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Dreicer Electric Field Definition and Runaway Electrons in Solar Flares

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Abstract

We analyze electron acceleration by a large-scale electric field E in a collisional hydrogen plasma under the solar flare coronal conditions based on approaches proposed by Dreicer and Spitzer for the dynamic friction force of electrons. The Dreicer electric field $E_{\rm Dr}$ is determined as a critical electric field at which the entire electron population runs away. Two regimes of strong ($E \lesssim E_{\rm Dr}$) and weak ($E \ll E_{\rm Dr}$) electric field are discussed. It is shown that the commonly used formal definition of the Dreicer field leads to an overestimation of its value by about five times. The critical velocity at which the electrons of the "tail" of the Maxwell distribution become runaway under the action of the sub-Dreiser electric fields turns out to be underestimated by $\sqrt{3}$ times in some works because the Coulomb collisions between runaway and thermal electrons are not taken into account. The electron acceleration by sub-Dreicer electric fields generated in the solar corona faces difficulties.

Key words: acceleration of particles – Sun: flares – Physical Data and Processes

1. Introduction

Solar flares are a conversion process of free magnetic energy to kinetic and thermal energy. Moreover, they are a major particle accelerator in the solar system (Reames 2015). Almost all electrons contained in flare coronal loops should be accelerated (miller et al. 1997). This suggests that the very effective electron acceleration mechanism should be implemented during the flare energy release, for example, associated with the large-scale electric field generation (Zaitsev et al. 2016; Fleishman et al. 2022).

In a fully ionized plasma, the collisional friction force is inversely proportional to the square of the electron velocity v if it exceeds the most probable thermal velocity $v_{\rm Te}$ (see, e.g., Trubnikov 1965). As a result, a strong electric field acceleration force can overcome the collisional damping, accelerating high energy (runaway) electrons to relativistic speeds. The Dreicer electric field is the fundamental concept of this phenomenon (Dreicer 1958, 1959; Harrison 1960, Trubnikov 1965, Gurevich & Dimant 1978; Knoepfel & Spong 1979; Kaastra 1983; Benz 2002; Aschwanden 2004; Bellan 2006; Zhdanov et al. 2007; Fleishman & Toptygin 2013; Marshall & Bellan 2019). According to the generally accepted definition, the Dreicer electric field $E_{\rm Dr}$ (or Dreicer field) is a critical electric field at which electrons in a collisional plasma with $v \approx v_{\rm Te}$ can be accelerated, i.e., the entire electron population runs away (e.g., Holman 1985). The field $E_{\rm Dr}$ was named after Harry Dreicer who derived the corresponding expression for the critical electric field in 1958 (Dreicer 1958, 1959).

For the first time, the idea of runaway electrons was outlined by the Nobel Prize laureate Wilson (Wilson 1924) to explain thunderbolts in the Earth's atmosphere and was further developed by Gurevich (e.g., Gurevich & Zybin 2001). Gurevich's theory was applied by Tsap et al. (2022) (see also Tsap et al. 2020) in relation to the acceleration of electrons in the lower solar atmosphere during flares. However, the origin of strong electric fields was not considered.

The mechanism for ion runaway is different from electron runaway (e.g., Gibson 1959; Gurevich 1961; Furth & Rutherford 1972; Holman 1995; Fleishman & Toptygin 2013). The positive test charge experiences two opposite forces: acceleration due to E, and friction with the moving electrons. If the test charge has the same charge as the bulk ions, these two forces must be equal and opposite when the electric field $E < E_{\rm Dr}$. However, if the ionic charge Z differs from the charge of bulk ions Z_b, the forces scale differently with Z: electric field acceleration scales as Z, while friction on the drifting electrons scales as Z^2. Therefore, for $Z > Z_b$, the dominant force on the test charge will be electron friction, and the charge will be dragged to high energies as its velocity equilibrates with the electron mean flow. For $Z < Z_b$, friction becomes unimportant, so the test charge accelerates along E. Note that the total drag force on an ion does not monotonically fall off below $v_{\rm Te}$, but has a minimum and in a multispecies plasma "partial runaway" can occur. As to the solar flares, Holman (1995) has shown that the ions will be freely accelerated to energies greater than $\sim$1 MeV only if they are able to overcome the electron drag or if the entire electron population is freely accelerated, i.e., the electric field exceeds the Dreicer field.

Despite the concept of the Dreicer electric field being quite common in solar physics, there are some essential inconsistences. In particular, the formulae for the Dreicer electric field can differ by a factor of 4.7 (e.g., Aschwanden 2004, Equation (11.3.2); Bellan 2006, Equation (13.85)). Therefore, this issue requires a more detailed analysis.

The purpose of this work is to clarify the reason for the existing inconsistences and to discuss the consequences of the results in light of the electron accelerations in solar flares.

2. Dynamic Friction Force and Dreicer Electric Field

Let us consider two regimes in the motion of electrons under the action of an electric field. In the limit of the strong field regime ($E \lesssim E_{\rm Dr}$), the encounters between alike particles do not contribute to dynamical friction. In the weak field regime ($E \ll E_{\rm Dr}$), as distinguished from the previous case, the acceleration of runaway electrons is possible only in the "tail" of the Maxwellian distribution function, and we take into account the Coulomb collisions of accelerated electrons not only with thermal ions but also with thermal electrons of the background plasma.

2.1. Strong Field Regime

Following Dreicer (Dreicer 1958, 1959) (see also Trubnikov 1965) for the Maxwellian distribution function of electrons at the initial moment of time and ion (proton) gas with zero temperature, neglecting the interaction between alike particles and using the standard notation, the solution of the Boltzmann equation

$$\frac{df}{dt} + \frac{e\mathbf{E}}{m}\frac{df}{d\mathbf{V}} = \left.\frac{df}{dt}\right|_c, \tag{1}$$

can be represented as the displaced Maxwellian distribution function

$$f(t, v) = n\left(\frac{m}{2kT}\right)^{3/2} \exp\left\{-\frac{m}{2kT}(\mathbf{V} - \mathbf{v}(t))^2\right\}. \tag{2}$$

Here the average electron velocity $\mathbf{v}(t)$ is the solution of an equation of motion which includes the effects of collisions and has the form

$$m\frac{dv}{dt} = e(E - E_c G(x)), \quad x = \frac{v}{v_{\rm Te}}, \quad v_{\rm Te} = \sqrt{2kT/m}, \tag{3}$$

where the Chandrasekhar function

$$G(x) = \frac{\Phi(x) - x\Phi'(x)}{2x^2}, \quad \Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy,$$

and the critical electric field E_c is

$$E_c = \frac{4\pi n e^3}{kT}\ln\Lambda. \tag{4}$$

Note that Dreicer (Dreicer 1958, 1959) used the function $\Psi(x) = 2G(x)$ instead of $G(x)$.

It should be stressed that the electric field E_c (Norman & Smith 1978; Holman 1985; de Jager 1986; Benz 2002, Equation (9.2.6); Aschwanden 2004, Equation (11.3.2); Tsap & Kopylova 2017) or $E_c/2$ (Kuijpers et al. 1981; Moghaddam-Taaheri & Goertz 1990) is called the Dreicer electric field E_D in the papers devoted to the electron acceleration in solar flares. Meanwhile, the Chandrasekhar function $G(x)$ reaches its maximum at $x \approx 1$ ($v \approx v_{\rm Te}$) and $G(1) \approx 0.214$ (see, e.g., Trubnikov 1965). As a result, the condition of the acceleration of runaway electrons with $v \approx v_{\rm Te}$, in view of Equations (3) and (4), takes the form (see also, Dreicer 1958, 1959; Trubnikov 1965; Golant et al. 1977, Equation (7.174); Bellan 2006, Equation (13.85))

$$E > E_{\rm min} = E_{\rm Dr} \approx E_c G(1) \approx \alpha\frac{4\pi n e^3}{kT}\ln\Lambda = \alpha\frac{e}{r_{\rm De}^2}\ln\Lambda, \tag{5}$$

where $\alpha = 0.214$ and the Debye radius $r_{\rm De} = \sqrt{kT/(4\pi n e^2)}$. The inequality, $E > E_{\rm Dr}$, can be considered as a condition for runaway acceleration when all electrons accelerate to high energy.

It is interesting to note that sometimes for the definition of the Dreicer electric field, kinetic effects connected with the velocity distribution functions of charged particles are not taken into account and the thermal electron velocity $\bar{v}_{\rm Te} = v_{\rm Te}/\sqrt{2}$, instead of the most probable one $v_{\rm Te}$, is used (e.g., Holman 1985; Tsap & Kopylova 2017). In this case, $x = 1/\sqrt{2} \approx 0.71$ and the Chandrasekhar function $G(0.71) \approx 0.198$ (Spitzer 1962). Since $G(1) > G(0.71)$, the acceleration of the entire electron population is formally impossible in this case because, according to Equation (3), the braking force

$$F_D(x) = eE_c G(x),$$

reaches its maximum at $x \approx 1$.

Thus, if we proceed from the definition that the Dreicer electric field $E_{\rm Dr}$ is the minimum electric field $E_{\rm min}$ at which all electrons undergo free acceleration, then the Dreicer field $E_{\rm Dr} = E_{\rm min}$. This approach seems to be more justified than the approach based on $E_D = E_c$ and agrees with the definition of the Dreicer electric field $E_{\rm Dr}$ proposed in Bellan (2006, Equation 13.85); Zhdanov et al. (2007, Equation 1.107); Marshall & Bellan (2019). The commonly used formal Dreicer electric field is (Holman 1985; Benz 2002, Equation (9.2.7); Aschwanden 2004, Equation (11.3.2))

$$E_D = \frac{4\pi n e^3}{kT}\ln\Lambda = \frac{e}{r_{\rm De}^2}\ln\Lambda, \tag{6}$$

and it turns out to be approximately 4.7 times greater than the Dreicer electric field $E_{\rm Dr}$ because, according to Equations (5) and (6), the ratio $E_{\rm Dr}/E_D \approx \alpha$.

The obtained difference is partially caused by different approaches which are used for the dynamic friction force calculation. For example, according to Spitzer (Spitzer 1962), the dynamic friction force for the electron flux (test particle) caused by the Coulomb collisions with Maxwellian thermal protons is (Harrison 1960; Spitzer 1962, Equation (5.15); Knoepfel & Spong 1979)

$$F_{\rm ep} = \frac{4\pi n e^4}{kT} \ln\Lambda \frac{M}{m} G(\sqrt{M/m}\, x), \quad (7)$$

where M is the mass of a proton.

Assuming $\sqrt{M/m}\, x \gg 1$ $(G(y \gg 1) \approx 0.5y^{-2})$, instead of Equation (7), we have

$$F_{\rm ep} \approx \frac{4\pi n e^4}{m v^2} \ln\Lambda. \quad (8)$$

Note that the square root of $\sqrt{M/m} \approx 43$ and for the electron velocity $v = v_{\rm Te}$ $(x = 1)$ from Equations (6)–(8) we find the "Dreicer electric field"

$$E_{DS} = \frac{F_{\rm ep}}{e} \approx \frac{2\pi n e^3}{kT} \ln\Lambda = \frac{E_D}{2}. \quad (9)$$

Equation (9) agrees with the appropriate expressions in Kuijpers et al. (1981); Moghaddam-Taaheri & Goertz (1990).

It should be stressed that Spitzer (1962) did not take into consideration the velocity distribution of electrons exposed to an external electric field. In spite of this, the formulae for dynamic friction forces obtained with Spitzer's and Dreicer's approaches are coincided at $x = v/v_{\rm Te} \gg 1$, because Equations (3), (4) and (8) give

$$F_D = eE_c G(x \gg 1) \approx \frac{4\pi n e^4}{m v^2} \ln\Lambda \approx F_{\rm ep}.$$

In fact, in accordance with Equation (7), the friction force $F_{\rm ep}$ reaches its maximum value when the electron velocity v is equal to the thermal proton velocity $v_{Tp} = \sqrt{2kT/M}$ $(\sqrt{M/m}\, x = 1)$ and

$$F_{\rm ep}^{\max} \approx 2\alpha \frac{4\pi n e^4}{kT} \frac{M}{m} \ln\Lambda.$$

Since $F_{\rm ep}^{max} \gg F_D(1) \approx \alpha F_{\rm ep}(1)$, we can conclude that Spitzer's approach does not work for slow $(v \lesssim v_{\rm Te})$ electrons in the strong field regime (see also Figure 1, Holman 1995).

2.2. Weak Field Regime

In the general case, Spitzer (1962) has shown that the total dynamic friction force for the electron flux with the same initial velocity due to the Coulomb collisions with thermal electrons and protons of a Maxwellian hydrogen fully ionized plasma is (Harrison 1960; Spitzer 1962, Equation (5.15); Knoepfel & Spong 1979)

$$F_S = \frac{4\pi n e^4}{kT} \ln\Lambda \{2G(x) + \frac{M}{m} G(\sqrt{M/m}\, x)\}, \quad (10)$$

where the first term on the right-hand side of Equation (10) corresponds to the friction force caused by electron-electron collisions $F_{\rm ee}$. Then it follows from Equation (10) that at $x \gg 1$ we have (Golant et al. 1977, Section 7.11)

$$F_S = F_{\rm ee} + F_{\rm ep} \approx \frac{12\pi n e^4}{m v^2} \ln\Lambda, \quad (11)$$

where

$$F_{\rm ee} \approx \frac{8\pi n e^4}{m v^2} \ln\Lambda.$$

This allows us to find the critical velocity $v_{\rm cr}$ for runaway electrons based on the equality between the electric and the dynamic friction force, which has the form

$$F_S = eE.$$

Consequently, using Equation (11), we get

$$v_{\rm cr}^2 = \frac{12\pi n e^3}{mE} \ln\Lambda. \quad (12)$$

Equation (12) agrees well with the appropriate expression in Golant et al. (1977, Equation (7.176)). After that, in view of Equations (6) and (12), we find

$$v_{\rm cr}^2 = \frac{3}{2} \frac{E_D}{E} v_{\rm Te}^2. \quad (13)$$

It should be stressed that according to Knoepfel & Spong (1979), the square of the critical velocity is

$$v_c^2 = \frac{4\pi n e^3}{mE} \ln\Lambda = \frac{E_D}{2E} v_{\rm Te}^2. \quad (14)$$

Comparing Equations (13) and (14), it easy to conclude that the critical velocity in Knoepfel & Spong (1979) was underestimated by $\sqrt{3}$ times $(v_{\rm cr}/v_c = \sqrt{3})$ because authors did not take into account collisions between runaway and thermal electrons as distinguished from us and Golant et al. (1977).

A small difference between values of $v_{\rm cr}$ and v_c can be very important to estimate the number of runaway electrons in the "tail" of the Maxwellian distribution function. Indeed, as follows from Kaplan & Tsytovich (1972, Equaton (9.10); Holman 1985), the ratio of the accelerated electrons to their total number is

$$\frac{n_r}{n_e} \approx \exp\left[-\left(\frac{v_r}{v_{\rm Te}}\right)^2\right]. \quad (15)$$

Then, from Equation (15) we derive

$$\frac{n_{\rm cr}}{n_c} \approx \exp\left[-\frac{v_{\rm cr}^2 - v_c^2}{v_{\rm Te}^2}\right] = \exp\left[-\frac{2v_{\rm cr}^2}{3v_{\rm Te}^2}\right]. \quad (16)$$

Supposing $v_{\rm cr} = 3v_{\rm Te}$, we find from Equation (16) that $n_{\rm cr}/n_c \approx 2.5 \times 10^{-3}$ because the total friction force F_S is greater than $F_{\rm ep}$. Therefore, the difference in the number of runaway electrons can reach orders of magnitude in spite of the small difference between values of $v_{\rm cr}$ and v_c. This means that the electron acceleration in solar flare coronal loops by sub-

Dreicer electric fields faces difficulties (for details see Tsap et al. 2022).

3. Discussion and Conclusion

We have shown that the definitions of the Dreicer electric field differ in diverse works. This is partly explained by different approaches proposed by Dreicer (1958, 1959) and Spitzer (1962). In particular, Dreicer considered the interaction between the electrons with the displaced Maxwellian distribution and an ion gas at zero temperature, while Spitzer investigated the evolution of the electron flux with the same initial velocity in the Maxwellian plasma. These approaches complement each other, but Equation (5) for the Dreicer electric field $E_{\rm Dr}$ seems to be the most adequate because the distortion of the distribution function of electrons under the action of an electric field is taken into account in this case. Note that some authors are restricted to the approximation of pair collisions and do not take into account the kinetic effects connected with the velocity distribution of charged particles (e.g., Tsap & Kopylova 2017).

The energy of runaway electrons can be essentially different because of different definitions of the Dreicer electric field. This may be quite an important point for electron acceleration in solar flares. Indeed, the Dreicer electric field can be considered as a rough estimate of the peak electric field in the coronal collisional plasma. This suggests that the maximum energy of a runaway electron W_m under the action of electric field is

$$W_m = eE_{\rm Dr}L,$$

where L is the characteristic length of a coronal loop. Therefore, using Equation (5), we find

$$L = \frac{W}{eE_{\rm Dr}} = W\frac{r_{\rm De}^2}{\alpha e^2 \ln\Lambda} \approx 1.44 \times 10^9 \frac{W[\rm eV]T[\rm K]}{n_e[\rm cm]^{-3}\ln\Lambda}. \quad (17)$$

Assuming $W = 100$ keV, $T = 3\times10^6$–10^7 K, $n_e = 10^8$–10^{10} cm^{-3}, from Equation (17) we get $L = 2.5\times10^9$–6.7×10^{11} cm. Since the characteristic length of flare coronal loops $L = 3\times10^9$ cm (Stepanov & Zaitsev 2018) and $n_{\rm cr}/n_c \ll 1$ (see Equation (16)), the obtained estimates suggest that the electron acceleration by sub-Dreicer electric fields seems unlikely in solar flares (see also Fleishman & Toptygin 2013). However, we did not take into account the possible important role of the electron acceleration by the induced electric field for the betatron mechanism (Tsap & Melnikov 2023). The essential increase of the Dreicer electric field can be caused by the ion-neutral collisions (Stepanov & Zaitsev 2018) and the interaction of accelerated electrons with turbulent pulsations (Kaplan & Tsytovich 1972). Note that some details on the electron acceleration by the super-Dreicer field ($E \gtrsim E_{\rm Dr}$) are discussed in Fleishman & Toptygin (2013).

We used a quite rough approach for the estimates of accelerated electrons in the "tail" of the Maxwellian distribution function and did not take into consideration the Joule dissipation and plasma heating. This should lead to a reduction of the Dreicer field $E_{\rm Dr}$ due to a temperature increase and, hence, the number of runaway electrons should also be increased. Besides, runaway electrons can be generated due to collisions between runaways and thermal electrons. Such collisions might be infrequent, but if they do occur, there is a high chance that after the collision both electrons will have a velocity that is higher than the critical momentum. This amplification of the runaway electron population is called the avalanche mechanism (Smith & Verwichte 2008). In addition, for relativistic runaway the friction attains a minimum value, i.e., the friction force increases for electrons with velocities $v \approx c$ (see, e.g., Gurevich & Zybin 2001), and additional physical effects such as radiation losses become important. These issues need further detailed investigations.

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