



Enhancing Cosmological Constraints by Two-dimensional β -cosmic-web Weighted Angular Correlation Functions

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Abstract

In this study, we investigate the potential of mark-weighted angular correlation functions, which integrate β -cosmic-web classification with angular correlation function analysis to improve cosmological constraints. Using SDSS DR12 CMASS-NGC galaxies and mock catalogs with Ω_m varying from 0.25 to 0.40, we assess the discriminative power of different statistics via the average improvement in chi-squared, $\Delta\bar{\chi}^2$, across six redshift bins. This metric quantifies how effectively each statistic distinguishes between different cosmological models. Incorporating cosmic-web weights leads to substantial improvements. Using statistics weighted by the mean neighbor distance ($\bar{D}_{\text{nei}}$) increases $\Delta\bar{\chi}^2$ by approximately 40%–130%, while applying inverse mean neighbor distance weighting ($1/\bar{D}_{\text{nei}}$) yields even larger gains, boosting $\Delta\bar{\chi}^2$ by a factor of 2–3 compared to traditional unweighted angular statistics. These enhancements are consistent with previous 3D clustering results, demonstrating the superior sensitivity of the β -weighted approaches. Our method, based on thin redshift slices, is particularly suitable for slitless surveys (e.g., Euclid, CSST) where redshift uncertainties limit 3D analyses. This study also offers a framework for applying marked statistics to 2D angular clustering.

Key words: (cosmology:) cosmological parameters – methods: statistical – (cosmology:) large-scale structure of universe

1. Introduction

The cosmic structure originated from early tiny density perturbations. Under the combined influence of gravity and accelerated expansion driven by dark energy, these perturbations gradually give rise to the observed complex and significantly nonlinear structure, which exhibits a wide variety of morphologies and properties at different scales.

The spatial distribution of matter in the universe manifests as a complex, hierarchical network structure termed the “cosmic web”. This anisotropic pattern emerges from gravitational collapse processes dominated by dark matter (DM) dynamics, and has constituted a major focus of cosmological research since its first identification in the late 20th century (Bardeen et al. 1986; Bond et al. 1996; de Lapparent et al. 1986; Huchra et al. 2012; Tegmark et al. 2004; Guzzo et al. 2014; Suárez-Pérez et al. 2021).

The study of the cosmic web relies on large-scale galaxy surveys. Currently, several large-scale survey projects have been successfully implemented, such as the Sloan Digital Sky Survey (SDSS; York et al. 2000; Abazajian et al. 2003, 2004;

Eisenstein et al. 2005; Gott et al. 2005; Adelman-McCarthy et al. 2006; Percival et al. 2007; Adelman-McCarthy et al. 2008; Anderson et al. 2012; Sánchez et al. 2012, 2013; Anderson et al. 2014; Ross et al. 2015; Alam et al. 2017; Beutler et al. 2017; Chuang et al. 2017; Sánchez et al. 2017), the 2-degree Field Galaxy Redshift Survey (2dFGRS) (Colless et al. 2001), the 6-degree Field Galaxy Redshift Survey (6dFGRS) (Beutler et al. 2011, 2012) and the WiggleZ Dark Energy Survey (Parkinson et al. 2012), providing valuable data support for unveiling the mysteries of the cosmic web.

Various methods have been developed to describe and analyze the cosmic web. The most popular techniques for describing the cosmic web include the density-based classification (Klypin & Holtzman 1997; Springel et al. 2001; Knollmann & Knebe 2009), T-web (Hahn et al. 2007; Forero-Romero et al. 2009; Suárez-Pérez et al. 2021), V-web (Hoffman et al. 2012; Forero-Romero et al. 2014), DisPerSE (Sousbie 2011), skeleton algorithms defined based on Lagrangian fluid dynamics (Feldbrugge et al. 2018), and so on. These tools explore the properties of the cosmic web by analyzing the velocity field, density field, or the intuitive geometric distribution of the cosmic web.

Among the various cosmic web descriptions, the β -skeleton method (Fang et al. 2019; Suárez-Pérez et al. 2021) utilizes an algorithm, which originates from the fields of computational geometry and geometric graph theory to describe the properties of the cosmic web. This algorithm constructs a graph to describe the connectivity properties from a set of n -dimensional spatial points, and has been applied in a variety of fields such as machine learning, visual perception, image analysis and pattern recognition (Edelsbrunner et al. 1983; Amenta et al. 1998; Zhang & King 2002). Fang et al. (2019) first applied this algorithm to the cosmic large-scale structure (LSS), demonstrating that the cosmic web can be intuitively reconstructed from galaxy spatial distributions, enabling effective visualization of this fundamental structure. García-Alvarado et al. (2020) explored β -cosmic-web and found that it reveals significant advantages in characterizing sparsely distributed galaxies. Based on the cosmic web, they further proposed a measure of entropy for describing the complexity of large-scale structures, and built up the relationship between the β -cosmic-web and the traditional T-web via machine learning methods. Recently, Yin et al. (2024) have pointed out that the environmental information of β -cosmic-web weighting schemes can be utilized to build up mark weighted correlation functions, which can substantially improve the accuracy of the constraints on the cosmological parameters compared to the traditional two-point correlation function.

In the next decade, stage-IV surveys such as Dark Energy Spectroscopic Instrument (DESI) (DESI Collaboration et al. 2016, 2022; Adame et al. 2025), Large Synoptic Survey Telescope (LSST; LSST Science Collaboration et al. 2009; Ivezić et al. 2019; Crenshaw et al. 2025; Kumar et al. 2025), Euclid Space Telescope (Laureijs et al. 2011; Euclid Collaboration et al. 2022, 2025c; Cuillandre et al. 2025; Euclid Collaboration et al. 2025a, 2025), and China Space Station Telescope (CSST; Zhan 2011; Cao et al. 2018; Gong et al. 2019; Zhan 2021; Fu et al. 2023; Luo et al. 2024; Sui et al. 2025; Shi et al. 2025a) will provide deep, wide-field observations in much wider range of redshift, producing unprecedented data sets for cosmic web studies. While these surveys provide abundant data for studying the cosmic evolution, the analysis of the data faces new challenges. The stage-IV surveys will unveil complex nonlinear structures, which require more advanced analysis methods. Moreover, some stage-IV surveys (e.g., CSST and Euclid) employ slitless grating spectrometers, which efficiently expand the survey volume/depth but at the cost of reduced redshift measurement precision. Because this trade-off may significantly affect data analysis (Gong et al. 2019; Xiao et al. 2023; Gu et al. 2024b; Euclid Collaboration et al. 2025), the methodology must be adjusted to address the resulting challenges of redshift uncertainty.

One possible way to avoid the difficulties brought by redshift uncertainty is to use the two-point angular correlation

function (2PACF; Connolly et al. 2002; Euclid Collaboration et al. 2025b; Wang et al. 2013; Alcaniz et al. 2016; Carvalho et al. 2016; de Carvalho et al. 2018; Carvalho et al. 2020; Franco et al. 2024; Venville et al. 2024; Wu & Xia 2025), which analyzes cosmic large-scale structures without requiring precise redshift information and is ideal for photometric surveys. Yet the limitation of the 2PACF is that it only captures Gaussian features of the density field, becoming increasingly limited as gravitational collapse enhances non-Gaussianity. Recently, mark-weighted correlation functions (MCFs) have been extensively studied (White 2016; Satpathy et al. 2019; Yang et al. 2020; Xiao et al. 2022; Massara et al. 2024; Storey-Fisher et al. 2024; Xu et al. 2025). MCFs extract non-Gaussian information by weighting galaxy clustering statistics with galaxy properties, offering a more complete characterization of nonlinear structure. This simple yet powerful method has proven effective in probing the complex features of LSS.

In this study, we introduce a novel mark-weighted statistical measure, the mark-weighted angular correlation functions (MACFs), which combine β -cosmic-web classification with angular correlation function analysis. This approach retains sensitivity to non-Gaussian features in the LSS while reducing sensitivity to redshift uncertainties.

The study is organized as follows: In Section 2, we briefly introduce the data sets used in this analysis. In Section 3, we detailedly introduce the methodology of building up β -cosmic-web from the 2D galaxy distribution, as well as the methodology for the related MACFs. The results and findings are presented in Section 4. We summarize and conclude in Section 5.

2. Datasets

To validate the sensitivity of MACFs with β -cosmic-web weights to cosmological parameters, we compare measurements from the SDSS BOSS DR12 data with those from mock catalogs. The mock simulations are produced using COMoving Lagrangian Acceleration (COLA) fast algorithm (Tassev et al. 2013; Wang et al. 2024b), and then calibrated using the subhalo abundance matching (SHAM) methods (Ding et al. 2024), so that to accurately reproduce CMASS galaxy statistics. For assessing the power of statistics, we compute covariance matrices from Multidark PATCHY (PerturbAtion Theory Catalog generator of Halo and galaxy distributions) (Kitaura et al. 2016) simulations.

2.1. The BOSS CMASS Galaxies

The Baryon Oscillation Spectroscopic Survey (BOSS) project is a crucial component of SDSS-III (Eisenstein et al. 2011; Bolton et al. 2012; Dawson et al. 2013). Its primary scientific goal is to precisely measure expansion and structure growth history via the BAO measurements from the spatial

distribution of luminous red galaxies (LRGs) and quasars (Eisenstein et al. 2001). To do this, redshift information for about 1.5 million cosmic galaxies within a sky area of approximately 10,000 square degrees is accurately measured. These galaxies are divided into two samples of LOWZ and CMASS. The LOWZ galaxy sample includes mainly brightest red galaxies at $z \leq 0.4$, while the CMASS sample targets a population of galaxies at higher redshifts, many of which are also in the category of LRGs.

To explore the statistical properties of MACFs, the data must be projected from the three-dimensional space onto a two-dimensional angular space. For simplicity, in this analysis we only use a sub-data set of the BOSS DR12 CMASS NGC galaxies lying in the redshift range of $z \in [0.45, 0.55]$. The process is done by the following steps:

1. First, we divide the three-dimensional galaxy sample into six overlapping redshift intervals, each with a thickness of $\Delta z = 0.05$. The intervals are $z \in [0.45, 0.50]$, $[0.46, 0.51]$, $[0.47, 0.52]$, $[0.48, 0.53]$, $[0.49, 0.54]$, $[0.50, 0.55]$, respectively.
2. Next, we project the galaxies in each redshift shell onto a two-dimensional plane at the shell's central redshift. The central redshifts for the shells are: $z_c = 0.475, 0.485, 0.495, 0.505, 0.515$, and 0.525 , respectively.

In this way, we create overlapping 2D projected bins of galaxies. Allowing these overlaps serves two purposes: (1) to better trace the evolution of clustering patterns across redshift, and (2) to reduce information loss caused by boundary effects between adjacent redshift bins.

2.2. COLA Mock Catalogs

COLA (Tassev et al. 2013) is a hybrid simulation method that integrates second-order Lagrangian perturbation theory (2LPT) and N -body algorithms, offering an effective solution for simulating DM particles. The perturbation theory results, which can successfully describe large-scale clustering processes, can replace the time integration calculations of N -body simulations on linear scales, so the large-scale and small-scale evolution processes are decoupled and separately treated. In the large-scale structure evolution, the evolution of DM particles can be described by the precise 2LPT theory results to describe their dynamical behaviors, while on small scales the evolution is carried out with well-established N -body simulation methods. To achieve this goal, COLA introduces a co-moving framework to place DM particles in a co-moving coordinate system that moves together with the “LPT observer.” Although COLA sacrifices some accuracy on small scales, it can greatly improve computational efficiency by 10–100 times, while the accuracy on relatively large clustering scales is still quite good.

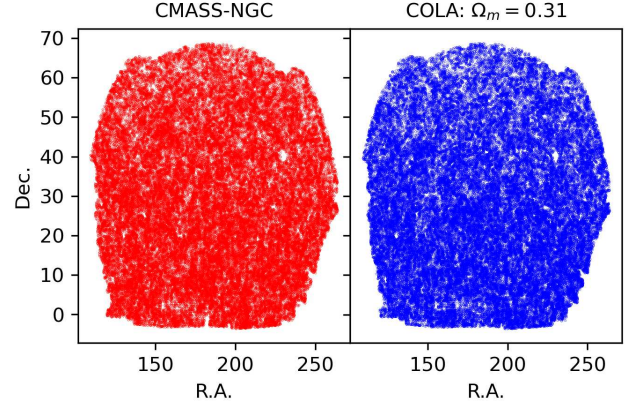


Figure 1. Comparison of the R.A.-decl. distribution of the CMASS-NGC galaxies (left panel) and galaxies in the $\Omega_m = 0.31$ COLA mock (right panel). The 2D shell data sets are derived from the 3D samples within the redshift interval $z \in [0.48, 0.53]$, projected onto the central redshift plane $z_c = 0.505$. The close agreement between the angular distributions of simulated and observed galaxies supports the validity of our simulation framework.

In this study we used the mock SDSS CMASS galaxy catalogs produced in Ding et al. (2024). The mocks are based on a set of simulation assuming a flat Λ CDM with Ω_m varying in region of $0.29 < \Omega_m < 0.33$, and the other parameter values fixed $\Omega_b = 0.048$, $w = -1.0$, $\sigma_8 = 0.82$, $n_s = 0.96$, and $h = 0.67$, which are close to the average constraints of Planck 2015 results (Planck Collaboration et al. 2016). Each set of COLA simulation uses 1024^3 particles in a cubic box with a side length of $800 h^{-1}$ Mpc. Ding et al. (2024) then applied the SHAM method to COLA simulations to efficiently generate mock galaxies. Three parameters, including the scatter magnitude σ_{scat} in SHAM, the initial redshift of the COLA simulations z_{init} and the time step da , are tested to achieve a good match. A good agreement between COLA-simulated galaxies and CMASS-NGC galaxies is achieved on scales ranging from 4 to $20 h^{-1}$ Mpc by setting $z_{\text{init}} = 29$ and $da = 1/30$. Finally, by comparing various statistical properties such as anisotropic 2PCF, three-point clustering, and power spectrum multipoles, the authors found their COLA mocks achieve equally good performance when compared to GADGET mocks (in the region of $s \geq 4 h^{-1}$ Mpc and $k \leq 0.3 h \text{Mpc}^{-1}$), while the computational cost is two orders of magnitude lower than the latter one.

Following the methodology of Yin et al. (2024), in this proof-of-concept analysis we only used three sets of mock simulations with $\Omega_m = (0.25, 0.31, 0.4)$ to assess the constraining power of the statistics. Following the same procedure applied to observational data, we process each simulation by dividing them into six overlapping redshift bins and projecting onto 2D angular planes. For illustration purpose, Figure 1 compares the 2D positions of the observational and mock galaxies, in the $z \in [0.48, 0.53]$ subsample. This figure reveals very good similarity in angular

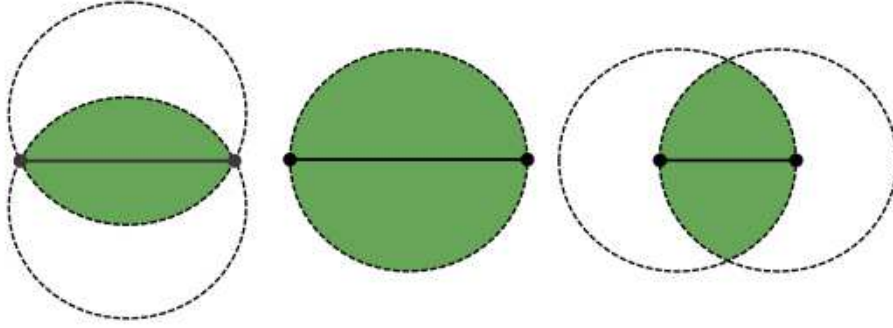


Figure 2. Lune-based definition of the empty regions of the β -skeleton. Left: $\beta < 1$, Center: $\beta = 1$, Right: $\beta > 1$.

galaxy distributions between COLA simulations and CMASS-NGC observations.

2.3. PATCHY Mocks

The PATCHY mock catalogs (Kitaura et al. 2016) combine efficient structure formation models with local, nonlinear, and scale-dependent stochastic biasing to generate realistic DM halo distributions. These mocks are constructed using augmented Lagrangian perturbation theory (2LPT for large scales plus spherical collapse for small scales), which evolves Gaussian density fluctuations into a full DM density field while self-consistently calculating peculiar velocities. The catalogs are carefully calibrated against the high-precision BigMultiDark N -body simulation (Kitaura et al. 2014; Rodríguez-Torres et al. 2016), which contains 3840^3 particles in a volume of $(2.5 h^{-1}\text{Gpc})^3$ and adopts a Λ CDM cosmology with parameters $\Omega_m = 0.307$, $\Omega_b = 0.048$, $\sigma_8 = 0.82$, $n_s = 0.96$, and $h = 0.67$. The calibration is performed using analytic statistical biasing models.

For our analysis, we project the 3D PATCHY simulations into six distinct 2D shell layers that correspond to the CMASS-NGC observational slices. To reduce computational cost, we use only the first 300 realizations out of the total 2048 PATCHY mocks, which is sufficient for this proof-of-concept study. These mocks are then employed to compute the covariance matrices for the MACFs.

3. Methodology

In this section, we present our methodology. We begin by outlining the theoretical framework and construction process of the β -skeletons, followed by an introduction to our novel mark-weighted correlation function statistics.

3.1. Two-dimensional β -cosmic-web

The β -skeleton algorithm originates from the fields of computational geometry and geometric graph theory. It is similar to the minimum spanning tree (MST) algorithm (Barrow et al. 1985), with a key distinction being that the structural

graph constructed relies on β -parameter, which makes it flexible in handling point sets with complex neighbor relationships (Kirkpatrick & Radke 1985; Correa & Lindstrom 2013).

The definition of β -skeleton is as follows. For a point set S in n -dimensional Euclidean space, any two points p and q in the point set S are considered connected if there is no third point in the empty regions, which is illustrated in Figure 2⁷:

1. For $0 < \beta < 1$, the empty region is formed by the intersection of two spheres with diameter d_{pq}/β , having p and q on their boundaries, where d_{pq} is the common chord of the spheres.
2. For $\beta = 1$, the empty region becomes a sphere with diameter d_{pq} .
3. For $\beta > 1$, there are two different definitions of the empty region: the Circle-based definition and Lune-based definition. In this study we adopt the Lune-based definition, i.e., the empty region is the intersections of two spheres with a diameter of βd_{pq} and their centers located at $p + \beta(q - p)/2$ and $q + \beta(p - q)/2$, respectively.

The β -skeleton defined in this way exhibits a series of unique and interesting mathematical properties:

1. *Hierarchical nature.* As β varies from 0 to ∞ , the constructed β -skeleton graph transitions from a complete graph to an empty graph. In particular, the graphs formed by the special case of $\beta = 1$ is called Gabriel Graph, which is known to contain the Euclidean minimum spanning tree, while in image analysis the $\beta = 1.7$ Circle-based graphs are found to accurately reconstruct the entire boundary of any smooth surface, without generating any edges that do not belong to the boundary, as long as the samples are sufficiently dense with respect to the local curvature of the surface.
2. *Inclusion relation.* For any parameter value satisfying $\beta_1 < \beta_2$, for fixed points p and q , the empty region of the

⁷ The β -skeleton code utilized in this study can be accessed at the following link: <https://github.com/xiaodongli1986/LSSCode>.

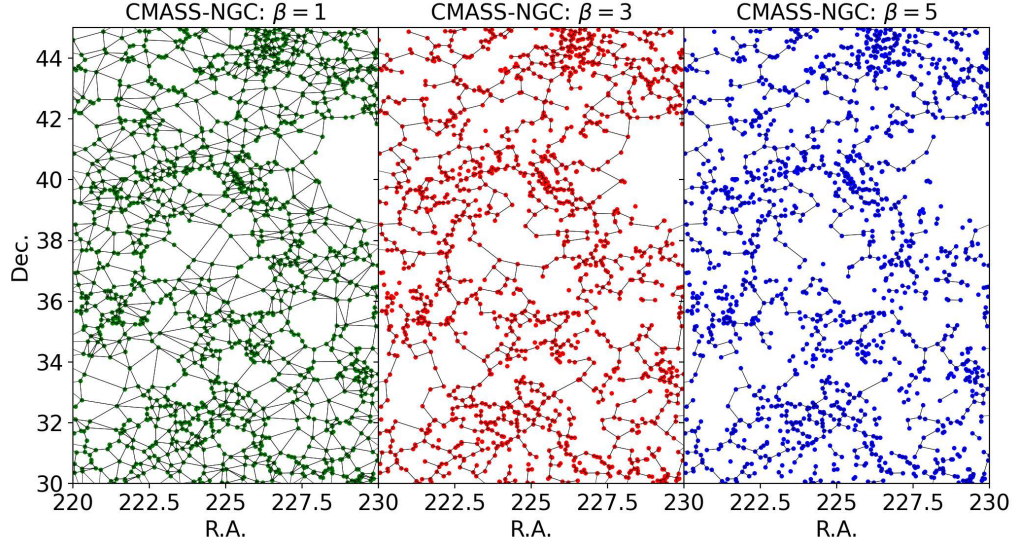


Figure 3. Visual comparisons of 2D β -cosmic-webs from CMASS-NGC data, with β set to 1, 3 and 5, respectively. The structures are derived from galaxies with $z \in [0.48, 0.53]$. Following Yin et al. (2024), we adopt the $\beta = 3$ results as our baseline analysis, as its topological structure closely agrees with our conceptual understanding of the cosmic web.

β -skeleton with β_2 contains the empty region of the β -skeleton with β_1 as the set parameter value. This property ensures that the connectivity of the web gradually decreases as increasing β .

3. *Parametric control.* The parameter β provides an intuitive and powerful “sparse-dense” adjustment knob allowing researchers to finely tune the web’s connection pattern according to the specific needs of scientific research, so that to conveniently visualize the web structures of the set.
4. *Interpretability.* β -skeleton is directly based on the geometric distribution of matter, which makes the results of the model highly intuitive and understandable.

In Figure 3, we visualize the 2D β -cosmic-web for CMASS-NGC galaxy data using $\beta = 1, 3$, and 5. The data set includes galaxies within the redshift interval $[0.48, 0.53]$. For visualization purposes, we plot only 1500 galaxies with R.A. in $[220^\circ, 230^\circ]$ and decl. in $[30^\circ, 45^\circ]$, corresponding to a slice of approximately $236.1 \times 354.1 \text{ } (h^{-1}\text{Mpc})^2$. We observe that $\beta = 1$ produces a network with dense connections, whereas for $\beta = 5$, the connections become significantly sparser. Specifically,

1. The $\beta = 1$ web has 2695 links (overconnected). It contains a substantial number of extraneous and unnecessary galaxy connections that do not reflect true connection patterns.
2. The $\beta = 3$ web yields 1467 connections (nearly a 1:1 ratio with the number of galaxies) and is considered to realistically capture the connection relationships among galaxies.

3. The $\beta = 5$ web reduces to 1150 links (relatively sparse), omitting rich structural details.

Following Yin et al. (2024), we adopt the $\beta = 3$ web as the foundation for our analysis, as its topological structure aligns well with the commonly accepted conceptualization of the cosmic web.

Figure 4 displays the $\beta = 3$ webs derived from CMASS-NGC data and COLA simulations with varying Ω_m values (0.31, 0.25, and 0.4). Consistent with Figure 3, only galaxies within the redshift range $z \in [0.48, 0.53]$, R.A. $\in [220^\circ, 230^\circ]$, and decl. $\in [30^\circ, 45^\circ]$ are shown. This figure shows that the results of the $\Omega_m = 0.31$ mock are statistically most consistent with the observations. In terms of connectivity, the cosmic web constructed from the CMASS-NGC subsample exhibits 1467 connections. The mock catalog for $\Omega_m = 0.31$ yields a comparable number, with 1534 connections. In contrast, larger discrepancies are observed for the mocks with $\Omega_m = 0.25$ and 0.4, which produce 1548 and 1359 connections, respectively.

We compare the β -cosmic-web statistics derived from COLA simulations and observational data in Figure 5. Although some differences are present, they are relatively minor—intuitively, the curves lie too close to each other to reliably distinguish between different cosmological models. Moreover, the shapes of the probability distributions of L_{con} and $\bar{D}_{\text{nei}}$ are influenced by various systematics, making direct cosmological inference difficult (Fang et al. 2019; García-Alvarado et al. 2020). As a result, the corresponding probability distributions are not well-suited for placing strong cosmological constraints. Instead, the most promising application of the β -cosmic-web in cosmological analysis remains the MCFs (Yin et al. 2024).

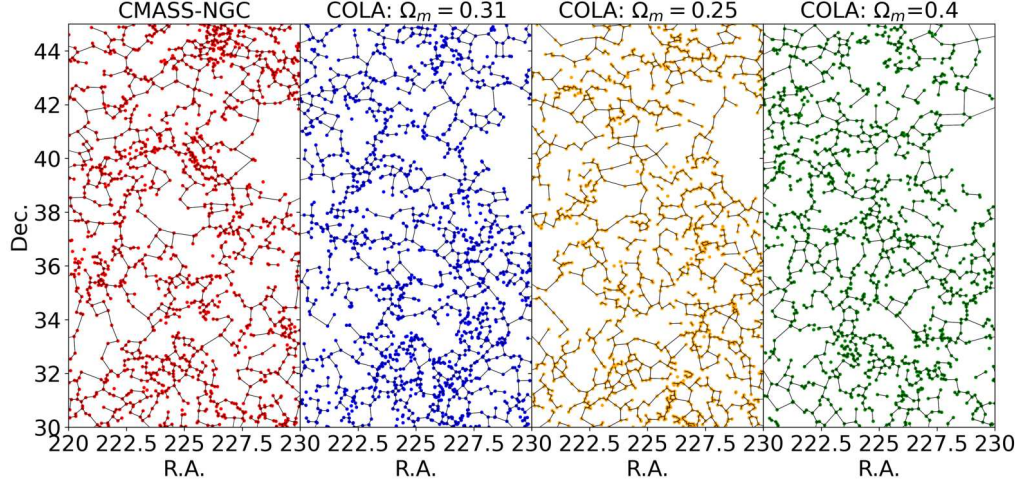


Figure 4. The $\beta = 3$ web of the CMASS-NGC galaxies and the COLA mocks with $\Omega_m = 0.31, 0.25, 0.4$. In all plots we chose galaxies with $z \in [0.48, 0.53]$, R.A. $\in [220^\circ, 230^\circ]$ and decl. $\in [30^\circ, 45^\circ]$. By comparing the connectivity of these webs, we find COLA simulation with $\Omega_m = 0.31$ shows the best agreement with the observational data.

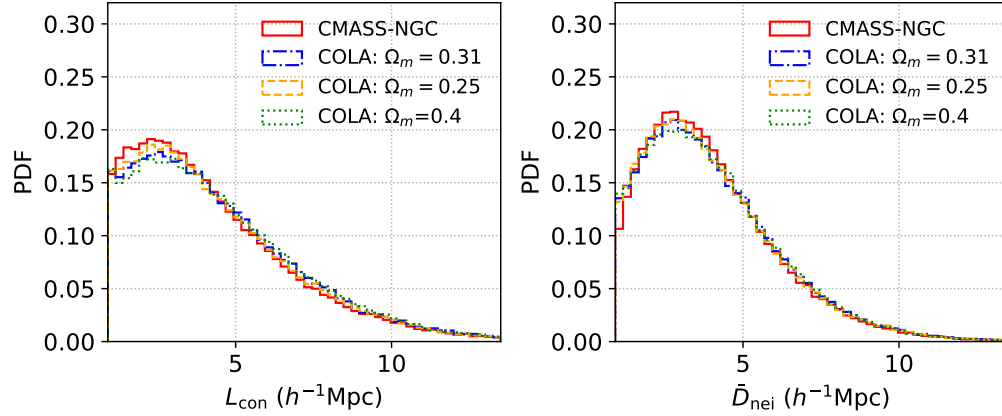


Figure 5. Comparison of the probability distributions of connection length, L_{con} (left), and the averaged connection length between each galaxy and its neighboring galaxies, $\bar{D}_{\text{nei}}$ (right), in 2D β -cosmic-webs of $\beta = 3$ for CMASS-NGC and COLA simulations with $\Omega_m = 0.31, 0.25$, and 0.4 .

3.2. Mark Weighted Angular Correlation Functions Using 2D β -cosmic-web

In brief, the procedure for computing MCFs follows the standard 2PCF framework, with the key difference being the incorporation of weights derived from the β -skeleton statistics. While the traditional 2PCF is defined as

$$\xi(\mathbf{r}) = \langle \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}) \rangle, \quad (1)$$

the MCF takes the form

$$W(\mathbf{r}) = \langle \delta(\mathbf{x}) f(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}) f(\mathbf{x} + \mathbf{r}) \rangle, \quad (2)$$

where $f(\mathbf{x})$ represents the assigned weights, and $\delta(\mathbf{x}) = \delta\rho/\bar{\rho}$ denotes the number density contrast.

Following the methodologies developed in Yin et al. (2024), we use three different quantities in the weighting:

1. Connection number (N_{con}): the number of neighboring galaxies around each galaxy.
2. Mean neighbor distance ($\bar{D}_{\text{nei}}$): the average distance between each galaxy and its neighboring galaxies.
3. Inversed neighbor distance ($1/\bar{D}_{\text{nei}}$): the reciprocal of mean neighbor distance.

Therefore, the weighting scheme is defined as

$$f(\mathbf{x}) = \begin{cases} N_{\text{con}}(\mathbf{x}), \\ \bar{D}_{\text{nei}}(\mathbf{x}), \\ 1/\bar{D}_{\text{nei}}(\mathbf{x}), \end{cases} \quad (3)$$

where different choices correspond to different physical interpretations: the number of connections N_{con} , the average neighbor distance $\bar{D}_{\text{nei}}$, or its reciprocal. Isolated galaxies (i.e., those without neighbors) are assigned a weight of $f(\mathbf{x}) = 0$. In particular, the reciprocal distance measure $1/\bar{D}_{\text{nei}}$ in 2D

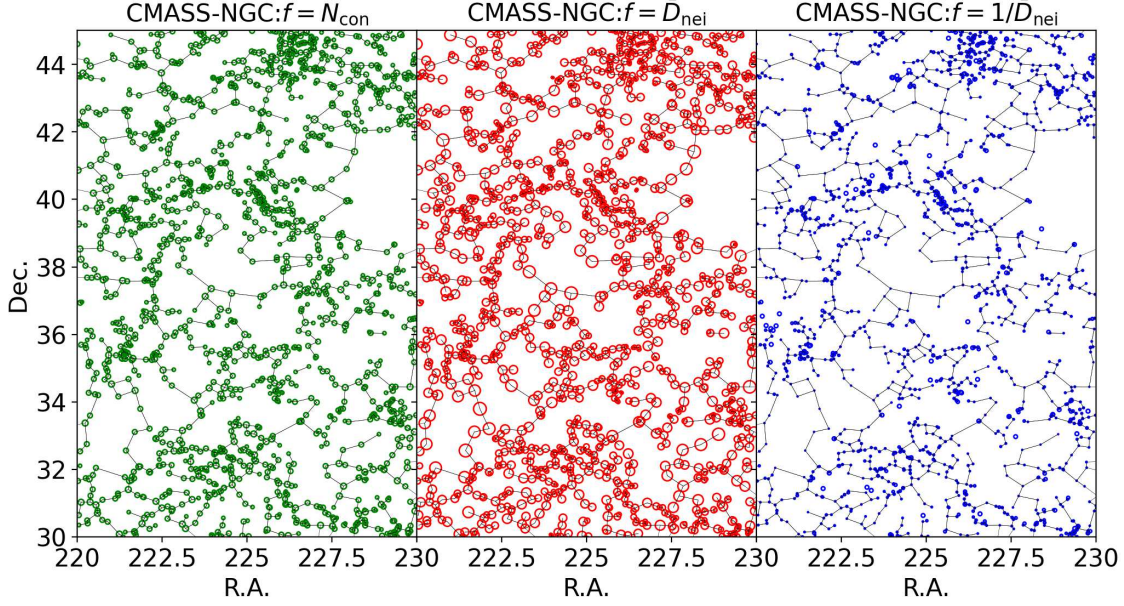


Figure 6. Visualization of the 2D β -cosmic-web weights for CMASS-NGC galaxies with $\beta = 3$, represented by circle sizes. The region spans R.A. $\in [220^\circ, 230^\circ]$ and decl. $\in [30^\circ, 45^\circ]$. The weights are derived from a 2D shell projected from 3D spatial data within redshift slice $\Delta z = 0.05$ at $z \in [0.48, 0.53]$, centered at $z_c = 0.505$. Three weighting schemes are shown side-by-side, from left to right: N_{con} , $\bar{D}_{\text{nei}}$, and $1/\bar{D}_{\text{nei}}$, corresponding respectively to uniform weighting, emphasis on low-density environments, and strong prioritization of high-density structures.

projections can amplify extreme values and potentially lead to divergence. To ensure numerical stability, we introduce a cutoff threshold f_{cut} , defined as the 95th percentile of the $1/\bar{D}_{\text{nei}}$ distribution. Values exceeding this threshold are truncated at f_{cut} , effectively suppressing outliers while preserving key nonlinear clustering features.

Figure 6 visualizes 2D β -cosmic-web weights of CMASS-NGC observational galaxies. The weights of galaxies are denoted by the circle sizes. In different weighting schemes, we find different environment-dependence of the weights. In the left panel, the sizes of the circles are not very different from each other, indicating that the dependence of $N_{\text{con}}(\mathbf{x})$ on the environment is weak. When $\bar{D}_{\text{nei}}(\mathbf{x})$ is used as weight, it upweights the low-density regions, while its inverse $1/\bar{D}_{\text{nei}}(\mathbf{x})$ highlights the overdense regions.

Based on these weights, MCFs are then computed through Landy-Szalay estimator (Landy & Szalay 1993):

$$W(s, \mu) = \frac{WW - 2WR + RR}{RR}, \quad (4)$$

where WW denotes the weighted count of galaxy-galaxy pairs, WR corresponds to galaxy-random pairs, and RR represents random-random pairs. s represents the distance between pairs, and μ is defined as cosine of the angle between the line of sight (LOS) direction and the line connecting the pair galaxies.

The monopole of MCF, which only depends on the clustering scale s , is obtained by integrating $W(s, \mu)$ over all

angles:

$$W_0(s) = \int_0^1 W(s, \mu) d\mu. \quad (5)$$

Assuming a flat cosmological model, the comoving distance between galaxies at redshifts z_1 and z_2 can be expressed as

$$s = |\mathbf{s}| = \sqrt{r^2(z_1) + r^2(z_2) - 2r(z_1)r(z_2)\cos\theta}, \quad (6)$$

where $r(z_1)$, $r(z_2)$ represent the comoving distances from the observer for two galaxies at redshifts of z_1 and z_2 , respectively, and θ is the angular distance between galaxy pairs. In Λ CDM model, the comoving distance $r(z)$ of a galaxy with redshift z takes the form of,

$$r(z) = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_m(1+z')^3}}, \quad (7)$$

where H_0 is the present-day Hubble constant, and Ω_Λ is the dark energy density parameter.

Under the flat-sky approximation and thin redshift slice assumption, for a set of galaxies with the center of redshift bin denoted as z_c , the angular component of the comoving distance between galaxy pairs take the form of

$$\begin{aligned} s &\approx \sqrt{2r^2(z_c) - 2r^2(z_c)\cos\theta} \\ &= 2r(z_c)\sin\left(\frac{\theta}{2}\right) \approx r(z_c)\theta, \end{aligned} \quad (8)$$

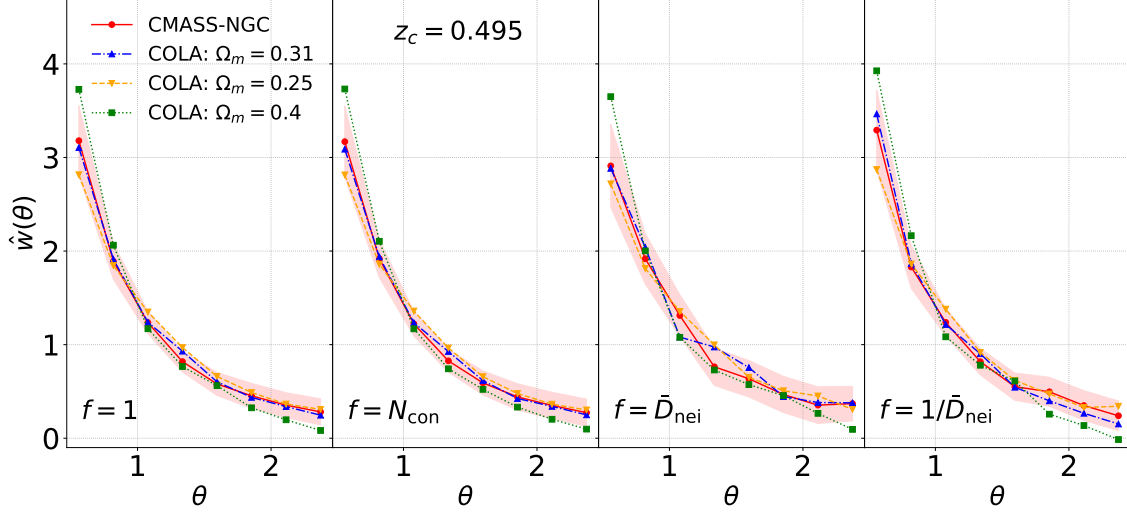


Figure 7. Comparison of normalized angular correlation functions $\hat{w}(\theta)$ for CMASS-NGC (red) and COLA simulations with $\Omega_m = 0.31$ (blue), 0.25 (orange), 0.4 (green) using galaxies with $z \in [0.47, 0.52]$, computed in the range of $\theta \in [0.45, 2.52]$. The shaded region represents 3σ confidence interval. Results of the $\Omega_m = 0.31$ COLA mock are most consistent with the observational data, while both $\Omega_m = 0.25$ and $\Omega_m = 0.4$ show significant discrepancies ($>3\sigma$) from the deviation from the CMASS-NGC data.

where θ represents the angular separation between the two galaxies. The last approximation is valid when $\theta \ll 1$, which holds in our analysis.

Since we consider a fixed redshift interval of $\Delta z = 0.05$ and angular separations across six redshift bins, all within the range $\theta \in [0.43^\circ, 2.62^\circ]$, the flat-sky approximation and thin redshift slice assumptions are well satisfied. Consequently, the clustering scale s is directly related to the angular separation θ between two galaxies, as given by Equation (8).

In this study, we focus exclusively on the monopole component, W_0 . The 2D β -cosmic-web-based MACFs, denoted by $\hat{w}(\theta)$, are computed using a normalized form (to mitigate the influence of galaxy bias and improve the accuracy of the analysis) defined as

$$\hat{w}(\theta) = \frac{W_0(s)}{\int_a^b W_0(s) ds}, \quad (9)$$

where $\theta = s/r(z_c)$, in accordance with Equation (8). We examined the impact of varying the integration limits a and b , and adopted $a = 10 h^{-1}\text{Mpc}$ and $b = 58 h^{-1}\text{Mpc}$ as our fiducial values. This range reflects a conservative compromise, chosen to retain meaningful clustering information while mitigating potential systematic uncertainties. For the analysis, the clustering separation s is divided into eight bins of equal width.

Figure 7 compares the $\hat{w}(\theta)$ values measured from CMASS-NGC galaxies (red) and from COLA mocks with $\Omega_m = 0.31$ (blue), 0.25 (orange), and 0.4 (green), using galaxies in the redshift range $z \in [0.47, 0.52]$. Measurements from the standard 2PACF and the MACFs weighted by N_{con} , $\bar{D}_{\text{nei}}$, and $1/\bar{D}_{\text{nei}}$ are shown. Note that, the case $f=1$ is equivalent to

2PACF measurements. COLA with $\Omega_m = 0.31$ aligns with CMASS-NGC within the 3σ confidence interval, while $\Omega_m = 0.25$ or $\Omega_m = 0.4$ simulations show evident discrepancy from the data. The trend is similar for other redshift intervals (e.g., Figure 8 compares the measurements of galaxies with $z \in [0.48, 0.53]$).

To assess the constraining power of different statistics, we define the χ^2 as

$$\chi^2 = (\Delta \mathbf{p})^T \cdot \mathbf{C}^{-1} \cdot \Delta \mathbf{p}, \quad (10)$$

where $\Delta \mathbf{p}$ characterizes the difference between the measurements from the data ($\mathbf{p}_{\text{data}}$) and the COLA mocks ($\mathbf{p}_{\text{model}}$)

$$\Delta \mathbf{p} = \mathbf{p}_{\text{model}} - \mathbf{p}_{\text{data}}. \quad (11)$$

The covariance matrix corresponding to the normalized weighted angular correlation function statistic $\hat{w}(\theta)$ is represented by $\mathbf{C}$. By varying the value of Ω_m , the variation of the χ^2 values can be utilized to evaluate the sensitivity of a specific statistic to the cosmological parameter, thereby enabling us to compare the power of different statistics. The covariance matrix is computed through

$$\mathbf{C} = \langle (\mathbf{p} - \bar{\mathbf{p}})(\mathbf{p} - \bar{\mathbf{p}})^T \rangle \quad (12)$$

$$= \frac{1}{N_{\text{mock}} - 1} \sum_{i=1}^{N_{\text{mock}}} (\mathbf{p}_i - \bar{\mathbf{p}})(\mathbf{p}_i - \bar{\mathbf{p}})^T. \quad (13)$$

Here, the vector $\mathbf{p}_i$ denotes the i -th PATCHY mock out of a total of N_{mock} mocks, and $\bar{\mathbf{p}}$ represents the average of $\mathbf{p}$ over all PATCHY mocks. In this analysis, we employ 300 PATCHY simulation sets to estimate the covariance matrix. The vector $\mathbf{p}$ collects all measured quantities. For a given z -bin, each statistic yields eight measured quantities, so combining two

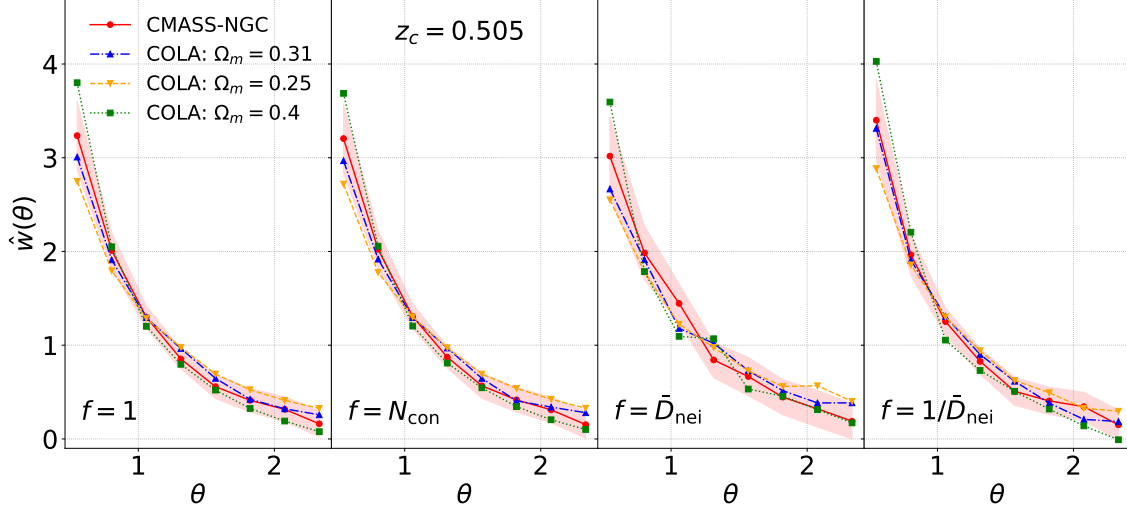


Figure 8. Same as in Figure 7, but for measurements in the redshift interval $[0.48, 0.53]$. We still find that the mock with $\Omega_m = 0.31$ best agrees with the observational data.

weighting schemes results in a total of 16 measured elements. Therefore, the number of PATCHY simulations is sufficient to accurately estimate the covariance matrix.

To reduce statistical fluctuations and enhance the robustness of our results, we use the average of the χ^2 values from six subsamples (each covering different but overlapping redshift ranges) to assess the performance of different statistics:

$$\overline{\chi^2} = \frac{1}{n} \sum_{i=1}^n \chi_i^2, \quad (14)$$

where $n = 6$, and χ_i^2 denotes the chi-square value calculated for the i th individual 2D shell of galaxies.

4. Results

Figure 9 shows the correlation coefficient of the $z \in [0.48, 0.53]$ subsample. To precisely quantify the relationships between different statistical measures, the figure displays both the autocorrelation of the correlation functions and the cross-correlation coefficients between the standard 2PACF and the $\hat{w}(\theta)$ weighted by N_{con} , $\bar{D}_{\text{nei}}$, and $1/\bar{D}_{\text{nei}}$, respectively. Among them, the MACF weighted by N_{con} is most correlated with 2PACF, since this weighting scheme is most close to uniform weighting.

The sensitivity of $\hat{w}(\theta)$ to a cosmological parameter is then evaluated using

$$\Delta\chi^2 \equiv \chi^2 - \chi_{\min}^2, \quad (15)$$

which directly measures the statistical power to discriminate between different cosmologies. To enhance the stability and robustness of our results, we compute the mean $\Delta\chi^2$ cross the six overlapping redshift bins, defined as:

$$\Delta\overline{\chi^2} \equiv \overline{\chi^2} - \overline{\chi_{\min}^2}, \quad (16)$$

where $\overline{\chi^2}$ and $\overline{\chi_{\min}^2}$ represent the average χ^2 values over all bins, as defined in Equation (14). This averaging procedure serves two purposes: (1) to reduce statistical fluctuations arising from the relatively small sample size, and (2) to improve robustness against random variations in our proof-of-concept analysis, which currently includes only three simulations. By adopting this approach, we effectively suppress statistical noise while preserving sensitivity to cosmological parameters.

Figure 10 presents the variation of $\Delta\overline{\chi^2}$ as Ω_m varies from 0.25 to 0.4. In all cases we find $\Omega_m = 0.31$ leads to the minimal χ^2 , so it always serves as the baseline. We find that:

1. The incorporation of weighted statistics provides significant improvement over using the 2PACF alone, as evidenced by the enhanced $\Delta\overline{\chi^2}$. This demonstrates greater statistical power for constraining cosmological parameters.
2. Among all tested combinations, the most sensitive probe of Ω_m is achieved by combining 2PACF with the $\hat{w}(\theta)$ statistic weighted by $1/\bar{D}_{\text{nei}}$ (red squares in Figure 10), enlarging the value of $\Delta\chi^2$ by 2–3 times.

To quantitatively assess the improvement gained by incorporating β -cosmic-web-weighted $\hat{w}(\theta)$ statistics compared to the conventional 2PACF approach, we introduce the relative sensitivity metric:

$$r = \frac{\Delta\overline{\chi^2}}{\Delta\overline{\chi^2}_{\text{2PACF}}} - 1, \quad (17)$$

where $\Delta\overline{\chi^2}_{\text{2PACF}}$ refers to the results obtained using only the 2PACF measurements, that is, by calculating $\hat{w}(\theta)$ using the weighting scheme $f = 1$ as defined in Equation (9).

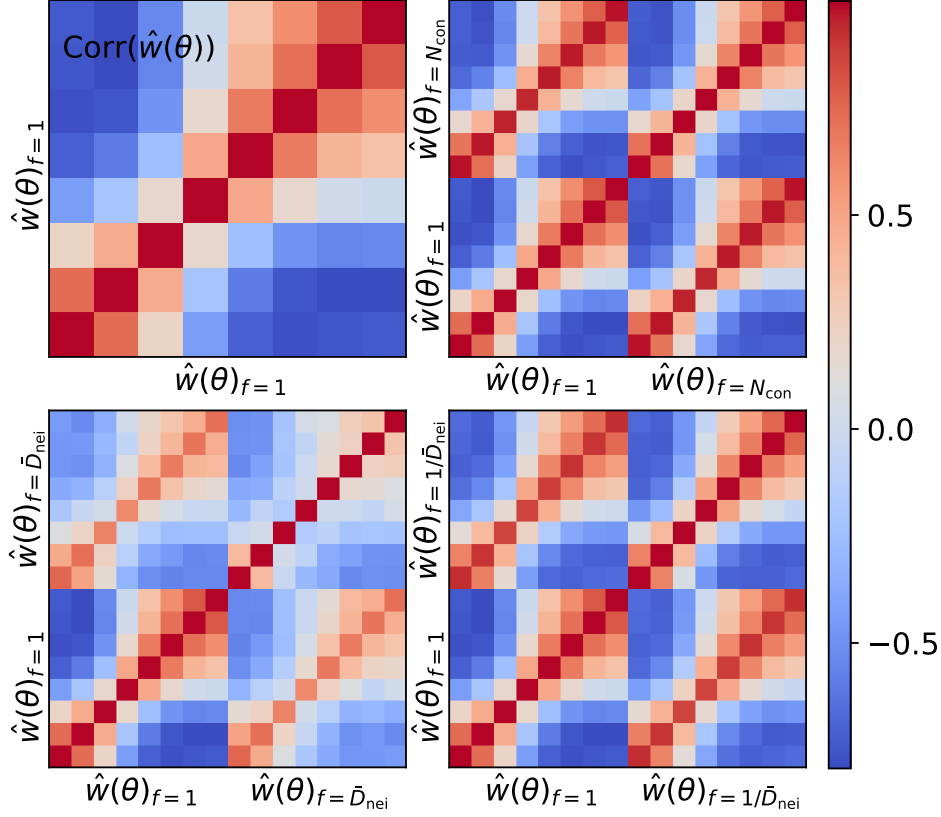


Figure 9. Normalized covariance matrix showing the correlation coefficients of $\hat{w}(\theta)$ computed for various combinations with different weighting schemes, obtained from PATCHY simulations. Here, we show results only for the sample with $z \in [0.48, 0.53]$.

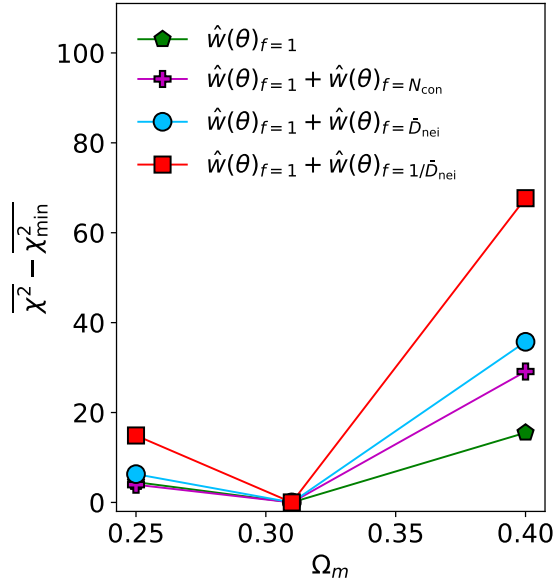


Figure 10. The variation of $\Delta\chi^2$ as Ω_m changes from 0.25 to 0.4. It reveals a minimum $\chi^2_{\min}$ at $\Omega_m = 0.31$, distinctively lower than values at other two Ω_m . Combination of 2PACF and $\hat{w}(\theta)$ weighted by $1/\bar{D}_{\text{nei}}$ (represented by red squares) yields to the highest sensitivity to Ω_m .

Table 1

Sensitivity of Each Statistic to Ω_m is Quantified by $\Delta\bar{\chi}^2$ and the Relative Deviation r (Equation (17))

Statistic	$\Delta\bar{\chi}^2 / r (\Omega_m = 0.25)$	$\Delta\bar{\chi}^2 / r (\Omega_m = 0.4)$
$\hat{w}(\theta)_{f=1}$	4.52 / 0%	15.51 / 0%
$\hat{w}(\theta)_{f=1} + \hat{w}(\theta)_{f=N_{\text{con}}}$	3.94 / -13%	29.12 / +88%
$\hat{w}(\theta)_{f=1} + \hat{w}(\theta)_{f=\bar{D}_{\text{nei}}}$	6.30 / +39%	35.73 / +130%
$\hat{w}(\theta)_{f=1} + \hat{w}(\theta)_{f=1/\bar{D}_{\text{nei}}}$	14.89 / +229%	67.68 / +336%

Note. The percentage values indicate the improvement in $\Delta\bar{\chi}^2$ relative to the 2PACF results, indicated by $\hat{w}(\theta)_{f=1}$. The $1/\bar{D}_{\text{nei}}$ -weighted $\hat{w}(\theta)$ yields the largest enhancement, with improvements of 229% at $\Omega_m = 0.25$ and 336% at $\Omega_m = 0.4$.

The statistical improvements are summarized in Table 1. Our analysis reveals that combining 2PACF with MACFs significantly enhances the statistical confidence. Specifically:

1. Incorporating $\bar{D}_{\text{nei}}$ -weighted statistics boosts the $\Delta\bar{\chi}^2$ values by 39%-130%, indicating substantially improved parameter constraints.
2. The $1/\bar{D}_{\text{nei}}$ weighting scheme shows even more dramatic improvement, enhancing $\Delta\bar{\chi}^2$ by 229%-336%.

Statistically, this enhancement is equivalent to increasing our data set by the same amount.

5. Concluding Remarks

In this study, we investigate whether cosmological constraints can be improved using the β -cosmic-web weighted angular correlation functions. We compare the statistical measurements from the SDSS DR12 CMASS-NGC galaxies and the COLA mocks with different values of Ω_m (0.25, 0.31 and 0.4), and use different statistics to distinguish between these Λ CDM cosmologies. We quantify the statistical power of each method using the value of $\Delta\chi^2$.

Our results show that adding the β -cosmic-web weighted statistics can effectively enhance the statistical power. Specifically, weighting the statistics by the mean neighbor distance ($\bar{D}_{\text{nei}}$) increases the $\Delta\chi^2$ values by approximately 40%–130%. An even more significant enhancement is achieved with the inverse mean neighbor distance weighting ($1/\bar{D}_{\text{nei}}$), which boosts $\Delta\chi^2$ by a factor of 2–3 (corresponding to an improvement of about 230%–340%) compared to unweighted traditional angular statistics.

The procedure of this analysis follows that of Yin et al. (2024), who investigated the statistical power of β -cosmic-web weighted correlation functions. Their results showed that including a single weighted statistic enhances $\Delta\chi^2$ by factors of a few, with multiple weights yielding improvements approaching an order of magnitude. Consistent with these findings, the present study also demonstrates that the $1/\bar{D}_{\text{nei}}$ weighting scheme performs best, indicating that dense regions carry substantial cosmological information. In contrast, weighting by N_{con} provides minimal improvement. Although this study focuses on constraints of Ω_m , it is expected that these statistics are sensitive to other cosmological parameters, such as those related to dark energy (Riess et al. 1998; Perlmutter et al. 1999; Weinberg 1989; Li et al. 2011; Yoo & Watanabe 2012; Weinberg et al. 2013; Gu et al. 2024a; Wang et al. 2024a).

The statistics developed in this study are based on thin redshift slices, making them well-suited for slitless spectroscopic surveys (e.g., Euclid, CSST) and wide-band photometric surveys, where redshift uncertainties hinder traditional 3D clustering. The 2D angular correlation function offers greater robustness to such uncertainties, enabling broader applicability. Additionally, since marked statistics have not been previously explored using 2D angular clustering, this study provides new theoretical insights.

In this proof-of-concept study, we only considered three weighting schemes. Since the dependence of clustering on the environment is complex and varied, we expect cosmological constraints could be further enhanced by exploring additional weighting approaches. For instance, the weighting scheme ρ^α ,

which assigns weights based on the local density raised to the power α , emphasizes either dense or underdense regions, improving the contrast between galaxy clusters in different environments and yielding tighter constraints (Yang et al. 2020; Lai et al. 2024). Alternatively, weights based on the density gradient, $(|\nabla\rho|/\rho)^\alpha$, or combinations of both schemes can be used in 2D marked angular statistics (Xiao et al. 2022). Additionally, cosmological constraints can be improved by employing reconstructed density fields (Wang et al. 2024b; Shi et al. 2025b). To further advance slitless survey analyses, machine learning methods and scattering transforms applied to 2D redshift-sliced data also deserve exploration. These avenues will be pursued in future study.

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