



Investigating the Impact of Interstellar Medium Density Discontinuity on SNR-PWN Composite Systems Through 2D RMHD Simulations

Qi Xia, Hong-Yu Wang, Yu-Jun Xian, and Jun Fang

Department of Astronomy, Yunnan University, Kunming 650091, China; fangjun@ynu.edu.cn

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Abstract

This study employs two-dimensional axisymmetric relativistic magnetohydrodynamic simulations to investigate the evolution of supernova remnant (SNR) and pulsar wind nebula (PWN) composite systems in two distinct interstellar medium (ISM) configurations: a uniform density distribution and a medium with a sharp density discontinuity. Compared to the uniform density distribution, the ISM with this density discontinuity better reflects the actual conditions and explains the overall morphological characteristics of specific types of SNR-PWN composite systems. These systems exhibit asymmetries, such as an SNR shell with differing radii or an inner PWN located nearer to the shell on one side. The simulation results suggest that the density discontinuity in the ISM is a contributing factor to both the shell asymmetry and the PWN displacement. Specifically, this density variation directly causes the inconsistency in the forward shock speeds of the SNR between high and low density regions, resulting in discrepancies in the shell layer radii. Furthermore, the asymmetric morphology of the PWN and its positional offset emerge through interactions with the reverse shock. The PWN tends to shift toward the SNR shell on one side. The greater the density jump in the background field, the more pronounced the shell radius differences and PWN offset become.

Key words: magnetohydrodynamics (MHD) – relativistic processes – shock waves – methods: numerical – ISM: supernova remnants

1. Introduction

The interstellar medium (ISM) refers to the material between stars, encompassing interstellar gas, dust, magnetic fields, and cosmic rays, collectively forming the dynamic environment of interstellar space. In the late stage of stellar evolution, stars undergo supernova explosions, releasing enormous amounts of energy and ejecting the progenitor star's material into the ISM at extremely high velocities. The high-energy ejecta interact with the surrounding ISM, generating shocks that propagate outward. As the shocks sweep through the ISM, the medium is compressed and heated, eventually forming an extended celestial object known as a supernova remnant (SNR), which consists of both the ejecta and shock-swept material.

Following a supernova explosion, the rapid rotation of the associated pulsar drives electron–positron pairs in their magnetosphere to flow outward and accelerate to ultra-relativistic speeds, forming a powerful pulsar wind (a high-energy particle flow). The interaction between this wind, the pulsar's magnetic field, and the surrounding ISM creates a pulsar wind nebula (PWN). PWNe can produce radiation spanning from radio wavelengths to hundreds of TeV, and in some cases reaching PeV energies (Zhu et al. 2018; Wu et al. 2023; Xia et al. 2023; Cao et al. 2024; Zhou et al. 2024).

Within this broad spectrum, the emission from radio to X-ray wavelengths is primarily generated through synchrotron radiation, while higher-energy radiation mainly originates from the inverse Compton scattering (ICS; Martín et al. 2012). As one of the most efficient particle accelerators and antimatter factories in the Milky Way (Adriani et al. 2009), PWNe serve not only as crucial objects for studying relativistic celestial bodies but also as ideal laboratories for exploring the physics of relativistic plasma. Currently, approximately 110 PWNe have been observed across various spectral bands, with about 20 of them yet to be confirmed (Olmi & Bucciantini 2023).

In addition to multi-wavelength observations and studies of radiation theory, numerical simulations of PWN dynamical evolution have emerged as a major research focus. Since the pioneering Rees & Gunn (1974) one-dimensional (1D) model in 1974 successfully predicted the internal structure of the Crab Nebula, numerical simulation studies of PWNe have made significant progress. Kennel & Coroniti (1984a, 1984b) developed a steady-state, spherically symmetric magnetohydrodynamic (MHD) model that incorporates more complex radiation emissions, calculating both the synchrotron spectrum and the spatial distribution of the Crab Nebula. Subsequent studies further expanded these models within both classical and relativistic frameworks, as well as in hydrodynamic (HD)

and MHD contexts, extending the evolution timeline beyond the free expansion phase (van der Swaluw et al. 2001; Bucciantini et al. 2003; van der Swaluw et al. 2004; Olmi & Bucciantini 2023).

During this period, observations of the Crab Nebula revealed more detailed internal structures, including the jet-torus, inner ring, knots, and wisps. Similar features, such as an equatorial ring and two opposing polar jets, have been observed in other young PWN systems (Hester et al. 1995; Helfand et al. 2001; Gaensler et al. 2002; Lu et al. 2002; Romani & Ng 2003; Camilo et al. 2004; Slane et al. 2004; Romani et al. 2005), which drive the development of two-dimensional (2D) and three-dimensional (3D) numerical models. On one hand, these models have been remarkably successful in reproducing the multi-band radiation characteristics and internal structural details of PWNe, such as optical, X-ray emissions, and synchrotron radiation (Del Zanna et al. 2006; Olmi et al. 2014), complete spectra from radio to gamma-rays (Volpi et al. 2008); polarization properties (Bucciantini et al. 2005b); variations in small-scale structures within the inner nebula (Camus et al. 2009; Olmi et al. 2015; Lyutikov et al. 2016); and the variable arc-shaped bright structures known as wisps (Hester et al. 2002; Melatos et al. 2005; Olmi et al. 2014, 2015) across multiple wavelengths. On the other hand, some models have investigated the formation mechanism of polar jets: these jets can be formed by magnetic hoop stress in the post-shock plasma, appearing immediately behind the polar front of the shock and positioned closer to the pulsar than the ring due to the oblate shape of the terminal shock itself (Bogovalov & Khangoulia 2002; Lyubarsky 2002; Del Zanna et al. 2004; Komissarov & Lyubarsky 2004; Bogovalov et al. 2005; Komissarov 2006; Spitkovsky 2006; Timokhin 2006).

In addition to the Crab Nebula as a research sample, many studies also focus on Bow Shock Pulsar Wind Nebulae (BSPWNe) as specimens. During the late evolutionary stage of a PWN, when the pulsar escapes from the SNR and propagates supersonically through the ISM, the interaction between the pulsar wind and ISM forms a BSPWN. In numerical simulation studies of BSPWNe, besides the internal dynamic evolution of the wind (Toropina et al. 2001; van der Swaluw et al. 2004; Bucciantini et al. 2005a; Toropina et al. 2019), the influence of surrounding ISM properties on their observable characteristics is also a key research focus; for instance, the opacity of neutral hydrogen in the ISM (Bucciantini 2002), and the ionization level of the medium (Olmi et al. 2018).

However, the model proposed by Kennel & Coroniti (1984a, 1984b) predicted the pulsar wind magnetization σ , which represents the ratio between the Poynting flux and the particle kinetic energy flux in the wind, approximately 3×10^{-3} for the Crab Nebula, while theoretical models of pulsar magnetospheres suggest this value should be much greater than 1. Currently, no known physical mechanism can

reduce σ to this level. To resolve this σ paradox, researchers have attempted to increase modeling dimensions, using 2D axisymmetric and 3D models to study the dynamical evolution of PWNe under different wind magnetization levels. Studies have shown that as the dimensionality of numerical simulations increases, the solution appears more feasible. From 1D to 2D, and then to 3D, the order of magnitude of wind magnetization has increased from 10^{-3} (Kennel & Coroniti 1984b) to 10^{-2} (Del Zanna et al. 2004; Komissarov & Lyubarsky 2004; Bogovalov et al. 2005; Del Zanna et al. 2006), and finally, Porth et al. (2013) and Olmi et al. (2014) achieved values greater than or equal to 1. However, since 3D simulations require substantial time and computational resources, compromises in resolution and evolution duration were necessary to save time. Therefore, with limited computational resources, 2D axisymmetric simulations still maintain unique advantages in balancing accuracy and simulation efficiency.

Furthermore, the interaction between SNRs and the ISM affects the morphologies and radiation properties of SNRs. Previous studies have successfully explained the unique observational morphologies of systems such as SN 1006, G349.7+00.2, and RCW 103 by simulating the dynamical evolution of SNRs in environments with specific directional density gradients (Fang et al. 2020; Yan et al. 2020; Lu et al. 2021). Some sources, however, possess extended TeV counterparts characterized by a compact bright core and an asymmetric faint component, such as Vela X and HESS J1825-137 (H. E. S. S. Collaboration et al. 2019). Blondin et al. (2001) suggested that this asymmetry might arise from inhomogeneities within the SNR, causing the reverse shock to reach one end of the PWN prematurely. These studies demonstrate that the interaction of PWNe or SNRs with inhomogeneities in the ISM significantly alters shock shapes, resulting in distinctive observational morphologies.

PWNe and SNRs both originate from supernova explosions. In SNR-PWN systems, the PWN typically resides within the SNR, and the entire system evolves in the ISM. The reverse shock generated during SNR evolution significantly influences the morphology and radiation characteristics of a PWN (Gelfand et al. 2009). However, existing numerical simulation studies often assume a uniform ISM distribution or only consider the direct interaction of either PWN or SNR with the ISM independently. This simplification overlooks the dynamical evolution of the composite system in non-uniform ISM background fields and fails to fully account for the interactions between a PWN and an SNR, as well as between the composite system and the ISM. Among the numerous PWNe detected in X-rays, most exhibit symmetric ring-jet or bow shock structures. However, some asymmetric morphologies are observed. For example, the SNR G189.1+03.0 (IC 443), a well-studied case interacting with a non-uniformly distributed medium (Mufson et al. 1986), exhibits two shells

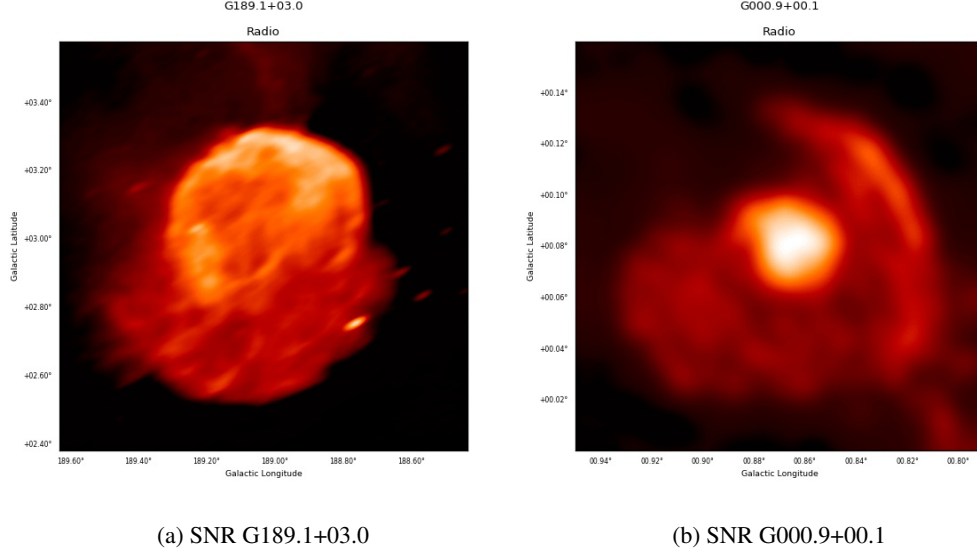


Figure 1. Left panel: The SNR G189.1+03.0 (IC 443), exhibiting two shells with differing radii. Right panel: The composite SNR G000.9+00.1, with a radio shell enveloping a centrally condensed PWN located nearer to the SNR shell on the right side. All images are observed in the radio band and sourced from the SNR Catalog (Ferrand & Safi-Harb 2012), <http://snrcat.physics.umanitoba.ca>.

with differing radii ($R_{\text{shellA}} < R_{\text{shellB}}$) (Braun & Strom 1986), as illustrated in the left panel of Figure 1. The composite SNR G000.9+00.1 consists of a radio shell and a centrally condensed PWN, as shown in the right panel of Figure 1, where the PWN is located nearer to the SNR shell on the right side. The formation mechanisms of these complex structures are not yet adequately explained.

Given the advantages of 2D simulations under resource constraints and the significant impact of ISM density variations on the morphology of PWNe and SNRs, this study utilizes 2D axisymmetric relativistic magnetohydrodynamic (RMHD) simulations to explore the evolution of SNR and PWN composite systems within two distinct ISM configurations: one with uniform density and another featuring a sharp density discontinuity. The results demonstrate that the density discontinuity significantly influences the dynamical evolution of these systems, contributing to the asymmetry of the shells and the displacement of the PWN. Specifically, this variation in density directly impacts the forward shock speeds across different density regions, leading to variations in shell layer radii. Additionally, the asymmetric morphology of the PWN and its positional offset are primarily driven by interactions with the reverse shock. The magnitude of discrepancies in shell layer radii and the degree of PWN offset toward one side of the SNR shell are correlated with the extent of the density jump in the background field.

The Athena++ numerical code is specifically developed for astrophysical problems and supports 1D/2D/3D simulations of compressible fluid dynamics and MHDs, including applications in special and general relativity (Stone et al. 2020; Gong et al. 2023). The code offers various coordinate

systems, grid refinement options, and mixed parallelization modes. Building upon this framework, we have developed our own numerical simulation model.

The paper is organized as follows: in Section 2, the model is briefly described; in Section 3, we show the results from the model and compare the results with the observations; in Section 4, the discussions and summary are given.

2. Model Description and Initialization

2.1. RMHD Equations and Distribution Functions

The pulsar wind, a highly relativistic plasma flow primarily composed of electron–positron pairs with strong magnetic fields, is modeled using the RMHD equations, which govern the conservation of mass, momentum, energy, and magnetic flux in a relativistic framework. These equations, adopted from Stone et al. (2020) where their numerical implementation in Athena++ is detailed, are as follows:

$$\frac{\partial \mathbf{D}}{\partial t} + \nabla \cdot (\mathbf{D}\mathbf{v}) = 0, \quad (1a)$$

$$\frac{\partial \mathbf{M}}{\partial t} + \nabla \cdot \mathbf{S} = 0, \quad (1b)$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \mathbf{M} = 0, \quad (1c)$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0. \quad (1d)$$

Here, the variables are defined in the laboratory frame: density $\mathbf{D} = \gamma\rho$, momentum $\mathbf{M}$, stress tensor $\mathbf{S}$, energy E , and magnetic field $\mathbf{B}$. The spatial components of the laboratory frame fluid four-velocity are given by $\mathbf{u} = \gamma\mathbf{v}$, where $\mathbf{v}$ is the three-velocity and γ is

the Lorentz factor. To apply these RMHD equations to the pulsar wind, we adopt specific assumptions about its energy flux and magnetic field distribution.

In this study, we model the pulsar wind by assuming that its energy flux and magnetic field depend on spatial coordinates. A pulsar releases energy through spin-down, with the total power given by its spin-down luminosity L_0 . In the split-monopole model proposed by Michel (1973), the pulsar wind energy flux varies with latitude as $\propto \frac{1}{r^2} \cdot \sin^2 \theta$. If the energy flux of the pulsar wind is distributed isotropically in space, the flux is expressed as

$$F(r) = \frac{L_0}{4\pi r^2}, \quad (2)$$

however, its distribution is anisotropic. To flexibly describe this anisotropy, an angle-dependent function $\mathcal{F}(\theta)$ is introduced, defining the pulsar wind energy flux as

$$\mathcal{F}(\theta) = \frac{\alpha + (1 - \alpha)\sin^2 \theta}{k}, \quad F(r, \theta) = \frac{L_0}{4\pi r^2} \mathcal{F}(\theta), \quad (3)$$

where the constant α is the anisotropy parameter, which controls the ratio between polar and equatorial energy flux, and k is a normalization factor. Energy conservation requires

$$\int_0^{2\pi} \int_0^\pi F(r, \theta) r^2 \sin \theta \, d\theta \, d\phi = L_0. \quad (4)$$

Substituting Equations (3) into (4) and performing the integration, we obtain $k = 1 - \frac{1}{3}(1 - \alpha)$. Thus, the pulsar wind energy flux at any specific coordinate (r, θ) within the computational domain is defined as follows

$$\mathcal{F}(\theta) = \frac{\alpha + (1 - \alpha)\sin^2 \theta}{1 - (1 - \alpha)/3}, \quad F(r, \theta) = \frac{L_0}{4\pi r^2} \mathcal{F}(\theta). \quad (5)$$

The split-monopole model captures the field distribution from the poles to the equator, though it overlooks complex processes near the equatorial region. Subsequently, Coroniti (1990) reported that the pulsar wind exhibits a striped structure near the equator, with dissipation between the stripes reducing the magnetic field strength, necessitating a refinement of the model. To address this, the function $\mathcal{G}(\theta)$ is introduced to define the magnetic field strength, incorporating a parameter σ_0 , as

$$\mathcal{G}(\theta) = \sin \theta \tanh \left[b \left(\frac{\pi}{2} - \theta \right) \right], \quad B(r, \theta) = \sqrt{\frac{\sigma_0 L_0}{c}} \frac{\mathcal{G}(\theta)}{r}, \quad (6)$$

where σ_0 defines the magnitude of B and typically ranges from 10^{-3} to 0.1 (Kennel & Coroniti 1984a; Bucciantini et al. 2004; Del Zanna et al. 2004, 2006). The form of $\mathcal{G}(\theta)$ combines the classical split-monopole model with the dissipation effect: $\sin \theta$ reflects the classical magnetic field distribution, while $\tanh \left[b \left(\frac{\pi}{2} - \theta \right) \right]$ models the dissipation near the equator. The dissipation parameter b , controlling the width of the

dissipation region, typically ranges from 1 to 10 based on the extent of the striped structure. In our simulations, the parameters are fixed at $\alpha = 0.1$, $b = 10$, and $\sigma_0 = 0.025$.

The energy flux of the pulsar wind consists of both the particle kinetic energy flux and the magnetic energy flux

$$F(r, \theta) = \left(\frac{B(r, \theta)^2}{4\pi} + nm_e c^2 \gamma_0^2 \right) c, \quad (7)$$

where n is the particle number density of pulsar wind, m_e is the electron mass, γ_0 is the Lorentz factor of the pulsar wind, B is the magnetic field strength, and c is the speed of light. Substituting $F(r, \theta)$ and $B(r, \theta)$ into Equation (9) and simplifying, we obtain the particle number density

$$n(r, \theta) = \frac{L_0}{4\pi c^3 m_e \gamma_0^2} \frac{1}{r^2} [\mathcal{F}(\theta) - \sigma_0 \mathcal{G}^2(\theta)]. \quad (8)$$

We note that Equations (5), (6), and (8) are derived from Olmi et al. (2014), with detailed derivations available in that paper.

2.2. Density Discontinuity of ISM

Observational evidence indicates that the ISM is a turbulent and inhomogeneous environment characterized by stochastic fluctuations in velocity, density, and magnetic field strength (Lee & Jokipii 1976; Armstrong et al. 1981, 1995; Minter & Spangler 1996). Numerical simulations of well-known SNRs such as Cas A, SN 1006, and IC 443 have shown that their morphologies are influenced by the ambient turbulent ISM (Blondin et al. 2001; Lee et al. 2008; Velázquez et al. 2017), which comprises multiple thermal phases. In the solar neighborhood, ISM densities span a wide range: the cold neutral medium (CNM) at $20\text{--}50 \text{ cm}^{-3}$, molecular clouds from 10^2 to 10^6 cm^{-3} , and the diffuse warm neutral medium (WNM) at $0.2\text{--}1.0 \text{ cm}^{-3}$ (Heiles & Troland 2003; Kim & Ostriker 2015). Interstellar clouds exhibit substantial density contrasts, with components such as molecular clouds, the CNM, and the WNM spanning several orders of magnitude. Density discontinuities naturally arise at the boundaries between interstellar clouds with different densities. This leads to discontinuities that are not captured by models assuming either a uniform ISM or a turbulent medium with continuous variations. To explore the influence of such discontinuities on the evolution of PWN-SNR composite systems, we model the ISM density field as a piecewise constant function, as defined below

$$n_{\text{ISM}} = \begin{cases} K \times 1.0 & \text{if } 0 < \theta < \frac{\pi}{2} \\ 1.0 & \text{if } \frac{\pi}{2} < \theta < \pi \end{cases} \text{ cm}^{-3}, \quad (9)$$

where the constant K represents the density contrast ratio, taking values of 1, 2, and 4, with larger K indicating a more pronounced density discontinuity in the ISM. This approach differs from previous SNR simulations that often assume either a uniform ISM or a turbulent field with a Kolmogorov-like

power spectrum (Yu et al. 2015; Velázquez et al. 2017). Instead, we adopt a piecewise constant density distribution, with distinct values in different regions and a sharp density discontinuity at $\theta = \frac{\pi}{2}$, emphasizing the role of discontinuities themselves rather than the absolute values of the ambient density. This setup highlights the effects of density discontinuities on the evolution and morphology of the PWN-SNR system, rather than focusing on specific density values.

2.3. Simulated Configurations

This work investigates the evolution of PWN-SNR systems under varying ISM density conditions using 2D RMHD simulations in a spherical-polar coordinate system with the Athena++ code (Stone et al. 2020). The simulation starts from the early expansion phase of the composite system, with the PWN and SNR radii set to $r_i = 0.1$ pc and $r_e = 2.0$ pc at $t = 0$, respectively. We employ the Harten-Lax-van Leer discontinuity (HLLD) solver for the flow calculations. The computational domain extends from $r_{\min} = 0.1$ pc to $r_{\max} = 8.0$ pc, with the angular domain spanning from 0 to π and reflection conditions applied on the polar axis. A spatial grid consisting of 4096 cells in the radial direction and 512 cells in the polar angle θ is used. The background plasma has a temperature of 1.0×10^4 K and an adiabatic index $\gamma = 4/3$ for all relativistic gas.

2.4. Initial Conditions

The supernova explodes at the center of the simulation domain with an explosion mass of $M_{ej} = 3.0 M_{\odot}$ and an explosion energy of $E_{ej} = 1.0 \times 10^{51}$ erg. The ejection velocity is calculated as $v_{ej} \approx 0.025c$ using the formula $v_{ej} = \sqrt{\frac{10E_{ej}}{3M_{ej}}}$ (van der Swaluw et al. 2001, 2004), which assumes a uniform density and a linear velocity profile as a function of radius for the stellar ejecta. The background magnetic field is uniformly distributed with a strength of 1×10^{-6} G, aligned along the z -axis with both polar and azimuthal components equal to zero.

Meanwhile, the pulsar is positioned at the center of the computational domain ($(r, \theta) = (0, 0)$) with an initial luminosity of $L_0 = 5.0 \times 10^{39}$ erg s $^{-1}$. The pulsar wind, with a relativistic Lorentz factor $\gamma_0 = 100$, is injected from the inner boundary at r_i .

3. Results

Figure 2 illustrates the evolution of the PWN-SNR composite system over time in a uniform ISM ($K = 1$). To clearly demonstrate the changes in density distribution, we introduced a conversion factor and took the logarithm of the computed results. This allowed us to create slice plots at different times in the RZ plane. The figure illustrates the density distribution at various times. The core low-density area represents the PWN, while the purple and yellow-green boundaries indicate the reverse shock of the SNR. The yellow semicircular area represents its forward shock. Additionally,

Rayleigh–Taylor instability occurs at the contact discontinuity between the two shocks, manifesting as finger-like structures that elongate and distort as the evolution progresses, further complicating the interaction dynamics.

The supernova explosion releases a significant amount of energy, causing the surrounding material to be heated and ejected at high speeds. The temperature of the ejected material can reach millions of Kelvin, resulting in particles moving at high velocities and significantly increasing the internal pressure. As the SNR expands in interstellar space, it sweeps up more material. The material behind the shock wave is compressed and heated, which is represented in the figure as the bright yellow region on the outer layer. At the same time, a reverse shock wave forms, causing the ejected material to decelerate. The reverse shock initially expands outward with the forward shock and then moves toward the center of the remnant. During this stage, the PWN undergoes free expansion, with the expansion rate in the Z direction surpassing that in the R direction. By 978 yr, the extremities of the PWN encounter the reverse shock, fully merging by 1305 yr. Under the influence of the high-temperature, high-pressure ejected material and the reverse shock, the expansion of the PWN decelerates and continues to be compressed. The internal material density significantly increases, resulting in a uniform dense core characterized by fragmented structural features. By 1794 yr, the scale of the dense core reaches approximately 1 pc.

As the PWN is compressed, pulsar wind particles are accelerated under the influence of the pulsar’s gravity and magnetic field, forming a high-temperature and high-pressure wind. The continuous injection of pulsar wind leads to an increase in the density, energy, and pressure of the nebula. This process continues until the pressure reaches the Sedov solution, corresponding to the forward shock of the SNR, marking the transition into the Sedov–Taylor phase from 2120 to 2772 yr. During this phase, the PWN continues to expand. Typically, the radius of the nebula undergoes several cycles of contraction and expansion, with an oscillation timescale of a few thousand years. However, in this instance, only one oscillation occurs, which is directly related to the specific initial conditions selected for our simulations. Throughout its evolution, the PWN maintains clear symmetry about the $Z = 0$ axis. This symmetry persists during both the expansion and compression phases of the nebula, without any dissipation. Notably, during the reverse expansion phase, some low-density areas within the PWN (within $r < 1$ pc) do not exhibit strict symmetry. However, these isolated low-density regions are nearly invisible in the nearly circular central area.

To clearly demonstrate the impact of density distribution of ISM on the overall morphology of PWN, we plotted density distribution slices under different ISM density configurations ($K = 1, 2, 4$), as shown in Figure 3. The images clearly demonstrate that when the ISM density is non-uniform, the shapes of the SNR’s double shock structures undergo significant changes.

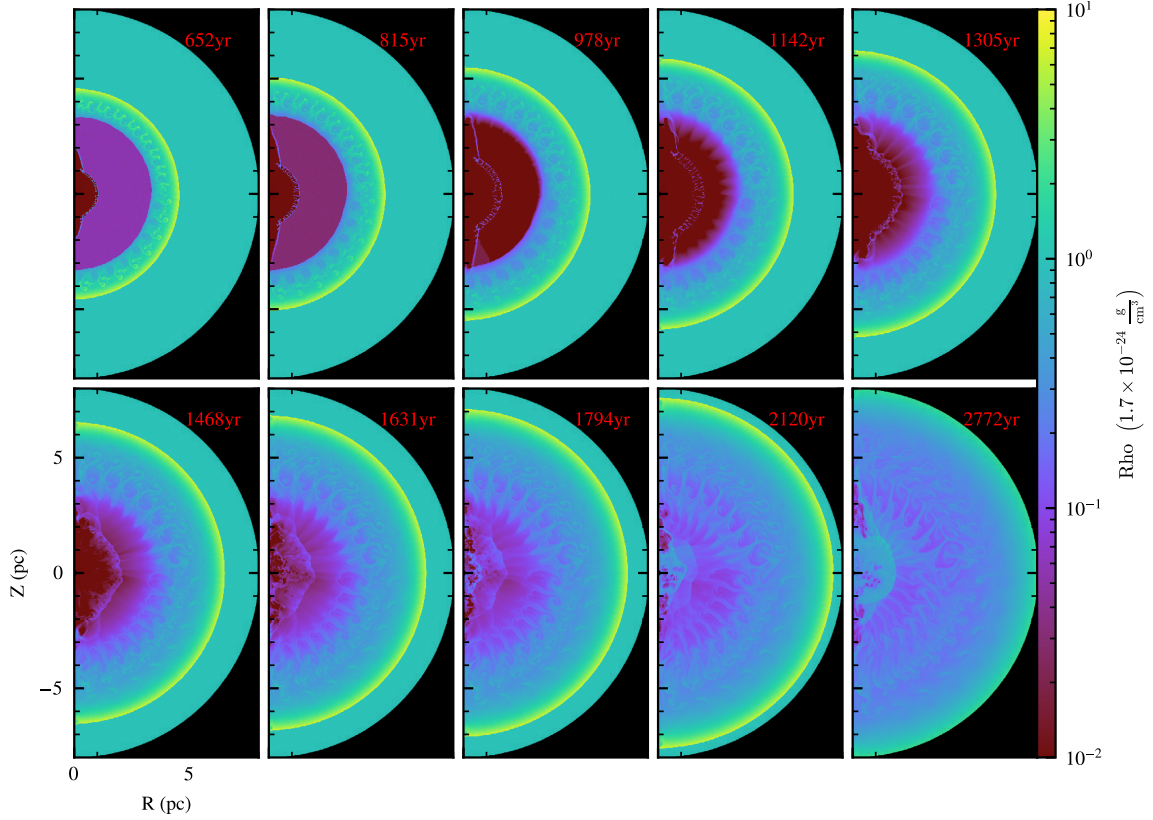


Figure 2. Density distribution slices of the SNR-PWN composite system evolving over time in a background field with uniform density ($K = 1$) at different times, shown in the RZ plane. The color bar represents the logarithm of density, with units of $1.7 \times 10^{-24} \text{ g cm}^{-3}$.

A higher ISM density impedes the outward propagation of the forward shock while accelerating the inward propagation of the reverse shock in that region. This difference in propagation speeds between the two regions leads to the formation of a vortex structure at the density discontinuity, which grows in size over time. Comparing the same column of images, it can be observed that after the same duration of 1142 yr, regions with higher K values exhibit smaller radii for both the forward and reverse shocks. Additionally, the difference in the radii of the SNR shell layers is more pronounced in these areas.

After the reverse shock collides with the PWN, the shape of the PWN is significantly altered. In regions of higher density, the reverse shock propagates more rapidly, compressing more material toward the low-density side. This asymmetry drives a gradual displacement of the PWN toward the lower-density side relative to its initial location. As the material accumulates in the low-density region and reaches the Sedov solution, it expands again, further influencing the displacement of the PWN. The displacement of the PWN increases with longer evolution times when the K value remains constant. Conversely, for the same duration of evolution, an increase in K value results in greater displacement due to more intense and prolonged compression in the high-density regions. This

non-uniform distribution of ISM disrupts the overall symmetry of the PWN, which would otherwise be observed in a uniform density environment. The asymmetry is primarily driven by the differential propagation of the shocks across varying densities, with higher K values exacerbating the displacement over time.

Figure 4 illustrates how the $\frac{B^2}{2}$ distribution of the system evolves over time under the assumption of a uniform ISM density. The image not only clearly marks the three stages of system evolution but also shows the symmetry of the SNR shell and internal PWN along the $Z = 0$ axis. Compared to Figure 2, the $\frac{B^2}{2}$ map distinctly enhances the contrast between the internal PWN (bright yellow) and the surrounding environment (yellow-green), making the fragmentation process and shape changes of the PWN more visually apparent. This distinction is primarily due to the PWN being closer to the pulsar magnetosphere and the strong magnetic fields generated by the high-speed charged particles within it, which significantly elevate the $\frac{B^2}{2}$ inside the PWN compared to its surroundings. When logarithmically processed, the differences in $\frac{B^2}{2}$ between the PWN and its environment are amplified, resulting in a more pronounced color contrast.

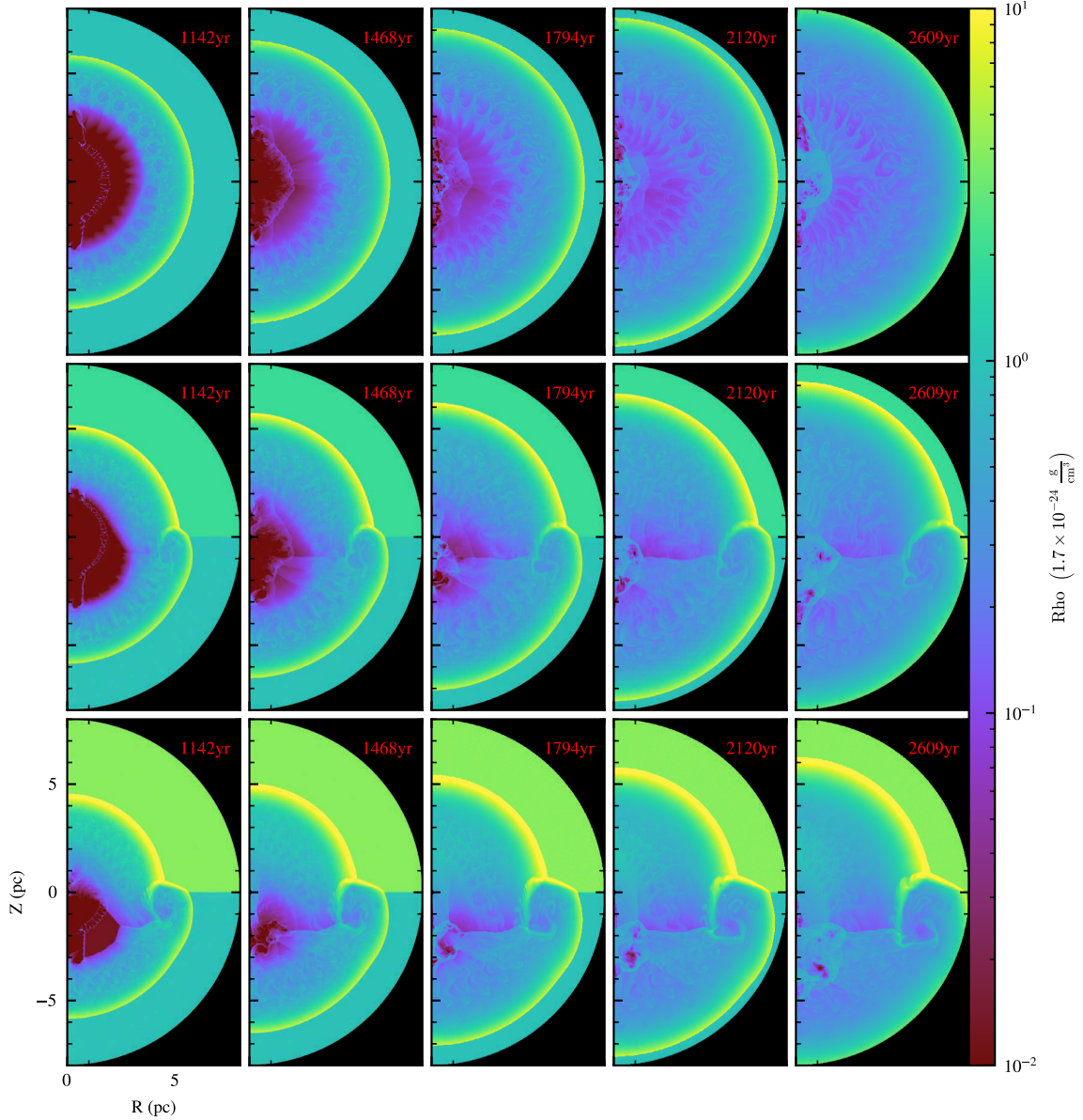


Figure 3. Density distribution slices of the SNR-PWN composite system evolving over time in different density configurations ($K = 1, 2, 4$, from top to bottom respectively) at different times. Others are same as in Figure 2.

Moreover, the Rayleigh–Taylor instability is more pronounced in Figure 4 due to the density differences between the ejected material, the ISM, and the PWN. The ejected material continuously penetrates into the ISM and is swept by the reverse shock, forming finger-like structures. These structures undergo constant changes and growth during the free expansion phase, gradually approaching the forward shock while their shapes continue to distort. As the PWN encounters the reverse shock, a distinct double-shock structure begins to emerge, and the region between the double shocks expands. These finger-like structures extend radially both inward and outward, gradually approaching the PWN and the forward shock.

Figure 5 illustrates the evolution of $\frac{B^2}{2}$ in the PWN-SNR system under three different conditions ($K = 1, 2, 4$). As it propagates radially outward, the forward shock generally maintains a spherical shape. However, in high-density regions, the radius of the shock is smaller compared to that in low-density regions, closely matching the radius at the boundary of instability in the latter. Vortex structures, forming at the interface where the medium density changes, intersect with the forward shock and propagate outward with it, continuously increasing in scale. Notably, the Z-coordinate of the vortex center remains unchanged regardless of variations in K values or evolution time.

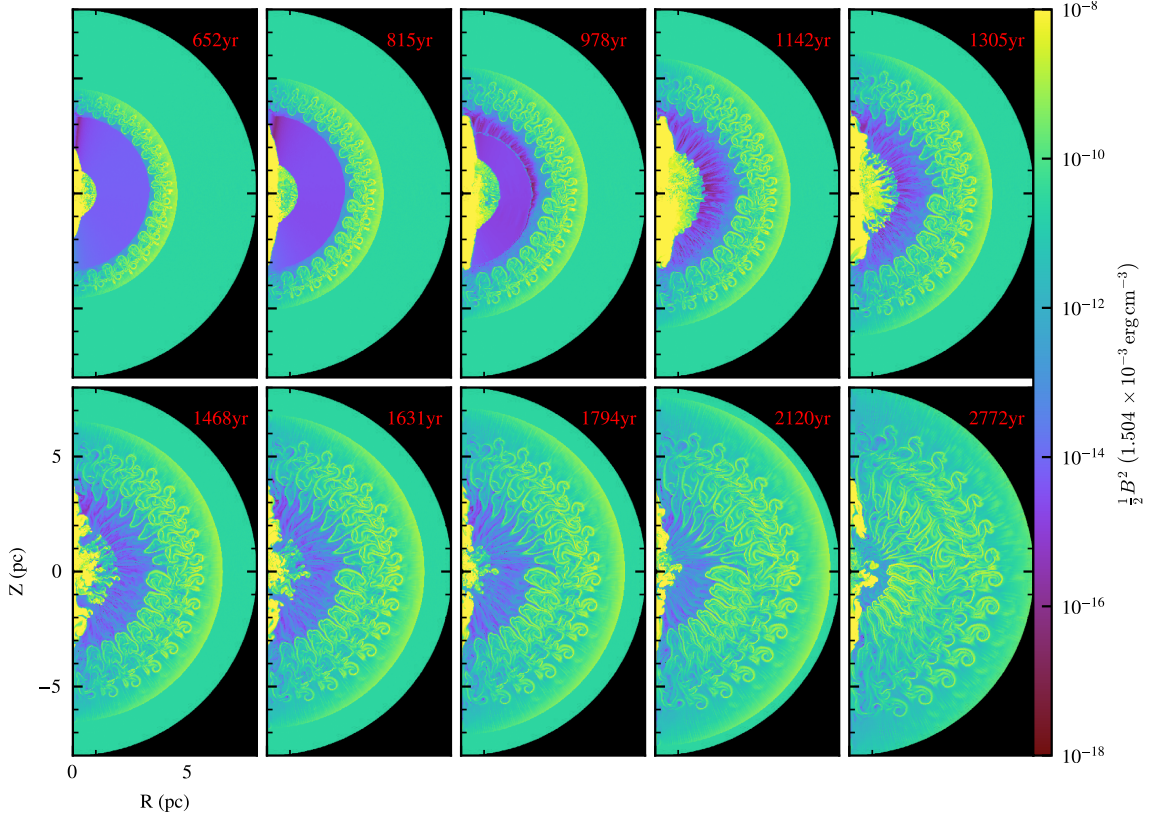


Figure 4. Time evolution of $\frac{B^2}{2}$ distribution slices of the SNR–PWN composite system in a uniform-density background, shown in the RZ plane. The color bar indicates the logarithm of $\frac{B^2}{2}$, in units of $1.504 \times 10^{-3} \text{ erg cm}^{-3}$.

Under the influence of the reverse shock, the PWN is compressed into the low-density region, resulting in a narrower shape in high-density areas and a wider shape in low-density areas. As the low-density region of the PWN encounters the reverse shock, it undergoes further compression, pushing most of its structure into the low-density area. By the time it reaches the Sedov–Taylor phase, the PWN primarily expands only in the low-density region, while expansion in the high-density area becomes nearly imperceptible. In contrast to the free expansion phase, the entire nebula becomes highly concentrated and expands uniformly outward during this stage. The degree of concentration within the nebula decreases significantly, exhibiting a fragmented structure with partial concentration in some regions. Additionally, as the evolution time and the parameter K increase, the center of the nebula progressively shifts away from the explosion center. By 2609 yr, the explosion center is nearly located at the edge of the PWN, and the symmetry of the PWN shape between high- and low-density regions is completely disrupted. Comparative analysis reveals that a higher K value causes the PWN to encounter the reverse shock earlier, accelerating the overall evolution process. As a result, the scale of the PWN is larger at the same time points during the Sedov–Taylor phase (at 2120 yr and 2609 yr).

4. Summary and Discussion

This study investigates the evolution of an SNR and PWN composite system in two different backgrounds: one with uniform ISM density and another with a sharp discontinuity in ISM density across $Z = 0$, based on a 2D axisymmetric RMHD model. In both scenarios, the simulation results clearly demonstrate that the PWN undergoes free expansion, reverse compression, and the Sedov–Taylor phase, with fluid dynamic phenomena such as Rayleigh–Taylor instability also being observed.

Significant differences in evolutionary characteristics between uniform and non-uniform backgrounds are highlighted. In the uniform background, the overall morphologies of the PWN and the SNR shell maintain symmetry about the $Z = 0$ axis, persisting over time. In contrast, with a sharp discontinuity in the ISM density, the evolution of the composite system exhibits distinct differences. During the free expansion phase, before the PWN encounters the reverse shock of the SNR, its evolution is unaffected by the ISM density distribution. The PWN retains its symmetrical feature about the $Z = 0$ axis while continuing to expand, as illustrated in the first column of Figure 6. The forward shock of the SNR maintains a spherical shape as it expands outward; however, the shock radius is slightly larger in low-

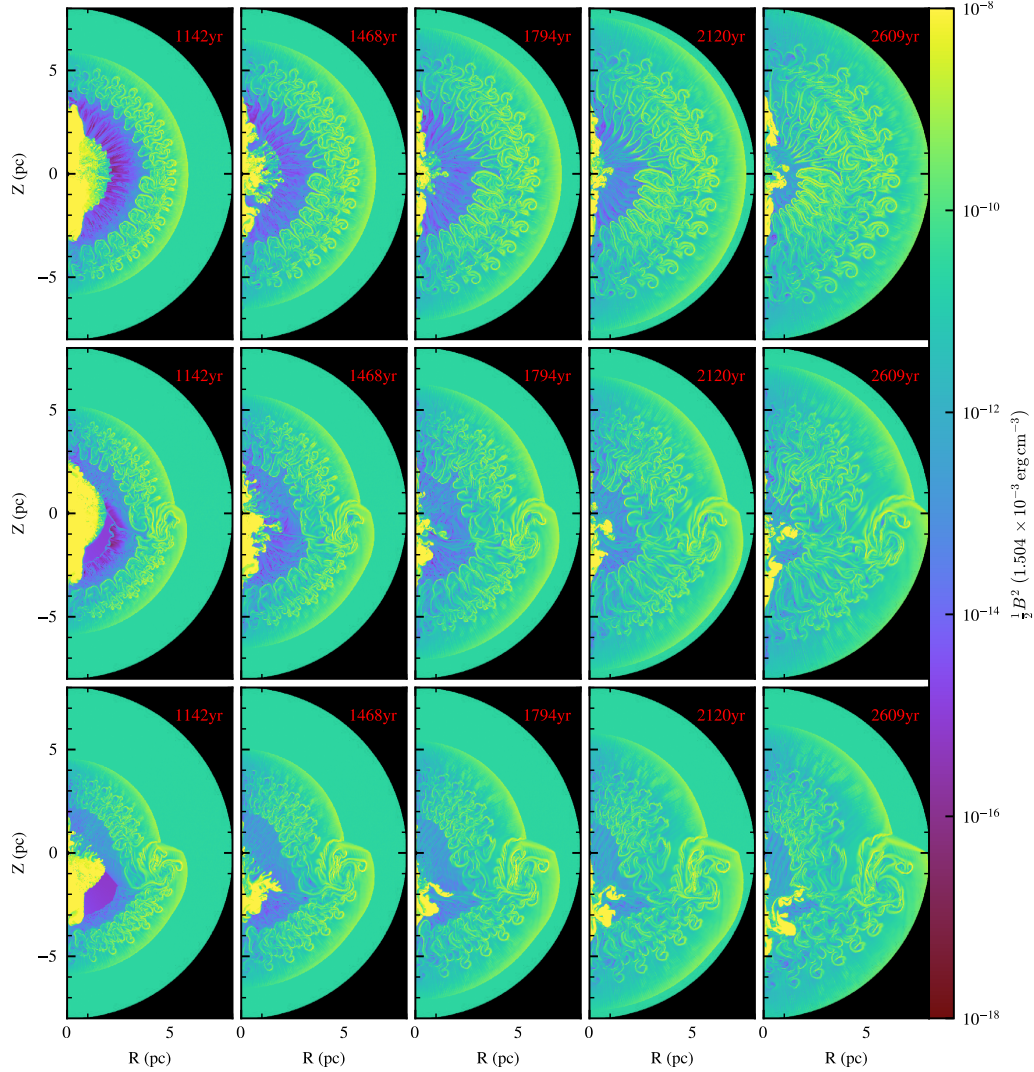


Figure 5. Time evolution of $\frac{B^2}{2}$ distribution slices of the SNR-PWN composite system in different density configurations ($K = 1, 2, 4$, from top to bottom) at different times. Others are the same as in Figure 4.

density regions than in high-density ones, which results in the formation of two shell layers with distinct radii. Additionally, a vortex structure forms at the density discontinuity, which gradually enlarges and expands outward with the forward shock as the system evolves. The greater the density jump, the larger the scale of the vortex structure becomes, while its central coordinates show no significant change.

Upon encountering the reverse shock, the PWN enters the reverse compression phase. During this phase, the reverse shock interacts with the PWN earlier in high-density regions and compresses it, leading to the development of asymmetrical features until the PWN is fragmented. As the evolution progresses, material is continuously compressed from high-density regions to low-density regions. The greater accumulation of material on the low-density side causes the PWN to become asymmetrical.

Notably, in low-density regions, the reverse shock also compresses material toward the system center, but the compression intensity is significantly less intense than in high-density ones. Ultimately, under the combined influence of the reverse shocks in two regions, the structure of the PWN moves from the high density side to the low density side. The greater the density jump, the more pronounced the shift of the PWN structure toward the low-density region, resulting in a greater distance from the location of the supernova explosion. In the Sedov-Taylor stage, the center of the PWN no longer coincides with the explosion center (i.e., the location of the pulsar at $((r, z) = (0, 0))$). At this stage, the pulsar is located at the edge of the PWN or even completely separated from it.

Our 2D RMHD simulations provide a reasonable explanation for two asymmetric morphological features observed in

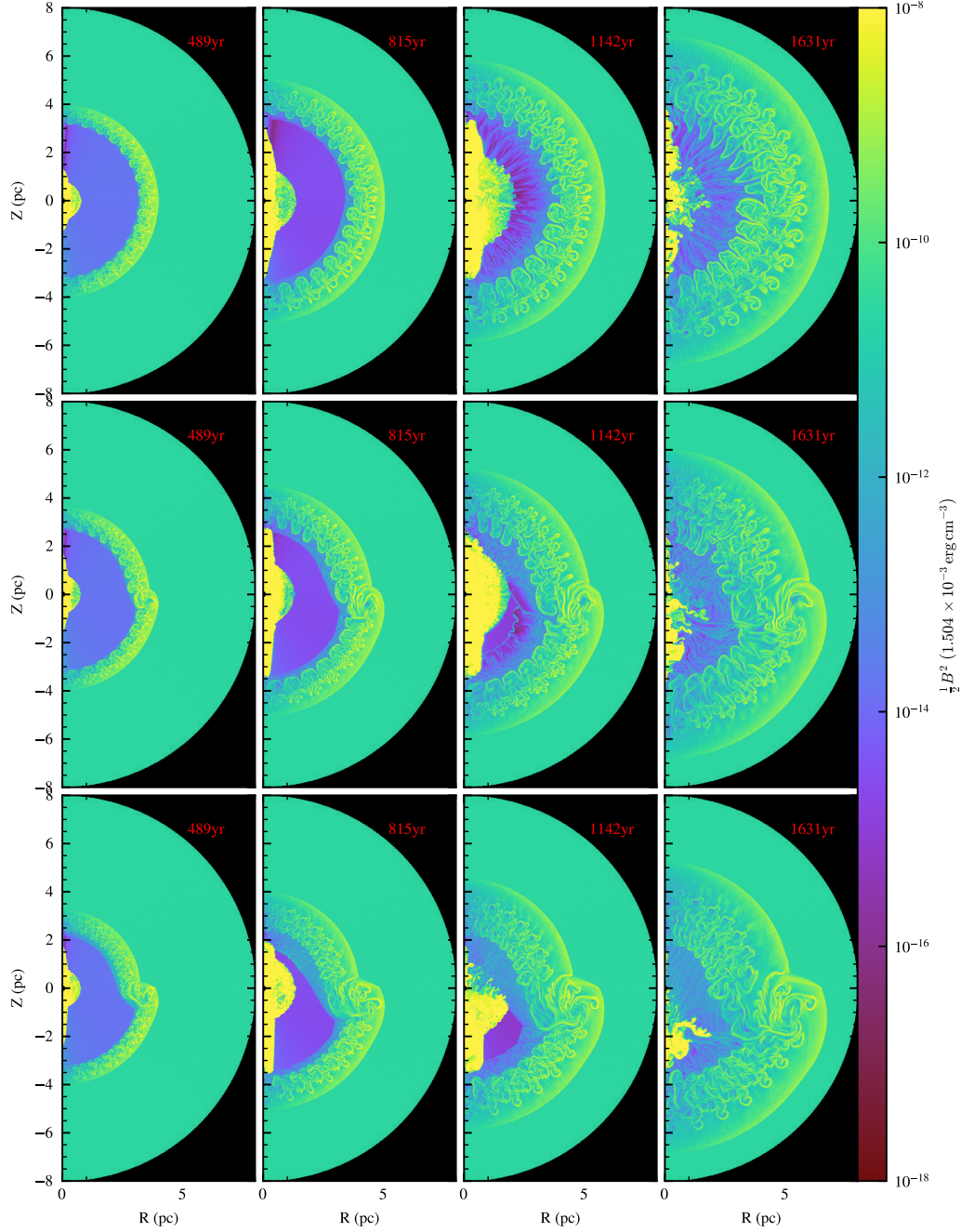


Figure 6. Others are the same as in Figure 5 but earlier.

PWN-SNR systems: the presence of a single SNR exhibiting two shells of different radii and the positional offset of the inner PWN. For example, observations of IC 443 reveal an SNR with shells A and B, with $R_{\text{shellA}} < R_{\text{shellB}}$, as shown in the left panel of Figure 1. We suggest that this asymmetry may arise from a density distribution divided roughly by the Galactic Latitude $+3.00$ horizontal line, with the ISM density being higher in the upper half than in the lower half.

Consequently, the forward shock propagates more slowly in the high-density upper region, leading to the formation of the smaller-radius shell A, while in the low-density lower region it advances more rapidly, producing the more extended shell B. In contrast, observations of the composite SNR G000.9+00.1 reveal the inner PWN located nearer to the SNR shell on the right side, as shown in the right panel of Figure 1. We propose that this offset may arise from a density distribution divided roughly along the diagonal line

from the upper-left to the lower-right corner of the image, with the ISM density being higher in the lower-left region (left of the line) than in the upper-right region (right of the line). During the system's evolution, the reverse shock propagates more rapidly in the high-density lower-left region and continuously compresses the PWN from the high-density lower-left region toward the low-density upper-right region, leading to the observed displacement.

Although the simulations focus on the distributions of density and $\frac{B^2}{2}$ rather than directly modeling emission, the resulting asymmetries are consistent with structures observed in radio images. Regions of higher $\frac{B^2}{2}$ generally correspond to stronger synchrotron emission, as the latter scales with magnetic field strength. Additionally, the density structure indirectly affects emission by modulating shock propagation and particle acceleration. However, detailed modeling of brightness profiles requires incorporating high-energy particle acceleration and radiative transfer processes, which will be explored in future work through coupling MHD simulations with radiation mechanisms.

In conclusion, the distribution of density in the ISM directly influences the propagation speeds of both forward and reverse shocks, thereby significantly affecting the morphology and positioning of the SNR and PWN composite system. Specifically, the discontinuity in ISM density directly leads to inconsistencies in the propagation speeds of the forward shock across different density regions, not only disrupting the symmetry of the SNR shell but also resulting in the formation of two shell layers with distinct radii. Meanwhile, the asymmetrical features and positional offset of the PWN (from the initial location) primarily emerge through interactions with the reverse shock at varying speeds. The more pronounced the density jump is, the greater the discrepancies in the shell layer radii and the more significant the offset of the PWN from initial location.

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