



Pixel-level Modeling of Group-scale Strong Lens CASSOWARY 19

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Received 2025 April 8; revised 2025 April 18; accepted 2025 April 21; published 2025 May 30

Abstract

We present the first high-precision model for the group-scale strong lensing system CASSOWARY 19 (CSWA19), utilizing images from the Hubble Space Telescope. Sixteen member galaxies identified via the red-sequence method, and the main halo, all modeled as the dual Pseudo Isothermal Elliptical profile, are incorporated into a parametric lens model alongside an external shear field. To model the system, we adopt the PYAUTOLENS software package, employing a progressive search chain strategy for realizing the transition of source model from multiple Sérsic profiles to a brightness-adaptive pixelization, which uses 1000 pixels in the source plane to reconstruct the background source corresponding to 177,144 image pixels in the image plane. Our results indicate that the total mass within the Einstein radius is $M_{\theta_E} \approx 1.41 \times 10^{13} M_{\odot}$ and the average slope of the total mass density $\rho(r) \propto r^{-\gamma}$ is $\bar{\gamma} = 1.33$ within the effective radius. This slope is shallower than those measured in galaxies and groups but is closer to those of galaxy clusters. In addition, our approach successfully resolves the two merging galaxies in the background source and yields a total magnification of $\mu = 103.18^{+0.23}_{-0.19}$, which is significantly higher than the outcomes from previous studies of CSWA19. In summary, our research demonstrates the effectiveness of the brightness-adaptive pixelization source reconstruction technique for modeling group-scale strong lensing systems. It can serve as a technical reference for future investigations into pixel-level modeling of the group- and cluster-scale strong lensing systems.

Key words: galaxies: groups: general – gravitational lensing: strong – (cosmology:) dark matter

1. Introduction

Strong gravitational lensing (hereafter strong lensing) occurs when a massive foreground object bends spacetime and deflects light rays, producing multiple images or distorted arcs of background sources. This phenomenon was first predicted by Einstein (1936). Strong lensing systems are typically categorized by mass and scale into three types: galaxy-scale, group-scale, and cluster-scale. Quantitatively, the classification is based on the halo mass M_{200} . For instance, systems with M_{200} greater than $10^{14} M_{\odot}$ are classified as cluster-scale lenses, while those with M_{200} less than or equal to $10^{13} M_{\odot}$ are considered galaxy-scale lenses. Group-scale lenses occupy the intermediate range. The boundaries between these categories—particularly between group- and cluster-scale lenses—are not strictly defined. In general, systems with a large number of member galaxies tend to have higher M_{200} . A strong lensing system is typically considered cluster-scale if it contains more than 25 member galaxies (Wang et al. 2024). In this study, we define systems with more than one but fewer than 25 member galaxies as group-scale lenses, based on the

complexity of lens modeling. It is important to note, however, that having fewer than 25 member galaxies does not necessarily imply $M_{200} < 10^{14} M_{\odot}$.

Strong lensing serves as a unique astrophysical laboratory with a wide range of scientific applications. One key application is measuring time delays between multiple images of strongly lensed quasars or supernovae, which provides a powerful probe to constraining the Hubble constant (Refsdal 1964; Schechter et al. 1997; Fassnacht et al. 2002; Fohlmeister et al. 2007; Suyu et al. 2010; Kelly et al. 2015; Millon et al. 2020; Wong et al. 2020; Shajib et al. 2023; Pascale et al. 2024). Besides, the statistics of strong lensing can be used in the cosmological analyses to estimate the cosmic curvature and other cosmological parameters (Turner et al. 1984; Biesiada 2006; Cao & Zhu 2012; Cao et al. 2012; Chen et al. 2019; Wang et al. 2020; Wei et al. 2022; Li & Chen 2023). As a natural cosmic telescope, strong lensing magnifies background sources, enabling the detailed study of distant galaxy structures (Yuan et al. 2011; Wuyts et al. 2014; Jones et al. 2015; Johnson et al. 2017b, 2017a; Cava et al. 2018; Dunham et al. 2019;

Meštrić et al. 2022; Sharon et al. 2022; Stacey et al. 2025). In addition, strong lensing is highly sensitive to the mass distribution of lensing objects, offering an independent and robust probe of both dark and luminous matter (Dalal & Kochanek 2002; Vegetti & Koopmans 2009; Suyu & Halkola 2010; Vegetti et al. 2010a; Hezaveh et al. 2016; Schuldt et al. 2019; Gilman et al. 2020; Meneghetti et al. 2020; Du et al. 2020, 2023; Wang et al. 2022, 2024; Lange et al. 2024). For example, the Sloan Lens ACS Survey (SLACS) has modeled a large sample of galaxy-scale lenses and found that their average total density slope γ is close to 2 (Auger et al. 2010), whereas cluster-scale systems typically exhibit shallower profiles (Newman et al. 2013b). Newman et al. (2015) extended this picture by incorporating group-scale lensing systems, suggesting that the variation in total density slopes reflects scale-dependent interactions between dark matter and baryons. Group-scale strong lensing systems thus represent a crucial intermediate regime between galaxy- and cluster-scale lenses. Studying their dark matter distributions and galaxy properties provides valuable insight into the physical processes governing structure formation across cosmic scales.

Several surveys have targeted group-scale strong lensing systems, including the Sloan Bright Arcs Survey (Diehl et al. 2009; Kubo et al. 2009), the CFHTLS-Strong Lensing Legacy Survey—Arcs (Limousin et al. 2009a; More et al. 2012), and the Cambridge Sloan Survey of Wide ARcs in the Sky (CASSOWARY) (Belokurov et al. 2009; Stark et al. 2013). Among them, CASSOWARY is a catalog of strong lensing arcs identified from the Sloan Digital Sky Survey (SDSS), with a typical lens redshift of ~ 0.4 . High-precision modeling is essential for detailed investigations of group-scale lenses. However, owing to the complex configurations often found in these systems, a standardized framework for their modeling is currently lacking. In this work, we introduce a practical modeling framework specifically designed for group-scale strong lensing systems. In galaxy-scale lens modeling, extended image pixels have long been used to constrain the parametric source model (Auger et al. 2009; Shu et al. 2016, 2017; Cao et al. 2025). With recent advances in modeling techniques, non-parametric pixelated source reconstructions have become widely adopted in the field of galaxy-scale strong lensing. These models can fit lensed image features down to the noise level and leverage thousands of pixels along the arcs to tightly constrain the lens mass distribution (Warren & Dye 2003; Suyu et al. 2006; Nightingale & Dye 2015; He et al. 2024; Nightingale et al. 2024). Historically, modeling of group- or cluster-scale strong lenses typically relied solely on the positions of multiple lensed images to constrain the lens model. This approach was primarily adopted due to computational resource limitations. As a result, the wealth of information contained within the surface brightness distribution of the lensed images was not fully exploited (although recent studies have started to incorporate this detailed information, see e.g., Acebron et al. 2024; Urcelay et al. 2024; Xie et al. 2024).

In this work, we select CASSOWARY 19 (hereafter CSWA19) as a case study to demonstrate the pixel-level modeling of a group-scale strong lensing system. CSWA19, with Sloan ID SDSS J090002.79+223403.6, was first identified by Diehl et al. (2009), who measured the source redshift and modeled the lens mass using a singular isothermal sphere (SIS), obtaining an Einstein radius of $7''.0 \pm 0.8$ and an enclosed mass of $(11.6 \pm 2.7) \times 10^{12} M_{\odot}$. Stark et al. (2013) later adopted the same SIS model and reported a magnification of $\mu = 6.5$. Subsequently, Leethochawalit et al. (2016) employed the Light-Trace-Mass (LTM) method (Zitrin et al. 2015), deriving a lower magnification of $\mu = 4.3$. Their mass model enabled source reconstruction from Keck/OSIRIS near-infrared spectroscopy, resolving the source plane velocity field and H_{α} distribution, which revealed an early-stage merging pair. Although follow-up spectroscopic observations and analysis of the source have been conducted (Jones et al. 2018; Rigby et al. 2018; Mainali et al. 2023), a detailed lens modeling study remains lacking. In our work, we analyze high-resolution Hubble Space Telescope (HST) imaging data of CSWA19, incorporating 177,144 image pixels into the χ^2 statistic. This represents a major advancement over previous models of CSWA19, which solely relied on a few multiple image positions (Leethochawalit et al. 2016). Compared to other group-/cluster-scale lensing studies that adopted pixel-based modeling, our analysis also involves a significantly larger number of pixels than Urcelay et al. (2024) (~ 1500 pixels) and Acebron et al. (2024) ($\sim 78,000$ pixels).

The structure of the paper is organized as follows. Section 2 presents the observational data of CSWA19, including optical/infrared imaging data and spectroscopy. Section 3 outlines our modeling procedure using PYAUTOLENS, covering the selection of group members, modeling of the foreground galaxy light, the lens mass distribution, the source reconstruction, and the configuration of the search chain. Sections 4 and 5 present our modeling results and discuss both the mass distribution of the lens and the intrinsic luminosity of the background source. Finally, Section 6 summarizes our findings and discusses future directions.

Throughout this work, we assume a Planck 2015 cosmological model (Planck Collaboration et al. 2016). In such a cosmology, $1''$ at redshift 0.48841 corresponds to 6.2099 kpc. All figures of this galaxy group are aligned to the WCS coordinate system: north is up, east is left. The reference center in our analysis is fixed at the center of the brightest group galaxy (hereafter BGG): R.A. = $9^{\text{h}}00^{\text{m}}02^{\text{s}}.79$, decl. = $+22^{\circ}34'03''.60$. Magnitudes are given in the AB system.

2. Data

2.1. Imaging Data

This study makes use of archival HST high-resolution, multi-color imaging (GO-11602; P.I.: Sahar Allam), from the WIDE FIELD CAMERA 3 (WFC3). The optical images were taken using WFC3/UVIS with filters F475W, F606W, and F814W, with a

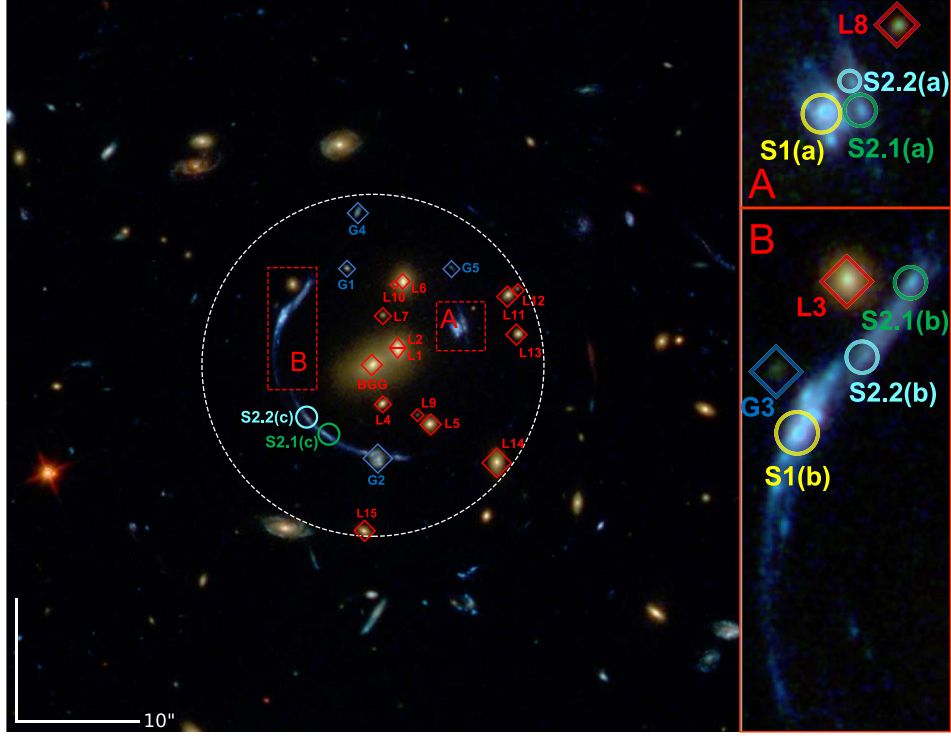


Figure 1. Color-composite image of the field of CASSOWARY 19 and its surrounding environment (1×1 arcmin²). The image is a combination of the F475W (blue), F606W+F814W (green), and F110W+F160W (red) images. We only consider member galaxies within a radius of $\sim 14''$ from the center (marked with a white dashed circle). The two right panels show the two images formed by the background source. The lens galaxies are marked with red diamonds, and non-member galaxies within the circle are marked with blue diamonds. The yellow circles mark the peak positions of the lensed images of the source galaxy 1. Accordingly, the green circles mark the peak positions of the lensed images of the source galaxy 2.

total exposure time of 5460, 2412 and 5598 s respectively in 2010 March. The near-infrared images were taken by WFC3/IR with filters F110W and F160W, with two exposures in each filter and a total exposure time of 2412 and 2812 s respectively in 2010 April. The UVIS field of view is approximately $2/8 \times 3'$, while that of the IR images is approximately $2/3 \times 2/1$. Details of the data reduction process are provided in Section 2.3.

2.2. Spectroscopic Data

Three spectroscopic measurements are available for four member galaxies within a radius of $\sim 14''$ from the reference center, based on the SDSS DR16. As shown in Figure 1, two of these galaxies, the BGG and L6, were observed by the BOSS spectroscopic survey, which uses a $2''$ -diameter fiber and provides spectral coverage from $3600 \sim 10400 \text{ \AA}$, with a resolution of $1560 \sim 2270$ in the blue channel and $1850 \sim 2650$ in the red channel. The third spectrum corresponds to a blended source identified as L1 and L2, observed by the SDSS Legacy survey using a $3''$ fiber with a wavelength range of $3800 \sim 9200 \text{ \AA}$ and resolution of $1850 \sim 2200$. While treated as a single object in SDSS, L1 and L2 are clearly resolved in the HST imaging (see Figure 1).

The spectroscopic redshifts of the BGG, L6 and L1/L2 are 0.48841 ± 0.00012 , 0.48367 ± 0.00013 and 0.48912 ± 0.00014 , respectively. Velocity dispersion and stellar mass are retrieved from the SDSS database and are listed in Table 1. For the purpose of lens modeling, we assume a common redshift of $z = 0.48841$ for all member galaxies.

For the background source, Diehl et al. (2009) obtained follow-up spectroscopic observations with the Astrophysical Research Consortium (ARC) 3.5 m telescope at the Apache Point Observatory, which shows emission and absorption lines. In this study, we adopt the average $z = 2.0325 \pm 0.0003$ of the two knots (corresponding to the A and B images in Figure 1) as the source redshift.

2.3. Data Preparation

The science image data obtained directly from the HST archive consist of default multi-extension FITS products processed with the DRIZZLEPAC/ASTRODRIZZLE (Avila et al. 2012) software to correct geometric distortions. For the UVIS images, we use the CTE-corrected, calibrated science data. The UVIS and IR science images obtained from the database have pixel scales of $0/03962$ and $0/12825$,

Table 1
Position, Magnitude, Effective Radius Fitting Results with 1σ Uncertainty for the Member Galaxies in the F160W Band, and the Dispersion, Stellar Mass Measurement from SDSS

ID	x	y	R_{eff}		AB-mag		σ	$\log(M_*/M_\odot)$
	(")	(")	(kpc)		Best-fit	Marginalized	(km s ^{−1})	
			Best-fit	Marginalized				
BGG	0.0236	−0.0087	21.707	21.704 ^{+0.068} _{−0.056}	17.126	17.126 ^{+0.002} _{−0.002}	263 ⁺⁴⁴ _{−44}	12.01 ^{+0.09} _{−0.07}
L1	1.2015	2.0348	3.470	3.601 ^{+0.065} _{−0.068}	18.771	18.748 ^{+0.011} _{−0.011}	157 ⁺³³ _{−33}	11.56 ^{+0.09} _{−0.09}
L2	1.6308	1.9956	2.368	3.217 ^{+0.080} _{−0.090}	19.728	19.758 ^{+0.016} _{−0.016}	...	...
L3	6.5766	−6.5502	7.141	7.185 ^{+0.110} _{−0.098}	19.660	19.658 ^{+0.007} _{−0.007}	...	...
L4	−3.1112	0.7814	0.392	0.378 ^{+0.010} _{−0.011}	19.566	19.576 ^{+0.013} _{−0.013}	...	...
L5	−4.7812	4.6346	1.674	1.693 ^{+0.013} _{−0.012}	19.452	19.452 ^{+0.006} _{−0.005}	...	...
L6	6.7970	2.4680	5.211	5.176 ^{+0.024} _{−0.020}	18.468	18.470 ^{+0.002} _{−0.002}	155 ⁺³² _{−32}	11.61 ^{+0.10} _{−0.09}
L7	4.0817	0.8063	0.639	0.588 ^{+0.049} _{−0.070}	22.686	22.636 ^{+0.042} _{−0.081}	...	...
L8	4.6237	8.1924	0.121	0.197 ^{+0.125} _{−0.074}	21.381	21.885 ^{+0.616} _{−0.548}	...	...
L9	−4.0195	3.6123	0.423	0.307 ^{+0.143} _{−0.151}	23.639	23.909 ^{+0.293} _{−0.767}	...	...
L10	6.6180	1.7786	0.210	0.174 ^{+0.074} _{−0.049}	22.467	22.828 ^{+0.649} _{−0.659}	...	...
L11	5.7239	11.0402	1.526	1.534 ^{+0.023} _{−0.023}	20.150	20.150 ^{+0.013} _{−0.013}	...	...
L12	6.2343	11.7888	0.352	0.337 ^{+0.159} _{−0.156}	22.888	22.916 ^{+0.357} _{−0.604}	...	...
L13	2.5430	11.8803	0.418	0.425 ^{+0.010} _{−0.009}	20.070	20.079 ^{+0.012} _{−0.008}	...	...
L14	−7.9041	10.1086	2.886	2.888 ^{+0.013} _{−0.013}	18.940	18.940 ^{+0.004} _{−0.004}	...	...
L15	−13.5111	−0.7457	1.927	1.920 ^{+0.025} _{−0.024}	19.996	20.001 ^{+0.006} _{−0.004}	...	...

Note. The above position, magnitude, effective radius are all from the multi-Sérsic fitting of the F160W band, and the velocity dispersion and stellar mass are from the SDSS measurement. Note that only three member galaxies have spectroscopic data, corresponding to BGG, L6, and L1/L2. L1 and L2 are identified as a single source in SDSS, but can be resolved in our high-resolution HST images. The stellar masses from SDSS database are using the FSPS model (Conroy et al. 2009) with the Granada method in early-star formation version for dust extinction.

respectively. We use the REPROJECT package (Robitaille et al. 2020) to resample the images with flux conservation and aligned them to a common WCS coordinate system for subsequent reduction and modeling. The resampled images have a pixel scale of $0''.04$ for the UVIS images and $0''.128$ for the IR images.

After calculating and subtracting the uniform background using the PHOTUTILS package (Bradley et al. 2023), we obtain the final images used for modeling. Based on the reduced multi-band images, we crop and generate a 1×1 arcmin² color-composite image centered on the reference center (as described in Section 1), shown in Figure 1.

The noise map is calculated following Equations (12) and (13) of Schuldt et al. (2019), accounting for both background noise and Poisson noise:

$$\sigma_{\text{tot},i}^2 = \sigma_{\text{bkg},i}^2 + \sigma_{\text{poisson},i}^2 = \sigma_{\text{bkg},i}^2 + \left(\frac{\sqrt{d_i t_i}}{t_i} \right)^2 \quad (1)$$

where t_i is the exposure time, d_i is the intensity of pixel i in units of electrons per second, and $\sigma_{\text{bkg},i}$ is the constant background noise calculated from the empty area of the image.

As noted by Acebron et al. (2024), different from traditional cluster-scale lens modeling that only relies on multiple images positions, constructing an accurate point-spread function (PSF) is critical for modeling extended lensed features. Here, we use

the PSFR (Birrer et al. 2022) software package to construct the PSF. Five isolated stars with diffraction spikes and minimal contamination are visually selected and cross-matched with the GAIA catalog. Only one of them lies within the infrared field of view. For each star, we crop the image and corresponding noise map centered on the brightest pixel and use these as PSF samples. These are then stacked using PSFR to generate the final PSF model. With the consideration of the calculating efficiency, the PSF is cropped to 31×31 pixels for UVIS and 35×35 pixels for IR.

Considering both resolution and signal-to-noise ratio, we adopt the F475W-band image as constraint for lens modeling. This filter enhances the contrast of the bluer background source while minimizing contamination from the redder lens galaxies (see Figure 1). Conversely, the F160W-band image is employed to model the extended infrared light of the lens galaxies for the scaling relation described in Section 3. To accommodate this extended emission, the IR PSF is cropped to a larger size than that used for UVIS, despite the IR's coarser pixel scale.

3. Lens Modeling

In our work, we use the PYAUTOLENS software (Nightingale et al. 2018, 2021b) to model the surface brightness distribution of the lens galaxies and source galaxies, as well as the mass

distribution of the galaxy group. PYAUTOLENS is an open-source automated lens modeling tool based on the probabilistic programming software PYAUTOFIT (Nightingale et al. 2021a), which is currently mainly applied to galaxy-scale strong gravitational lensing, with brightness-adapted pixel-grid-based semilinear inversion of background source reconstruction (Nightingale & Dye 2015). We apply PYAUTOLENS’s pixelized source reconstruction to model a group-scale gravitational lens system—its first application to a system with multiple lens galaxies—thereby extending the method’s reach to more complex, larger-scale lenses and demonstrating its effectiveness in this regime.

Section 3.1 presents the selection of member galaxies within the galaxy group using the red-sequence method (Gladders et al. 1998). Section 3.2 shows the parametric modeling of the surface brightness distribution of the foreground galaxies, the subtraction of their light contamination, and the measurement of the lens galaxies’ luminosities in the F160W band. Section 3.3 describes the parametric modeling of the mass distribution. Section 3.4 subsequently details the source reconstruction techniques. Finally, Section 3.5 outlines our search strategy and sampling configuration.

3.1. Membership Selection

Before performing lens modeling and foreground galaxy light fitting, we first determine which galaxies are member galaxies of the group. Diehl et al. (2009) derived an Einstein radius of $7''.0 \pm 0.8$ for the system using a simplified SIS model. To balance computational efficiency and modeling accuracy, we only consider galaxies within $\sim 14''$ of the BGG’s center. As shown in Figure 1, we identify a total of 21 candidate galaxies in this region using SOURCE EXTRACTOR (Bertin & Arnouts 1996), including the BGG.

As in most group-scale lenses, not all members have spectroscopic confirmation. In this area, only BGG, L1, and L6 have spectroscopic redshift data (see Section 2.2). For the remaining galaxies, we apply the so-called red-sequence method as our criterion to identify member galaxies.

The red-sequence is a well-known color–magnitude relation for member galaxies in galaxy clusters, where most early-type galaxies show a tight relation between color and magnitude. This color–magnitude relation was first noted by de Vaucouleurs (1961), and latter systematically summarized by Bower et al. (1992). Gladders et al. (1998) developed an algorithm to fit the red-sequence slope and demonstrated its utility in detecting galaxy clusters (Gladders & Yee 2000).

Following Stott et al. (2009) and Gladders et al. (1998), we adopt their slope-fitting method, using $F475W - F814W$ as our color index since these bands bracket the $4000 \text{ \AA}$ break at the group’s redshift, effectively distinguishing red-sequence galaxies from star-forming galaxies (Stott et al. 2009). We

use SOURCE EXTRACTOR to measure Kron magnitudes and errors for all galaxies in the $2.8 \times 3'$ field.

We construct the red sequence through the standard iterative procedure of linear fitting, slope correction, single Gaussian fitting, and 3σ clipping (Gladders et al. 1998; Stott et al. 2009). The final red sequence is shown in Figure 2 as a red solid line, with the 3σ range shaded. This classification divides the 21 candidate galaxies into members (BGG and L1 $\sim$ L15, marked with red squares) and non-members (G1 $\sim$ G5, marked with blue squares). Their positions and labels are indicated in Figure 1. All member galaxies have F160W magnitudes brighter than 24 mag. In the subsequent modeling, we only consider member galaxies BGG and L1 $\sim$ L15.

3.2. Foreground Galaxy Light

All the light profiles of the foreground galaxies in this work are described by one or more elliptical Sérsic profiles (Sérsic 1963):

$$I(\xi) = I_0 \exp \left\{ -k \left[\left(\frac{\xi}{R_e} \right)^{\frac{1}{n}} - 1 \right] \right\} \quad (2)$$

where $\xi = \sqrt{qx^2 + y^2}/q$, and I_0 is the intensity at the effective radius R_e , n is the Sérsic index, and k is a constant that depends on n (Ciotti & Bertin 1999). The ellipticity is introduced as described in Nightingale et al. (2024) Equation (1):

$$e_1 = \frac{1-q}{1+q} \sin 2\phi, \quad e_2 = \frac{1-q}{1+q} \cos 2\phi \quad (3)$$

where q is the axis ratio, and ϕ is the position angle defined counterclockwise from the positive x -axis. Therefore, a single Sérsic profile has seven free parameters: I_0 , R_e , n , x , y , e_1 , and e_2 . To reduce parameter space complexity, we use the linear light profile in PYAUTOLENS (unless stated otherwise), where intensities are solved via linear algebra during model fitting rather than treated as free parameters in the nonlinear sampling (see He et al. 2024 for details). This approach employs a non-negative least squares solver (Bro & De Jong 1997), which enforces physically meaningful, positive intensity values.

In modeling the light of foreground galaxies, we iteratively determine the number of Sérsic components required to describe the light of each member galaxy. Each galaxy is initially modeled with a single Sérsic component, and additional Sérsic components are added one by one until the intensity of the newly added component is solved to be zero. Unless otherwise stated, the centers of all Sérsic components associated with a given galaxy are fixed to the same position.

For the F475W band fitting, our primary goal is to subtract the light contamination from foreground galaxies. Therefore, we only fit the galaxies located within or near the arc: BGG, L1 $\sim$ L10, and G1. Since the light distribution in the F475W

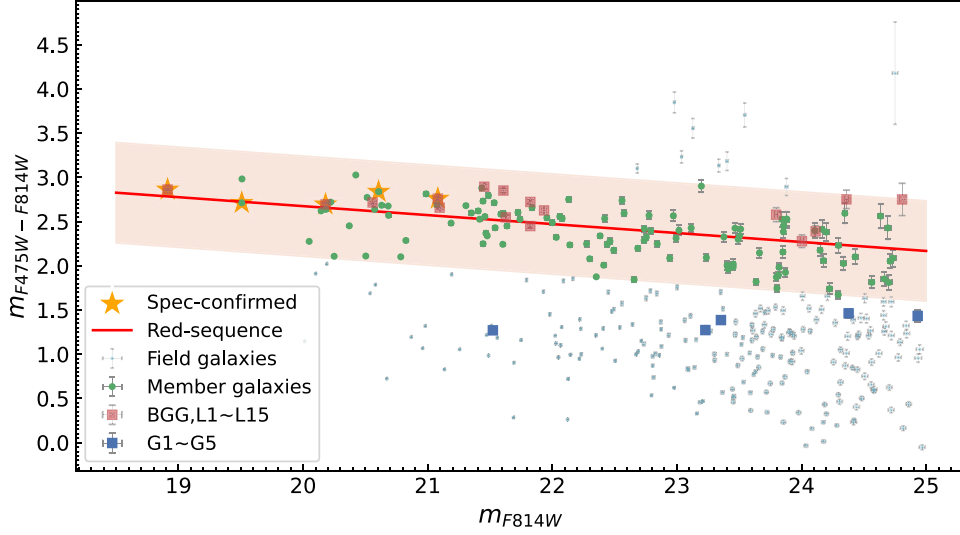


Figure 2. Red-sequence fitting of the galaxy group. The red solid line is the final converged red sequence, and the shaded area is the 3σ range. The 21 candidate galaxies are divided into two parts: member galaxies, i.e., BGG and L1 ~ L15 (marked with red squares), and non-member galaxies, i.e., G1 ~ G5 (marked with blue squares). The yellow pentagrams denote the five member galaxies with spectroscopic redshifts used for the initial guess.

band is relatively compact, most galaxies could be well modeled individually within local circular regions. An exception is made for the central galaxies BGG, L1, L2, and L4, whose light profiles overlap significantly and are thus fitted simultaneously. The F475W band fitting results are reliable, with only BGG and L5 requiring two Sérsic components, while the others are well described by a single component. The upper panel of Figure 3 shows the F475W band fitting results, displaying, from left to right, the observed image data, the Sérsic model image, and the normalized residual map. The residual image masks the arc and galaxies outside the modeling region.

For the F160W band, our goal is to obtain photometry for each lens galaxy, which is required for the scaling relation discussed in Section 3.3. Owing to the more extended light distribution in this band, we divide the galaxies into three modeling batches. We first fit BGG and L1 ~ L6 simultaneously, masking the arc and other galaxies, since the extended light of these inner galaxies contaminated both the arc and nearby faint or outer galaxies. A simple uniform background subtraction is applied during the data reduction (see Section 2.3), but residual non-uniform background remains during lens light modeling. This component may be attributed to inhomogeneous intra-cluster light. Following the approach of Martis et al. (2024), we iteratively estimate and subtract this background using PHOTUTILS within the modeling area. After removing the inner galaxy light and non-uniform background, the faint galaxies L7 ~ L10 and the distant galaxies L11 ~ L15 are fitted independently, in a manner consistent with the F475W band modeling.

The lower panels of Figure 3 show the F160W band fitting results; from left to right are the observed image data, the Sérsic

model image, and the normalized residual map, with the arc and local unknown fluctuations masked in the residual image. The fitting results are not as reliable as the F475W band, with some residuals remaining in the galaxy cores due to the more complex light structures present in the F160W band. We calculate the residual flux in the central regions of these galaxies, and in all cases, the ratio of residual flux to the total luminosity of the corresponding galaxy is less than 1%. Table 1 shows the Sérsic fitting photometry results of the foreground galaxies in the F160W band, where the coordinates are taken as the maximum likelihood values in the reference coordinate system, and the magnitudes are reported as the maximum likelihood values along with the median and 1σ error of the marginal distributions.

3.3. Lens Mass Parameterization

Regarding the lens mass model, the mass distribution of the galaxy group is assumed to be dominated by a group-scale dark matter halo centered on the BGG, with member galaxies residing within their own dark matter subhalos embedded in this main halo. For the above objects, we adopt a dual Pseudo Isothermal mass profile with pseudo-ellipticity,⁸ implemented in a development branch⁹ of PYAUTOGALAXY (Nightingale et al. 2023), the modeling sub-package of PYAUTOLENS. The

⁸ This profile was introduced into PYAUTOGALAXY by O'Donnell and will also be applied to the modeling of the galaxy cluster MACS J0138 in an upcoming study (J. H. O'Donnell et al. 2025, in preparation).

⁹ The corresponding implementation is available at <https://github.com/jhod0/PyAutoGalaxy/tree/dPIE>.

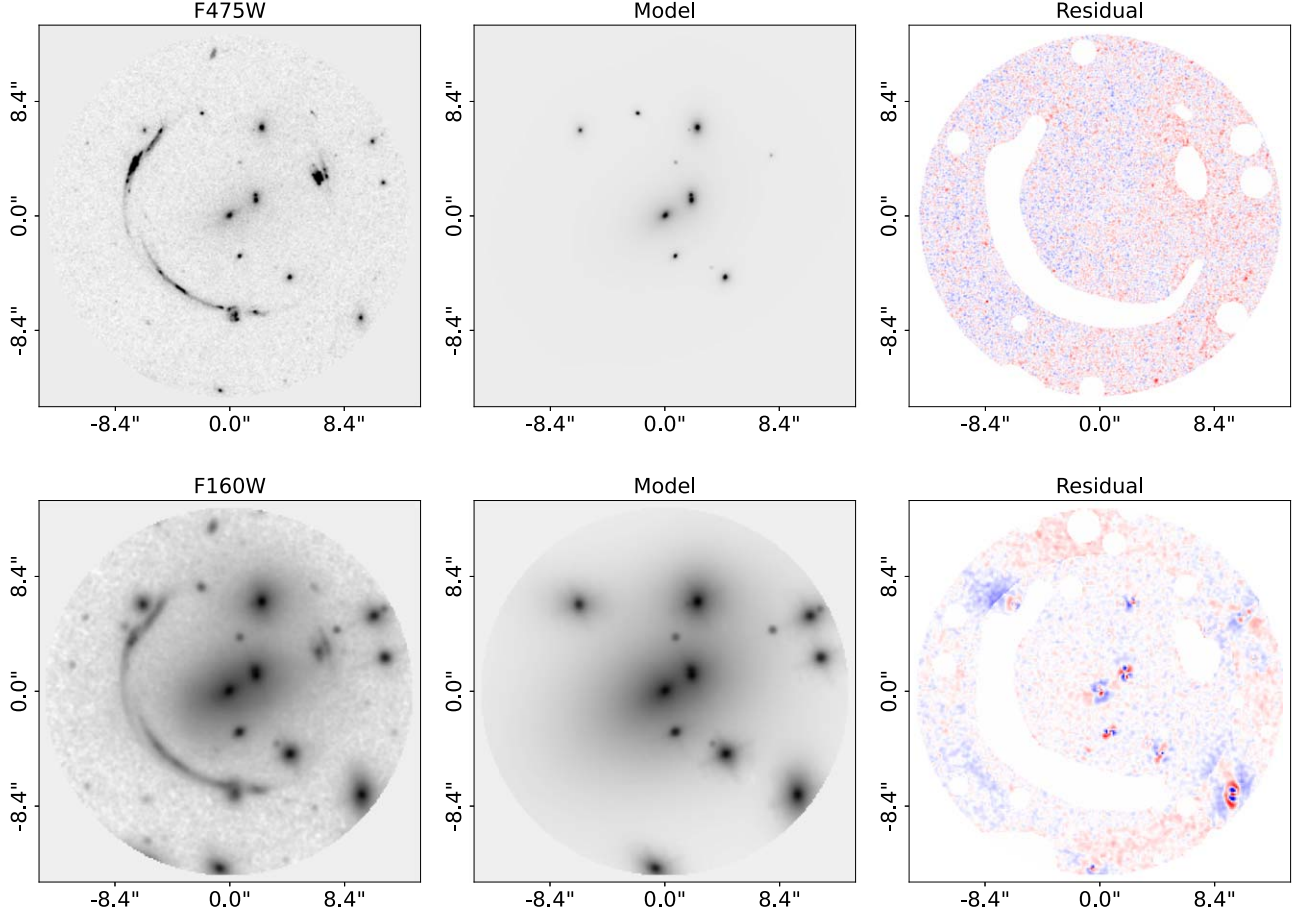


Figure 3. Multi-Sérsic light profile fittings to the F475W and F160W images for the lens galaxies. The upper panels show the F475W band fitting results, where the panels from left to right show the observed image, the Sérsic model image, and the normalized residual image (range $\pm 5\sigma$). The lower panels show the F160W band fitting results, and the panels from left to right are the observed image, the Sérsic model image, and the normalized residual image (range $\pm 10\sigma$). Only the BGG, L1 $\sim$ L10, and G1 are modeled with the F475W band image, because the purpose of the F475W band modeling is to subtract the light contamination of the foreground galaxies, and the light contamination of L11 $\sim$ L15 and G2 $\sim$ G5 is small or outside the lens modeling image area.

functional form of the convergence κ is:

$$\kappa(\xi) = \frac{E_0}{2} \frac{r_s + r_a}{r_s} \left(\frac{1}{\sqrt{r_a^2 + \xi^2}} - \frac{1}{\sqrt{r_s^2 + \xi^2}} \right) \times (1 - \mathcal{A}) + \frac{\alpha(\xi)}{\xi} \mathcal{A} \quad (4)$$

where r_a is the core radius, r_s is the scale radius. For the 3D density distribution, in the core region where $r < r_a$, it has a relatively flat mass distribution, in the transition region where $r_a < r < r_s$, it behaves like an Isothermal with $\rho \sim r^{-2}$, and in the outer region where $r > r_s$, it provides a drop-off with $\rho \sim r^{-4}$. The lens strength E_0 is defined as:

$$E_0 = 6\pi \frac{D_{LS} D_L}{D_S} \frac{\sigma_{\text{dPIE}}^2}{c^2} \quad (5)$$

As described by Elíasdóttir et al. (2007), this definition of E_0 is consistent with LENSTOOL (Kneib et al. 1996; Jullo et al. 2007), where E_0 is proportional to σ_{dPIE}^2 , and has units of length. $\mathcal{A}$ is the asymmetric term with respect to x and y , defined as:

$$\mathcal{A} = \frac{\epsilon(1 - \epsilon)x^2 - \epsilon(1 + \epsilon)y^2}{\xi^2} \quad (6)$$

where ξ is defined as:

$$\xi = \sqrt{x^2(1 - \epsilon) + y^2(1 + \epsilon)} \quad (7)$$

The ellipticity ϵ is defined as: $\epsilon = \sqrt{e_1^2 + e_2^2} = (1 - q)/(1 + q)$ where e_1 and e_2 are defined as in Equation (3). $\alpha(\xi)$ is the deflection angle in the circular case at radius ξ , defined as in Equation (A19) of Elíasdóttir et al. (2007).

This profile introduces ellipticity to the projected lensing potential rather than to the projected surface mass density of the

Table 2
Description of the Search Chain

Model	Search Chain	
	Source	Mass Components
Model 1	1 Sérsic	main halo + BGG + L1 + Shear
Model 2	1 Sérsic	main halo + BGG + L1 + Shear + 14 members
Model 3	2 Sérsic	main halo + BGG + L1 + Shear + 14 members
Model 4	3 Sérsic	main halo + BGG + L1 + Shear + 14 members
Model 5-0	Delaunay	fixed mass from model 4
Model 5-1	fixed Delaunay	main halo + BGG + L1 + Shear + 14 members

standard dual Pseudo Isothermal Elliptical (dPIE, Elíasdóttir et al. 2007) profile, an approximation often referred to as pseudo-ellipticity (Kovner 1987; Golse & Kneib 2002). This approximation allows for easier computation of the gradient and Hessian of the lens potential (Kovner 1987; Golse & Kneib 2002), although it is known that the surface density (Equation (4)) becomes dumbbell-shaped at higher ellipticities (Kassiola & Kovner 1993; Golse & Kneib 2002; Shajib 2019), which is unphysical. In our F160W-band modeling, the axis ratios of the member galaxies' light distributions are all greater than 0.6, justifying the use of this approximation.

In the circular case ($\epsilon = 0$, $\mathcal{A} = 0$), the above formulation reduces to the standard form of the dPIE convergence profile:

$$\kappa(\xi) = \frac{E_0}{2} \frac{r_s + r_a}{r_s} \left(\frac{1}{\sqrt{r_a^2 + \xi^2}} - \frac{1}{\sqrt{r_s^2 + \xi^2}} \right) \quad (8)$$

The dPIE profile is equivalent to the Pseudo-Jaffe model when $r_a = 0$ (Dalal & Kochanek 2002; Vegetti et al. 2010b), to the PIEMD model when $r_s \rightarrow \infty$ (Kassiola & Kovner 1993), and to the SIE model when both $r_a = 0$ and $r_s \rightarrow \infty$, in which case E_0 represents the Einstein radius.

Therefore, a complete description of the dPIE model with pseudo-ellipticity requires seven parameters: E_0 , r_a , r_s , x , y , e_1 , e_2 . For each member galaxy, we use a dPIE model to describe their total mass distribution. To reduce the complexity of the parameter space, we follow Grillo et al. (2015), Chirivì et al. (2018), and Wang et al. (2022), assuming that all member galaxies follow the following scaling relation:

$$\sigma_{\text{dPIE},i}^2 = \sigma_{\text{dPIE,ref}}^2 \left(\frac{L_i}{L_{\text{ref}}} \right)^{0.7} \quad \text{and} \quad r_{s,i} = r_{s,\text{ref}} \left(\frac{L_i}{L_{\text{ref}}} \right)^{0.5} \quad (9)$$

This specific scaling relation assumes that the total mass-to-light ratio increases with luminosity (Grillo et al. 2015): $M_{\text{total}}/L \propto L^{0.2}$, known as the tilt of the Fundamental Plane of early-type galaxies (Faber et al. 1987; Bender et al. 1992). Some works have shown that it can better reconstruct the observed positions of multiple images (Grillo et al. 2016; Kelly et al. 2016). In Equation (9), L_i is the F160W band luminosity

of the i th galaxy, and L_{ref} is the F160W band luminosity of the reference galaxy. The F160W band luminosity serves as a good proxy for their total mass (Grillo et al. 2015). We choose the second brightest galaxy L6 in the system as the reference galaxy instead of BGG, since BGG may have undergone different formation and evolution processes, thus not following the usual elliptical galaxy scaling relation (Caminha et al. 2019; Richard et al. 2021).

We also incorporate an external shear field to account for the tidal distortion induced by the surrounding environment or other mass components of CSWA19 beyond the modeled region. In PYAUTOLENS, this shear field is described by two elliptical components: $(\gamma_{\text{ext},1}, \gamma_{\text{ext},2})$. Their relationship with the shear magnitude γ_{ext} and the orientation measured counter-clockwise from north ϕ_{ext} is as follows:

$$\gamma_{\text{ext}} = \sqrt{\gamma_{\text{ext},1}^2 + \gamma_{\text{ext},2}^2} \quad \text{and} \quad \phi_{\text{ext}} = \frac{1}{2} \arctan \left(\frac{\gamma_{\text{ext},2}}{\gamma_{\text{ext},1}} \right) \quad (10)$$

For the member galaxies, we assume a circularly symmetric dPIE model, and fix their mass and light centers to further reduce the number of free parameters. For the BGG and the main halo of the group scale, we use an elliptical dPIE model. Both the BGG and member galaxies adopt a vanishing core dPIE, which approximates a singular isothermal profile in both the core and transition regions. We also fix the mass center at the inferred light center of BGG. The scale radius r_s of the main halo is fixed to a large value of 1000'', because this parameter describes the nature of the mass distribution of the main halo at large scales (Bergamini et al. 2021; Richard et al. 2021), which is usually much larger than the radius that a strong lens can provide constraints in practice (Limousin et al. 2007b).

We now explain the choices of the adopted priors for our initial model (see Model 1 in Table 2). First is the scale radius r_s for BGG and member galaxies, for which we set an upper limit to consider the tidal stripping of the dark matter halo (Limousin et al. 2007a, 2009b; Natarajan et al. 2009; Wetzel & White 2010). Based on prior experience, we set this upper limit to approximately 5 times the half-light radius in the F160W band and adopt a broader prior range to allow flexibility. For

the parameters of the main halo, we use uniform priors; specifically, we limit its core radius r_a to $3'' \sim 17''$. This wide range is inspired by the results of Richard et al. (2021) and Bergamini et al. (2021) for extended halos, to exclude unrealistically large core radii. The Gaussian prior on the velocity dispersion $\sigma_{\text{dPIE}}^{\text{BGG}}$ (also corresponding to E_0^{BGG}) is informed by the spectroscopic measurement from SDSS (see Section 2.2), with the prior mean set to the measured value and the standard deviation taken as three times the reported measurement uncertainty. Note that the theoretical velocity dispersion σ_{dPIE} defined by Elíasdóttir et al. (2007) is not exactly equal to the observed velocity dispersion, but differs by a projection coefficient c_p , which depends on the core radius r_a , the scale radius r_s , and the aperture radius R (see Figure C.3 and Equation (C.16) of Bergamini et al. 2019).

It is also worth noting that the member galaxy L1 is the third brightest and closest to BGG, which may deviate significantly from the scaling relation. We use the same elliptical dPIE model to describe its mass distribution as BGG, and allow its r_s and E_0 to vary freely.

3.4. Source Model

We fit lens models using two different approaches for the source galaxy. The simpler model uses one or more Sérsic profile—using the same elliptical formulation as described in Section 3.2—to characterize the background galaxy’s surface brightness. When multiple components are implemented, their centroids and ellipticities become free parameters in the fitting process.

Advanced modeling employs an adaptive pixelization to reconstruct the irregular surface brightness distribution of the background source. A uniform grid of size $y_{\text{pixels}} \times x_{\text{pixels}}$ is overlaid on the image plane, from which $J = y_{\text{pixels}} \times x_{\text{pixels}}$ cell centers are identified as those within the image mask. These centers are mapped to the source plane using deflection angles from the lens model and form the vertices of a Delaunay triangle mesh for source reconstruction, following the method of Vegetti & Koopmans (2009). This approach introduces only three nonlinear parameters: the x and y dimensions of the image-plane mesh and the regularization coefficient λ . The mesh naturally adapts to the lensing magnification pattern (Nightingale & Dye 2015).

However, adapting to magnification alone can lead to sub-optimal reconstructions if the source lies in low-magnification regions, away from the caustic. To address this, PYAUTOLENS includes a brightness-based pixelization scheme that instead adapts to the source’s surface brightness, using a model-predicted image of the lensed source or observed image itself with lens light subtracted called the “adapt image.” A normalized weight map is computed from this image, assigning higher weights to brighter regions. A total of I image-plane

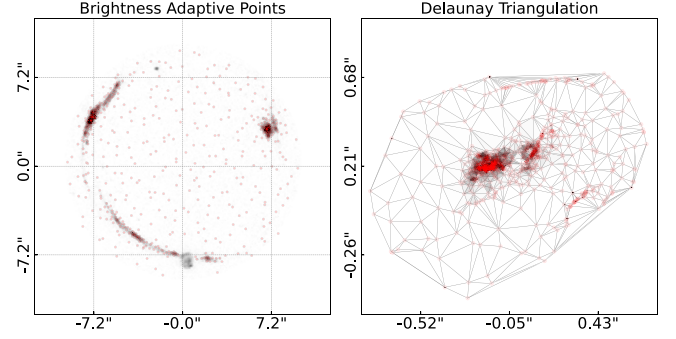


Figure 4. Schematic diagram of the construction of the brightness-adaptive grid. The left image shows the irregular grid point distribution on the image plane, constructed by the Hilbert space-filling curve, these 1000 points are distributed more densely in brighter areas and sparsely in unstructured areas, this grid point distribution comes from the best fitting values of Model 5-0, and we manually mask the area of G2; the right image shows the Delaunay triangle mesh on the source plane, the vertices of these triangles are mapped from the points on the image plane to the source plane through the mass model, the grid points shown here correspond to the mass distribution of the best fitting results of Model 5-1.

coordinates are then probabilistically sampled from this map using a generalized Hilbert space-filling curve (Hilbert 1891).

This scheme better captures the source’s structure (see Figure 4), yielding more detailed reconstructions with fewer vertices and significantly reduced computational cost. It has recently outperformed the previously used KMeans clustering method in modeling Euclid-resolution images (Wang et al. 2025). A full description of this new HilbertMesh method will be presented in Q. He et al. (2025, in preparation).

For a given mass model and set of source pixel centers, the mapping matrix f_{ij} is constructed. Assuming that there are J image-pixels in the masked observed data, this matrix maps the j th pixel on the image plane to the i th triangle on the source plane one by one. By default, PYAUTOLENS uses a 4×4 subgridding for each image plane pixel, meaning that the actual number of image plane sub-pixels used for mapping is $16 \times J$. In general, multiple image plane pixels are mapped to the same source plane triangle, and f_{ij} records the weights of each image pixel-vertex pair mapping, which are calculated by triangle linear interpolation:

$$\hat{z}_j = \frac{A_{ji_2i_3}z_{i_1} + A_{i_1ji_3}z_{i_2} + A_{i_1i_2j}z_{i_3}}{A_{i_1i_2i_3}} \quad (11)$$

where the j th sub-pixel falls into the triangle with vertices i_1 , i_2 , and i_3 , A is the area of the triangle formed by the three points, and z is the value of the corresponding point. f_{ij} is convolved with the PSF to obtain a blurred mapping matrix to include the effect of the PSF, and we assume that f_{ij} from here on includes the PSF convolution.

Therefore, assuming that the value of the source plane vertex is s_i and the flattened observed data is d_j , we can obtain the best

source reconstruction by minimizing the following equation:

$$\chi^2 = \sum_{j=1}^J \left(\frac{d_j - \sum_{i=1}^I f_{ij} s_i}{\sigma_j} \right)^2 \quad (12)$$

where σ_j is the error of the observed data, i.e., the noise map obtained in Section 2.3. We consider that d_j here has been subtracted the light of the foreground galaxies. The minimization solution of the above equation can be abstracted into the following matrix form:

$$\mathbf{s} = \mathbf{F}^{-1} \mathbf{D} \quad (13)$$

where $\mathbf{F}$ is the curvature matrix with $F_{ik} = \sum_{j=1}^J f_{ij} f_{kj} / \sigma_j^2$, and $\mathbf{D}$ is the data vector with $D_i = \sum_{j=1}^J d_j f_{ij} / \sigma_j^2$. Therefore, the source reconstruction can always be solved by linear algebra methods. Combined with the parametric mass model, this method is the so-called semilinear inversion (first proposed by Warren & Dye 2003). To avoid overfitting, a regularization term λH is introduced on the basis of the above:

$$\mathbf{s} = [\mathbf{F} + \lambda \mathbf{H}]^{-1} \mathbf{D} \quad (14)$$

where $\mathbf{H}$ is the regularization matrix, and λ is the regularization coefficient. The introduction of this regularization term is similar to introducing a smooth prior to the source reconstruction, thus avoiding overfitting (Warren & Dye 2003). Throughout the modeling we use a cross-like regularization ensuring a smooth behavior of the likelihood function, which guarantees a robust estimation of the errors of model parameters (see Nightingale et al. 2024 and appendix A of He et al. 2024).

In this work, we use the image with the foreground galaxies subtracted as the adapt image d_j . When the weight map w_j is calculated from the adapt image, two more parameters will be introduced:

$$w_j = \begin{cases} \hat{d}_j^{\omega_p}, & \text{if } w_j > \omega_f, \\ \omega_f, & \text{if } w_j \leq \omega_f. \end{cases} \quad (15)$$

where $\hat{d}_j = |d_j| / \max(d_j)$, ω_f is the weight floor, and ω_p is the weight power. The effect of the weight power is similar to increasing the contrast, while the weight floor ensures that the area around the arc has the lowest but not zero degree of grid point distribution.

It is worth mentioning that the number of parameters for this brightness-adaptive source is four: ω_f , ω_p , I , and λ . In actual tests, considering the computational efficiency and the accuracy of source reconstruction, the number of pixels I is fixed at 1000. In addition, adaptive regularization is not employed in the fitting (which controls smoothness variations across reconstruction regions). Instead, we use a constant regularization and fix the regularization coefficient to a relatively large value ($\lambda = 20.0$) in Model 5 to balance the physicality of the source reconstruction and the fineness of the model image. Although this coefficient could be treated as a free parameter,

the fitting process consistently favors smaller regularization to achieve a lower χ^2 , which leads to an unphysical source reconstruction with discontinuous structures. This issue likely originates from the oversimplification of the mass model (as discussed in 5.2.2), making such a large regularization necessary to ensure the physicality of the source reconstruction.

3.5. Sampling and Search Chain

We use the nested sampling algorithm DYNESTY (Speagle 2020), specifically its static sampler with random walk sampling to perform nonlinear parameter sampling fitting of each model. This method is less likely to fall into local optima than traditional MCMC algorithms. In PYAUTOLENS, when the fitting does not involve inversion, the likelihood function of the sampling is defined as:

$$-2 \ln \mathcal{L} = \chi^2 + \sum_{j=1}^J \ln(2\pi\sigma_j^2) \quad (16)$$

where χ^2 is defined as:

$$\chi^2 = \sum_{j=1}^J \left(\frac{d_j - m_j}{\sigma_j} \right)^2 \quad (17)$$

where m_j is the pixel value of the model image.

In the subsequent pixelized background source stage, the semilinear inversion framework described in Section 3.4 is used. PYAUTOLENS uses the Bayesian framework first defined by Suyu et al. (2006) where the likelihood function aims to maximize the Bayesian evidence ϵ :

$$\begin{aligned} -2 \ln \epsilon = & \chi^2 + \lambda \mathbf{s}^T \mathbf{H} \mathbf{s} + \ln[\det(\mathbf{F} + \lambda \mathbf{H})] \\ & - \ln[\det(\lambda \mathbf{H})] + \sum_{j=1}^J \ln(2\pi\sigma_j^2) \end{aligned} \quad (18)$$

The notion of the above equation comes from Equation (5) of Dye et al. (2008).

PYAUTOLENS provides standardized pipelines for automated modeling of galaxy-scale lenses, known as the *Source*, *Light*, and *Mass* (SLaM) pipelines, used in works such as Cao et al. (2022); Etherington et al. (2022). These pipelines link together a sequence of nonlinear searches that progressively increase model complexity, improving both efficiency and reliability while avoiding convergence to local optima in high-dimensional parameter spaces. For the system CSWA19, we build on the SLaM pipelines to make them appropriate for a group-scale lens and implement the custom search chaining strategy outlined in Table 2. The light from the foreground galaxies is assumed to have been cleanly subtracted in advance, so their light profile parameters are fixed and not refined in subsequent steps.

Model 1 begins with a simple single Sérsic background source, modeling only the main mass components: the primary halo, BGG, L1 (near the center), and external shear. Model 2

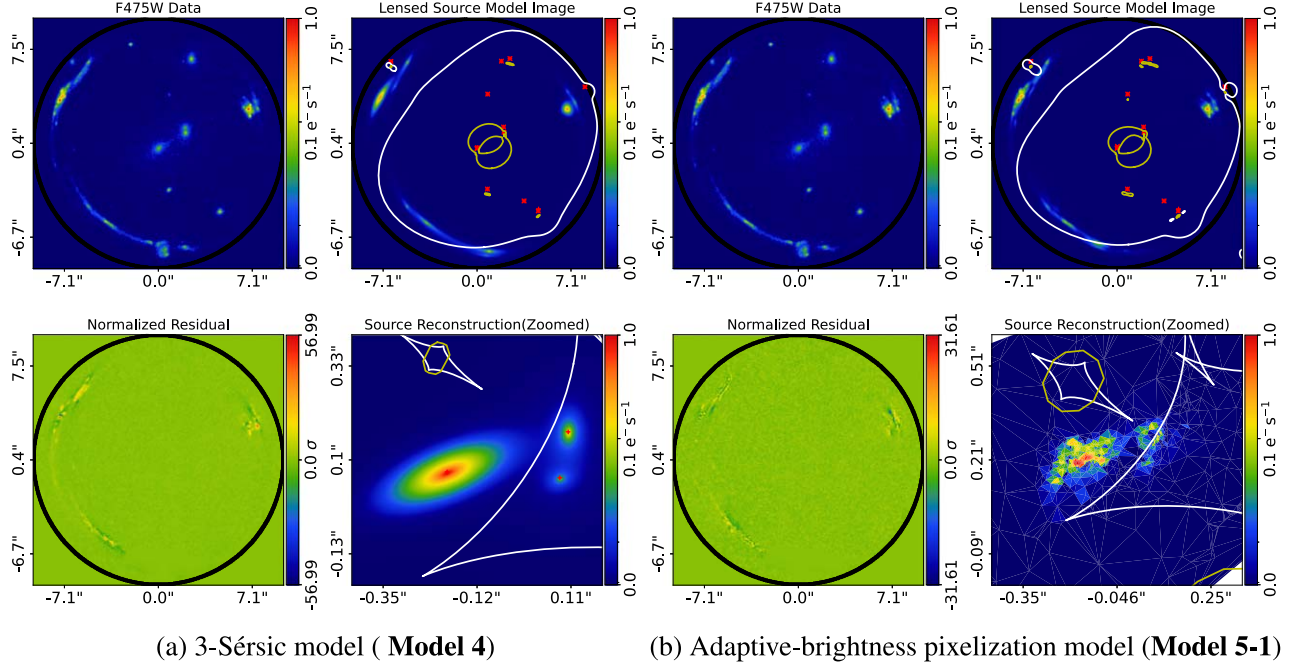


Figure 5. Comparison of lens modeling results. The panels in the left part (a) show the 3-Sérsic model results, and the panels in the right part (b) present the pixelized model results using a Delaunay mesh visualization for the source. Both parts share common features: 9.5'' radius circular mask (black circle), tangential critical curves and caustics (white lines) and radial critical curves and caustics (yellow lines) are overlaid, and red markers indicate mass component positions (including external L11 ~ L15 member galaxies not visible in frame).

builds on this by using the results of Model 1 as priors and adding the mass models of the remaining 14 member galaxies via a scaling relation. Model 3 introduces a second Sérsic component for the background source, again using the previous model as a prior. Model 4 adds a third Sérsic component. Once the three-Sérsic fit is complete, we consider Model 4 to provide sufficiently accurate priors for the final stage: an adaptive pixelized source reconstruction using a Delaunay mesh in Model 5. This is split into two steps: in Model 5-0, the mass model is fixed and only the pixelized source is fitted, to optimize the adaptive brightness parameters ω_f and ω_p ; in Model 5-1, these source parameters are fixed, and the mass model is refined based on the priors from Model 4.

We apply a circular mask with a radius of 9.5'', using only pixels within the mask as lensing constraints. Different weighting strategies are used throughout the modeling stages. In Model 1 and Model 2, we identify the region potentially corresponding to the S2 image and artificially inflate its noise by a factor of 10^8 in the noise map. This effectively removes it from the fit, allowing us to focus on modeling the extended structure of S1. From Model 3 onward, we increase the weight of the S2 image by a factor of 3 (due to its lower brightness relative to S1) to balance the constraints between S1 and S2. However, we retain high noise around the G2 galaxy and its surroundings due to the lack of spectroscopic confirmation of

G2's mass contribution, and therefore exclude this region from the fit. Notably, while G2 and its surroundings are excluded in the noise map, only G2 itself is masked in the construction of the adapt image. This enables us to test whether the nearby arcs can still be reconstructed despite being downweighted in the fitting process.

4. Result

In this section, we present the main modeling results of the above modeling process applied to CSWA19.

4.1. Modeling Result from Parametric to Pixelized

As shown in Table 2, we design a progressive search chain strategy to avoid introducing too many free parameters at once, and to gradually increase the complexity of the model. The parameter fitting results of each step are passed to the next step. We start with the simplest single Sérsic background source model, increase model complexity, and finally switch to the pixelized source model. In this process, the parameter space becomes more complex as the number of Sérsic components describing the source increases. In the transition from Model 1 to Model 4, the background source is described by 1–3 Sérsic components, and in the transition from Model 4 to Model 5, the background source is replaced by a pixelized model, so only the mass parameters are passed in this step. Finally, we take

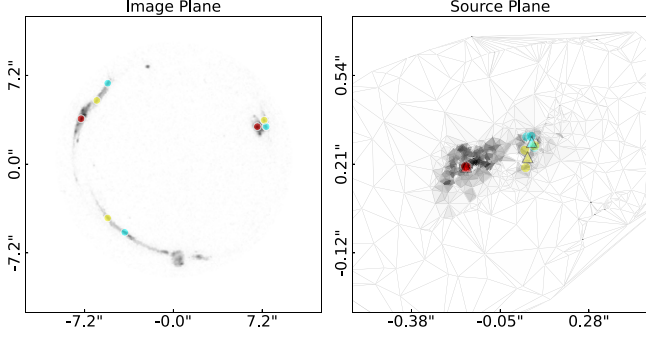


Figure 6. Reconstructed positions on the source plane of multiple image point-like positions. The bottom layer of the left image is the image data with the lens light subtracted, with the brightest points of S1(a) and S1(b) marked in red circles, the brightest points of S2.1(a), S2.1(b), and S2.1(c) marked in cyan circles, and the brightest points of S2.2(a), S2.2(b), and S2.2(c) marked in yellow circles. Based on the best fitting results of Model 5-1, the right image shows the relative positions of these multiple images traced to the source plane, the triangles represent the weighted average centers of these coordinates, and the bottom layer is the source plane Delaunay triangle mesh source reconstruction.

Model 5 as the adopted model result. As shown in Figure 5, we present the best-fitted modeling results of Model 4 and Model 5-1 in the search chain, which show the data image used as constraints, lensed source model image, normalized residual map and zoomed source reconstruction of their results.

We identify three groups of multiple images from visual inspection in Figure 1, although their positions are not used as lens constraints in our work. To verify the consistency between the extended source modeling result and the traced positions, we trace the peak points of these multiple images to the source plane based on the best-fitted results of Model 5-1 (see Figure 6). The red circles mark the brightest points of S1(a) and S1(b), the cyan circles mark the brightest points of S2.1(a), S2.1(b), and S2.1(c), and the yellow circles mark the brightest points of S2.2(a), S2.2(b), and S2.2(c). We can see the relative positions of these points on the source plane, where the triangles represent the weighted average centers of these points (weighted by magnification). The average separation of the source plane positions of S1 from the weighted center is $\sim 0''.003$, while for S2.1 it is $\sim 0''.02$ and for S2.2 it is $\sim 0''.04$.

Overall, our modeling result from both our 3-Sérsic model and adaptive-brightness pixelization model successfully resolve the two background sources S1 and S2, which is consistent with the results of Leethochawalit et al. (2016). S1 forms a double image, corresponding to the S1(a) and S1(b) regions on the image plane, while S2 forms a quadruple image, corresponding to the S2(a), S2(b), and S2(c) regions on the image plane. The fourth image may be near G2, but we are not sure about the specific situation around it.

The best-fitted and marginalized results for some important parameters in Model 5-1 with 1σ error are provided in Table 3,

Table 3

The Best-fit and Marginalized Values with 1σ Errors for the Model Parameters, Where the Adaptive-brightness Pixelization Model with Search Chain Model 5-1 is Adopted

Mass Component	Parameter	Fitting Result	
		Best-fit	Marginalized
BGG	$r_{s,BGG} ["]$	6.7617	$6.7637^{+0.0117}_{-0.0114}$
	$E_{0,BGG} ["]$	0.7395	$0.7395^{+0.0007}_{-0.0009}$
Halo	$x_{Halo} ["]$	0.1730	$0.1725^{+0.0004}_{-0.0004}$
	$y_{Halo} ["]$	0.0269	$0.0266^{+0.0012}_{-0.0012}$
	$e_{1,Halo}$	0.0831	$0.0831^{+0.0002}_{-0.0002}$
	$e_{2,Halo}$	-0.0120	$-0.0116^{+0.0002}_{-0.0003}$
	$r_{a,Halo} ["]$	16.6137	$16.6141^{+0.0205}_{-0.0192}$
	$E_{0,Halo} ["]$	29.4047	$29.4050^{+0.0368}_{-0.0341}$
L1	$r_{s,L1} ["]$	6.8246	$6.8248^{+0.0108}_{-0.0097}$
	$E_{0,L1} ["]$	1.5597	$1.5596^{+0.0020}_{-0.0023}$
Members	$r_{s,ref} ["]$	1.5048	$1.5058^{+0.0040}_{-0.0039}$
	$E_{0,ref} ["]$	0.2599	$0.2597^{+0.0004}_{-0.0004}$
External Shear	$\gamma_{ext,1}$	-0.0203	$-0.0201^{+0.0001}_{-0.0001}$
	$\gamma_{ext,2}$	0.0119	$0.0119^{+0.0001}_{-0.0001}$

and their joint posterior probability distributions are shown in Figure 7. As a direct result of lens modeling, the brightness-adaptive image mesh grid construction is shown in Figure 4. Note that the construction of this grid is unique based on the adapt image and parameter values. The left image of Figure 4 shows the data image with the lens light subtracted as the adapt image, with G2 masked, and the red circles show the adaptive grid points determined under the best-fitted results of Model 5-0. Under the mass profile of Model 5-1, these points can be traced to the source plane using the deflection angle to construct the Delaunay Triangulation.

5. Discussion

5.1. CSWA19 in the Scientific Context

In addition to using CSWA19 as a testbed to verify the ability of PYAUTOLENS to model a group-scale system, we are also interested in the specific properties CSWA19 has as a particular example and what scientific insights it can provide.

5.1.1. Mass Distribution of BGG and Main Halo

We calculate the cumulative mass distribution of BGG and the main halo as a function of the distance from the BGG center in the Model 5-1, and the results are shown in Figure 8. The positions of the effective radius and the Einstein radius are marked with gray lines in the figure. The effective radius of BGG, $R_{eff} = 21.7$ kpc, is measured from the best fitted multi-Sérsic model in Section 3.2 using the method outlined in Simard et al. (2011). The Einstein radius is calculated from the

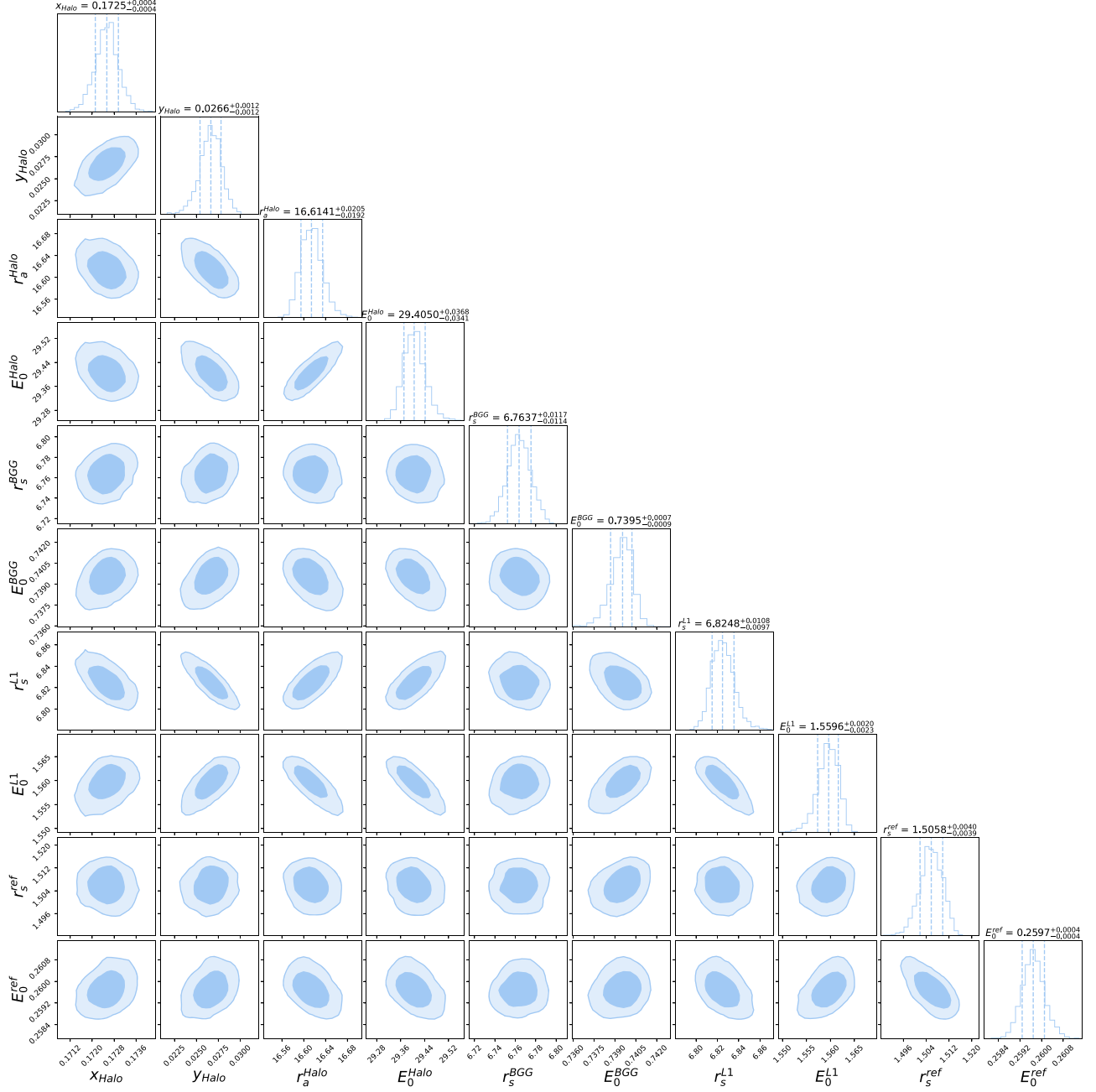


Figure 7. The joint posterior probability distribution of the adaptive-brightness pixelization model fitting results of search chain Model 5-1, showing the mass component parameters of the BGG, the main halo, L1, and the reference galaxy that we think are important. The shadow area of the two-dimensional distribution shows the 68.3% and 95.4% confidence intervals, while the histograms on the diagonal show the marginalized distribution of each parameter, with the corresponding quantile range marked.

equivalent circular radius of the area enclosed by the critical curve, which is defined as where the average surface density within the Einstein radius is equal to the critical surface density. The Einstein radius of the system is $\theta_E = 8''.0653^{+0.0001}_{-0.0001}$, corresponding to a physical scale of $50.0840^{+0.0006}_{-0.0007}$ kpc on the image plane. The vertical lines at the bottom of the image mark

the positions of the multiple images S1, S2.1, and S2.2, and it can be seen that the positions of the images are consistent with the positions of the Einstein radius. The mass within the Einstein radius is $1.4065^{+0.0002}_{-0.0001} \times 10^{13} M_{\odot}$ for total mass.

From the cumulative mass distribution (Figure 8), we find that the total mass at R_{eff} and R_{ein} is dominated by the main

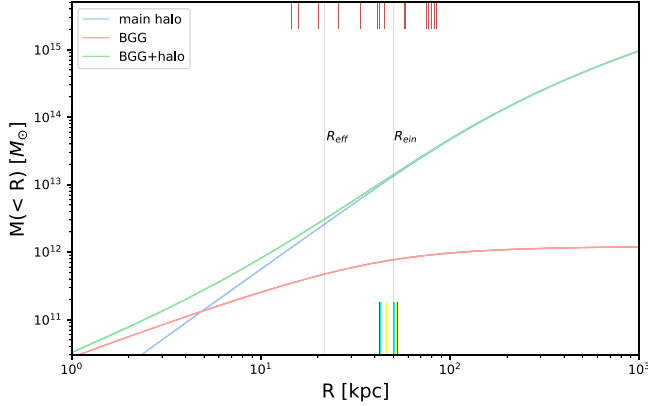


Figure 8. The cumulative mass profiles within the radius R (relative to the BGG center) for BGG (red solid line), main halo (blue solid line), and total mass (green solid line). The 1σ error range is narrower than the line width due to small statistical errors. The vertical lines at the bottom represent the multiple image positions in Figure 1, with the same color and label as in Figure 1. The red vertical lines at the top represent the positions of the member galaxies. We also use lines to mark the positions of BGG's effective radius $R_{\text{eff}} = 21.7$ kpc and the total Einstein radius $R_{\text{ein}} = 50.1$ kpc, R_{eff} is the best fitting value of BGG in Table 1, and R_{ein} is the Einstein radius of the best fitting results of Model 5-1.

halo, whereas the BGG component dominates at smaller radii. However, caution is required, as the fitting may underestimate the BGG mass. This limitation arises because strong lensing alone cannot reliably distinguish between the dark matter and stellar components of BGGs. Without complementary methods like stellar dynamics to securely decompose these contributions, the uncertainty persists. Nonetheless, the total mass estimate remains reliable, and the Einstein radius and enclosed mass derived in this work are consistent with previous results from Diehl et al. (2009), Stark et al. (2013).

5.1.2. Total Density Slope

The radial slope of the total mass density profile within the effective radius can be calculated by weighting the mass (Dutton & Treu 2014). Under the assumption of $\rho(r) \propto r^{-\gamma(r)}$ and the definition of $-\gamma(r) \equiv d \log \rho / d \log r$, the average slope $\tilde{\gamma}(r)$ within r is defined as:

$$\begin{aligned} \tilde{\gamma}(r) &= \frac{\int_0^r \gamma(x) \cdot 4\pi x^2 \cdot \rho(x) dx}{M(r)} = \frac{4\pi r^3 \cdot \tilde{\rho}(r)}{M(r)} - 3 \cdot \frac{\rho(r)}{\tilde{\rho}(r)} \\ &= 3 - 3 \frac{\rho(r)}{\tilde{\rho}(r)} = 3 - \frac{4\pi r^3 \rho(r)}{M(r)} \end{aligned} \quad (19)$$

where $M(r)$ is the cumulative mass within the radius r : $M(r) = \frac{4}{3}\pi r^3 \cdot \tilde{\rho}(r)$, where $\tilde{\rho}(r)$ is the average mass density within r , and $\rho(r)$ is the density at r . The 3D density

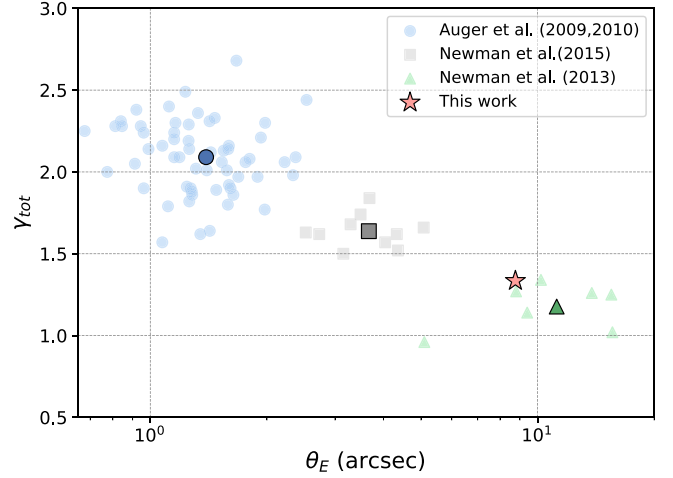


Figure 9. Trends of the total mass density slope within R_{eff} as a function of θ_E . These data points span a wide range of scales: galaxy-scale lenses from the SLACS survey (Auger et al. 2009, 2010) (blue circles), group-scale lenses in the work of Newman et al. (2015) (gray squares), and cluster-scale lenses in the work of Newman et al. (2013a, 2013b) (green triangles). The darker patterns corresponding to these different scale data points represent the average values of these data points. The results of CSWA19 are roughly located at the position of the red pentagram (with θ_E rescaled to a lensing ratio $D_{LS}/D_S = 0.7$).

distribution of dPIE (Elíasdóttir et al. 2007) is:

$$\begin{aligned} \rho(r) &= \frac{\rho_0}{(1 + r^2/r_a^2)(1 + r^2/r_s^2)} \\ &= E_0 \frac{\Sigma_{\text{crit}}}{2\pi} \frac{(r_s - r_a)(r_s + r_a)^2}{r_s \cdot (r_a^2 + r^2)(r_s^2 + r^2)} \end{aligned} \quad (20)$$

where Σ_{crit} is the critical surface density. Due to the lack of spectroscopic data and inclination angle measurements from stellar dynamics, we superimpose the density distributions of the BGG and main halo under this circular assumption. We then calculate the total density slope within R_{eff} to obtain $\tilde{\gamma}(R_{\text{eff}}) = 1.3327^{+0.0005}_{-0.0006}$.

Our result is contextualized with Newman et al. (2015), as shown in Figure 9. They found that the total density slope becomes shallower in larger systems and emphasized the importance of group-scale lenses in mapping the full evolutionary trend of $\tilde{\gamma}$. This trend is crucial in understanding how baryonic matter affects the dark matter distribution at different scales. Our result is located at the position of the red pentagram, which is consistent with the trend. Although CSWA19 belongs to group scale for its number of member galaxies, it is closer to the results of the cluster-scale within the trend.

5.1.3. Source Properties with Big Magnification

The total magnification μ_{tot} of the source is defined as the flux ratio between the image plane and the source plane. We found $\mu_{\text{tot},4} = 79.70^{+0.02}_{-0.03}$ in Model 4 and $\mu_{\text{tot},5-1} = 103.18^{+0.23}_{-0.19}$

in Model 5-1. The intrinsic magnitude of the background source in the F475W is thus ~ 25 mag.

This magnification is significantly higher than previous estimates: $\mu = 6.5$ reported by Stark et al. (2013), based on an SIS model from Diehl et al. (2009) using ground-based SDSS imaging; and $\mu = 4.3$ from Leethochawalit et al. (2016), who used HST data but constrained their LTM model solely with multiple image positions. The increase in magnification can be attributed to the intrinsically compact size of the background source, with effective radii in the range of $0''.07 \sim 0''.28$ (as inferred from Model 4), which are significantly stretched and distorted to a much larger scale. Notably, source component S2 is positioned very close to the caustic curve (Figure 5(a)) and even crosses it in the pixelized reconstruction (Figure 5(b)). Previous models, constrained primarily by multiple image positions and based on the low-resolution SDSS imaging, likely underestimated the magnification. Our results highlight the power of extended source modeling in revealing highly magnified, small-scale structures that may be missed by traditional methods.

5.2. Brightness-adaptive Pixelization as a Paradigm Shift

5.2.1. Challenges in Group-scale Lens Modeling

The modeling of group-scale strong lenses presents unique challenges compared to galaxy-scale or cluster-scale systems. Unlike cluster-scale lenses, which often exhibit multiple independent lensed images from different background sources, group-scale lenses typically have fewer multiply imaged sources and lack complete spectroscopic confirmation. In Section 4, we attempt to trace the brightest positions of multiple images back to the source plane despite the absence of spectroscopic confirmation (see Figure 6). We can clearly distinguish the positions of S1 and S2 as two sources on the source plane, but distinguishing S2.1 and S2.2 becomes difficult due to the larger separations (especially S2.2). This is evidence that selecting multiple image positions in the image plane can be inaccurate for group scale systems with large magnifications and extended arcs. It also reinforces the limitation of using point-like positions as constraints for group-scale lensing models, highlighting the reliability of extended image constraints (Xie et al. 2024).

Using extended image pixels as constraints offers a significant advantage over traditional point-like constraints. As demonstrated in previous works (Wang et al. 2022; Acebron et al. 2024; Urcelay et al. 2024; Xie et al. 2024), pixel-based constraints yield narrower statistical errors (see our result in Table 3 and Figure 7) compared to position-based constraints (refer to Figure 8 in Wang et al. 2022's works, Figure 7 in Acebron et al. 2024's work and Figure 3 in Xie et al. 2024's work). This improvement arises from the substantial increase in the number of independent constraints when using extended images.

Previous studies have employed either parametric light profiles (Urcelay et al. 2024; Xie et al. 2024) or pixelized reconstructions (Wang et al. 2022; Acebron et al. 2024). While parametric models (e.g., the Sérsic profile) struggle to represent complex and irregular brightness distributions, pixelized models demonstrate significantly lower residuals (see Figure 5), as parametric approaches often fail to reproduce fine-grained surface brightness features. Compared to purely parametric models, such as the 3-Sérsic model, pixelized models require very few nonlinear free parameters to describe the background source. Even though the linear inversion involves a large number of linear algebra operations, which increases computational cost during single-step calculations, fewer free parameters still speeds up the overall convergence time. Ideally, for models with more nonlinear parameters, whose nonlinear sampling becomes the dominant factor, the efficiency is higher with pixelized modeling.

5.2.2. Brightness-adaptive Pixelization as a Solution

We now assess the advantages of adapting source reconstruction to the brightness distribution in group-scale lenses, rather than to the lensing magnification as done in many previous methods. Magnification-based approaches typically place dense, uniform grids across the image plane to ensure accurate reconstructions. However, in systems like CSWA19—where the lensed source spans a wide circular region with an $8''$ radius—a large number of source pixels are required to capture the full emission. If each image pixel was mapped to a unique source pixel, the total would exceed 100,000, making such reconstructions computationally prohibitive.

The brightness-adaptive pixelization technique (Section 3.4) addresses this challenge by adjusting the grid density based on image brightness. This significantly reduces the number of source pixels and the dimensionality of the inversion matrices (Equations (12), (14)). In our analysis, just $I = 1000$ source pixels are sufficient to model the brightness distribution, compared to $J = 177,144$ image pixels used for residual calculations—yielding substantial computational savings without compromising reconstruction fidelity.

These gains are enabled by a Hilbert space-filling curve-based clustering algorithm (Figure 4), which concentrates source pixels in bright regions and sparsely samples faint ones. Although this method introduces two extra parameters (ω_f, ω_p in Equation (15)), our search chain (Section 3.5) optimizes them separately from the lens model to avoid inflating parameter space. Our results show that brightness-adaptive pixelization is a scalable and efficient technique for group-scale lensing studies.

Model 5-1 delivers a physically meaningful source reconstruction, resolving a merging galaxy pair and spiral-arm-like features. However, the normalized residuals remain high (up to $\pm 30\sigma$), indicating that finer structures are not fully recovered.

We attribute this to an overly simplistic lens mass model. Improvements likely require both a more complex lens galaxy parameterization (e.g., using multipoles Amvrosiadis et al. 2024) and relaxing the scaling relation assumptions for the member galaxies.

Looking ahead, brightness-adaptive pixelization offers a promising route for future group-scale lens modeling. As data sets grow larger and higher in resolution, methods that efficiently reconstruct complex source structures while remaining computationally tractable will be essential. Our results underscore the need for pixelized techniques that adapt to brightness, paving the way for the next generation of lens modeling.

6. Conclusion

In this work, we fully utilized high-resolution HST/WFC3 imaging to model the galaxy group-scale strong lens system CSWA19 at the pixel-level for the first time. We identified 16 member galaxies through red-sequence fitting (Figure 2) and subtracted their light contamination in the F475W and F160W bands (Figure 3). The foreground mass model—composed of a main halo, BGG, L1, and member galaxies in scaling relation—adopted pure dPIE profiles. The lensed source was reconstructed using PYAUTOLENS’s brightness-adaptive pixelization technique, which distributes source pixels along Hilbert space-filling curves (Figure 4). This method enabled efficient reconstruction of the total 177,144 HST pixels (pixel scale = 0″.04) in the WFC3/F475W image with only 1000 adaptive source pixels.

By progressively refining the model from multi-Sérsic parameterization to adaptive pixelization (Figure 5), we resolved the merging pair of background galaxies and measured a total magnification of $\mu_{\text{tot}} = 103.18^{+0.23}_{-0.19}$ —significantly exceeding previous estimates—which further demonstrates the advantage of extended image modeling over traditional point-position-based methods. The total mass within the Einstein radius was determined to be $M_{\theta_E} = 1.4065^{+0.0002}_{-0.0001} \times 10^{13} M_{\odot}$, with an Einstein radius of $\theta_E = 8″.0653^{+0.0001}_{-0.0001}$. Additionally, the total mass density slope within the effective radius was found to be $\tilde{\gamma}(R_{\text{eff}}) = 1.3327^{+0.0005}_{-0.0006}$, consistent with the $\gamma_{\text{tot}}-\theta_E$ correlation trend presented in Figure 9.

Future studies would benefit from integral field unit spectroscopy to resolve ambiguities in member galaxy identification (e.g., G2) and enable stellar-dynamical modeling of the BGG. While our current residuals are likely dominated by systematic uncertainties in the simplified mass model, advancements in computational frameworks—such as GPU-accelerated optimization with JAX (Bradbury et al. 2018)—could enable more flexible parameterizations. Currently, extended source modeling of group- and cluster-scale lenses is emerging as a transformative approach (Acebron et al. 2024;

Urcelay et al. 2024). Upcoming next-generation surveys like LSST, Euclid, and CSST will discover large samples of group-scale lenses, for which our methodology provides a scalable template to exploit high-resolution imaging. By integrating efficient modeling strategies with adaptive pixelization, these data sets will enable statistical studies of density slopes across mass scales, thus enhancing our understanding of the interplay between baryonic and dark matter at various scales.

Acknowledgments

This work has been supported by the National Key Research and Development Program of China (Nos. 2023YFB3002501 and 2022YFA1602903), the China Manned Space Project (No. CMS-CSST-2021-A01), and the National Natural Science Foundation of China (Nos. 12473002 and 11988101). L.W. acknowledges the support from the GHfund A (Nos. 202302017475 and 202407017555). We thank Andrew B. Newman (Carnegie Observatories and University of Southern California) for kindly providing the cluster-scale Einstein radii and normalization details critical to our comparative analysis. In this study, a cluster is used with the SIMT accelerator made in China. The cluster includes many nodes each containing two CPUs and eight accelerators. The accelerator adopts a GPU-like architecture consisting of a 64 GB HBM2 device memory and many compute units. Accelerators connected to CPUs with PCI-E, the peak bandwidth of the data transcription between main memory and device memory is 64 GB s^{−1}.

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