around  $z \sim 0.5$ . The whole class of models, either  $\Gamma = 0$ ,  $\Gamma > 0$  or  $\Gamma < 0$ , has this difficulty. To be concordant with the Big Bang



**Fig. 5** The model  $\Gamma > 0$ .  $\rho_\phi(t)$  dominates  $\rho_m$  and  $\rho_r$  for  $z \gtrsim 20$ .



**Fig. 6** The  $\Gamma > 0$  model for various  $\lambda$ .  $q(z)$  is negative within  $a \sim (0.5, 2.7)$  for  $\lambda = 0.4$ . A greater  $\lambda$  yields a shorter duration of acceleration.

scenario in this class of models, one would have to choose a smaller initial value of  $\rho_\phi$ , so that  $\rho_\phi(t)$  is sub-dominant to  $\rho_m(t)$  during the radiation or matter dominated era. Just as investigated by Sen & Sethi (2002) and Barreiro et al. (2000), by constructing the so-called quintessence  $\phi$  with a double exponential potential  $V(\phi)$ , one can achieve such a scaling solution followed by an exit into the accelerating expansion. However, as we have just shown, the dynamic evolution of this double exponential quintessence does not automatically ensure a proper future deceleration. If one still wants to achieve a transient acceleration, one has to use the double exponential quintessence for the scaling stage and the exit stage, and probably employ another new field for the return of deceleration. Still, one would have to give a physical motivation for such an artificial connection. It is still premature to compare the above models with the observed data associated with cosmology, such as SNe Ia (Amanullah et al. 2010), CMB (Komatsu et al. 2011) and BAO (Percival et al. 2010). We mention that the fluid dark energy model with a transient acceleration (Fabris et al. 2010) would also face the same problem as the above.

### 3 CONCLUSIONS

We have explored the possibility of transient acceleration in a class of scalar dark energy models with exponential potentials in the presence of the matter and radiation components. Through detailed examinations of the three cases, the non-coupling with  $\Gamma = 0$ , the coupling with  $\Gamma < 0$  and the coupling with  $\Gamma > 0$ , we find that this class of models with exponential potentials can provide a transient period of acceleration. However, extrapolating back to the earlier era of redshifts  $z \gtrsim 5$  ( $\Gamma = 0$ ),  $z \gtrsim 4$  ( $\Gamma < 0$ ) or  $z \gtrsim 20$  ( $\Gamma > 0$ ), respectively, the scalar energy  $\rho_\phi(t)$  will be dominant over matter and radiation, and the scaling of behavior is lost too soon. This would be inconsistent with standard Big Bang cosmology, in which the matter-dominated era extends from  $z \sim 3450$  to  $z \sim 0.5$ . The coupling between scalar dark energy and matter brings only minor modifications to the dynamic expanding behavior. A greater value of  $\Gamma$  tends to yield a shorter period of acceleration and an earlier return of deceleration.

Therefore, as they presently stand, this class of models with exponential potentials can only be used at most from the more recent past around low redshifts. In order to have a viable model of transient acceleration, one has to either use the exponential scalar potential to construct more sophisticated models with a proper dynamic behavior at high- $z$ , or seek other models.

**Acknowledgements** W. P. Cui's work has been part of the Undergraduate Research Project of USTC. Y. Zhang is supported by the National Natural Science Foundation of China (Grant No. 10773009), SRFDP and CAS.

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