# Investigations of Thermal Deformation Based on the Monitoring System of Tianma 13.2 m VGOS Telescope 

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#### Abstract

The thermal expansion of radio telescopes has been recognized as a significant systematic error in Very Long Baseline Interferometry (VLBI) data analysis. Although the thermal expansion model recommended by International Earth Rotation Service (IERS) Conventions 2010 can achieve millimeter accuracy for the International VLBI Service for Geodesy and Astrometry (IVS) routine telescopes, both the International Terrestrial Reference Frame 2020 (ITRF2020) in preparation and the VLBI2010 project encourage the scientific community to reconsider its modeling. To this end, we developed a monitoring system for the Tianma 13.2 m VLBI Global Observing System (VGOS) telescope. Based on the observed data, we refined the IERS expansion model, with results showing that the accuracy of our modified model was improved by 1.9 times. It suggested that the IERS thermal expansion model can achieve the declared millimeter accuracy, and refining its modeling can meet the requirement of 0.3 mm rms stability of the VGOS antenna reference point for the VLBI2010 project.


Key words: Astronomical Instrumentation - Methods and Techniques - methods: analytical - methods: data analysis - telescopes

## 1. Introduction

The International Earth Rotation Service (IERS) thermal expansion model (Nothnagel 2009; Petit \& Luzum 2010) has been widely used in Very Long Baseline Interferometry (VLBI) data analysis to reduce the annual signals of the height of the reference point of radio telescopes. However, partly due to some mismodeling of the VLBI antenna thermal deformation, the amplitude of the VLBI scale correlated to local station height is at least 2 times larger than that of the satellite laser ranging (SLR) scale, while the VLBI phase is almost similar to SLR one (Altamimi et al. 2011, 2016). Collilieux et al. (2005) and Altamimi et al. (2007) showed that the impact of this mismodeled thermal expansion on the VLBI scale may be of the order of 1.3 mm (at the Earth's surface using a mean Earth radius of 6375 km ) in amplitude. The thermal expansion model recommended by IERS (Petit \& Luzum 2010) has been standardized in VLBI data analysis, yet it is still necessary to verify or refine the present thermal expansion model, especially with the coming of a new system VLBI Global Observing System (VGOS) . To achieve the VLBI2010 project goal of 1 mm position accuracy in 24 h , the three-dimensional (3D) position of the VGOS telescope reference point must be either stable or modelable to 0.3 mm rms (Petrachenko et al. 2009). Currently, however, the IERS thermal expansion model can only achieve millimeter accuracy for IVS routine telescopes
(Nothnagel 2009). Therefore, it is crucial to investigate the thermal deformation of the VGOS antenna.

Given that the correlations between thermal expansion, station heights, and zenith delays in VLBI data analysis (Nothnagel et al. 2002; Le Bail et al. 2013), monitoring the displacement of the reference point with temperature change is a more direct and effective method to investigate the thermal expansion of the VLBI antenna structure. At the Hartebeesthoek Radio Astronomy Observatory (HartRAO) in South Africa, Nothnagel et al. (1995) observed a survey mark mounted on the telescope structure close to the VLBI reference point to estimate the time delay between the change in the surrounding air temperature and the expansion of the telescope structure, which is one parameter of the IERS thermal expansion model. Wresnik et al. (2007) replaced the ambient temperature with modeled temperature of the antenna structure in the IERS thermal expansion model, with results showing that the modified model agrees better than the IERS model with the local measurements. However, Wresnik's modified model is too complex to be widely used (Petrachenko et al. 2009).

Currently, the thermal height of VLBI telescopes is described by the height change of the telescope using an invar wire (Zernecke 1999) or elevation angles (Nothnagel et al. 1995). Usually, it is the only way to indirectly determine the reference point by observing some targets on the telescope
structure to fit the 3D circular trajectory (Eschelbach \& Haas 2003; Sarti et al. 2004; Dawson et al. 2007). Although the 3D circle-fitting method can determine the reference point with less than 1 mm rms , it requires a large number of observations of targets and good enough space coverage, which is labor-intensive and time-consuming. The transformation model as the modified version of the 3D circle-fitting method also cannot collect enough data to study thermal deformation with its low sampling rate and lack of data (Lösler \& Hennes 2008; Lösler 2008; Lösler et al. 2013). It takes more than 1 day to determine the reference point with less than 1 mm rms by using either of the two above indirect methods. The thermal deformation during the measurement period may bias the reference point estimate by several millimeters degrading its accuracy but not affecting its formal precision (Abbondanza \& Sarti 2012). According to the International Terrestrial Reference Frame 2014 (ITRF2014) calculation, Altamimi et al. (2016) believes that a local tie between physically inaccessible instrumental measurement reference points is unlikely to be precise to better than 3 mm . Therefore, indirect methods cannot be used to monitor the change of reference point position with temperature.

The purpose of this paper is to validate and refine the IERS model, based on monitoring the thermal deformation of the antenna structure with higher accuracy and sampling rate, with our modified model realizing the stability of 0.3 mm rms of the 3D position of the VGOS antenna reference point.

In order to observe the change of the reference point position, we developed a monitoring system for Tianma 13.2 m VGOS telescope to observe a prism rotating around the telescope azimuth axis and close to the reference point. The center of the circular trajectory of the prism is referred to as the approximate reference point (ARP). Based on the observed data, we validated and refined the IERS model. Compared to the observed data in vertical, residuals of the IERS model were less than 1.4 mm and the rms was 0.43 mm . By refining the IERS model, residuals of our modified model were less than 0.5 mm , and the rms was 0.15 mm . According to our calculation, the horizontal position change of the prism is negligible ( $\sim 0.1 \mathrm{~mm}$ ) and not significantly correlated with ambient temperature. We can conclude that the horizontal displacement of the reference point with ambient temperature is negligible, mainly because both the prism and the reference point are on the symmetric axis of the telescope. It suggested that the IERS thermal expansion model can achieve the declared millimeter accuracy, but only refining its modeling can achieve the VLBI2010 project goal which requires the VGOS antenna reference point must be either stable or modelable to 0.3 mm rms.

Section 2 introduces the monitoring system and evaluates its measurement accuracy. Section 3 analyzes the correlation between temperature and height, refines the modeling method, and finally verifies the IERS thermal expansion model and the
modified model. Section 4 discusses and concludes the corresponding results.

## 2. Observations and Data Reduction

### 2.1. The Monitoring System of Tianma 13.2 m VGOS Telescope

The geodetic reference point, or "invariant point", is the intersection point between the fixed rotation axis and the plane that contains the moving axis and is perpendicular to the fixed axis. For an elevation-over-azimuth mount, the fixed and moving axes are the azimuth and elevation axes, respectively. If the offset between the rotation axes is zero, the reference point is the point where the axes intersect (Petrachenko et al. 2009). According to the IERS thermal expansion model with the assumption that the telescope structures and their components expand or shrink linearly with temperature in a symmetric way (Nothnagel 2009; Petit \& Luzum 2010), for the Tianma VGOS telescope, a point close to the reference point and on the azimuth axis of the telescope (such as the ARP of the prism) has similar displacement caused by thermal deformation as the reference point.
The monitoring system developed by University of Information Engineering in Zhengzhou mainly consists of control and measurement systems. According to the survey scheme, the control system sends commands to the measurement instruments and controls them for observation. As shown in Figure 1, the measurement system consists of a TOPCON DS101AC total station in the middle of the telescope tower, a TOPCON AK11 retro-reflecting prism mounted under and 1.50 m far from the reference point, and an RT1000-WS three-parameter meteorological sensor beside the total station.

The three-parameter meteorological sensor can obtain indoor temperature, humidity, and air pressure in the telescope tower to correct the measurements of the total station. The temperature measurement accuracy is $0.1^{\circ} \mathrm{C}$. Note that in order to maintain a stable measuring environment for the measurement system, two air conditioners are installed in the tower. The temperature variation in the tower is less than $1^{\circ} \mathrm{C}$ to ensure that the total station and its survey pillar suffer from an insignificant thermal expansion effect.

The distance accuracy of DS101AC is specified by the manufacturer as $\sigma_{\text {Dist }}=1.5 \mathrm{~mm}+2 \mathrm{ppm}$ and the accuracy of both horizontal and vertical angles as $\sigma_{\mathrm{Hz}}=\sigma_{V}=1^{\prime \prime}$ for one measurement on two faces.

Because thermal deformation is a relative value, in this paper all observations are carried out in one face with the total station function of automatic target recognition (ATR). To evaluate the relative measurement accuracy of ATR mode, several experiments are performed at University of Information Engineering.
First, three prisms were observed by the total station with ATR mode for 3 days indoors (see Figure 2). The sampling interval of the measurement system was 2 minutes, and the measurement was repeated three times every prism. For static


Figure 1. The height of the VGOS telescope concrete foundation $h_{f}=8.32 \mathrm{~m}$, the height from the bottom of the steel structure to the reference point is $h_{a}=2.98 \mathrm{~m}$, and the height of the steel structure for the ARP is $h_{p}=1.48 \mathrm{~m} . \mathrm{S}$ stands for the total station, and T 1 is the prism.


Figure 2. Static measurement accuracy test of the total station with ATR mode. Three red circles highlight the prisms.
measurement, variations of coordinate time series can reflect the relative accuracy of ATR measurement mode. The results show that the indoor temperature varies from $21.6^{\circ} \mathrm{C}$ to $23.5^{\circ} \mathrm{C}$, and the variation of each coordinate component of the three prisms is within 0.1 mm .

Second, to estimate the accuracy of dynamic measurements, a prism is mounted on the H840 6-DOF parallel robot from PhysikInstrumente company. The repeatability of $X, Y$ and $Z$ components of the H 480 robot is $\pm 0.5, \pm 0.5$ and $\pm 0.4 \mu \mathrm{~m}$ respectively (Wang 2017). The prism rotates in the vertical


Figure 3. Dynamic measurement accuracy test of the total station with ATR mode to simulate the monitoring to the Tianma VGOS telescope. The red circle highlights the H 480 robot on which a prism is mounted. The robot is fixed on the fourth floor and the total station is under 13 m from the robot.
plane with the H480 robot and forms three circular trajectories. Results from the total station with ATR mode show that the rms of the variation of the $X, Y$ and $Z$ coordinate components of the prism are within 0.1 mm . The positions of the prism are projected onto the vertical plane and the circular trajectory is fitted by least square adjustment, with results showing that the rms of residuals is 0.05 mm .

Third, in order to simulate the monitoring of the reference point of the Tianma VGOS antenna, the H480 robot is installed on the fourth floor of a building, and the total station is located 13 meters under the H840 robot (see Figure 3). Five horizontal circular trajectories of the prism are observed by the total station with ATR mode and the result shows that the rms of the $X, Y$ and $Z$ coordinate components of the prism are 0.1 mm . The positions of the prism are projected onto the horizontal plane and the circular trajectory is fitted, with results showing that the rms of residuals is 0.1 mm .

To obtain thermal deformation of the Tianma VGOS telescope, a monitoring experiment was carried out from 2020 November 10 to 2021 October 18. The total station observed the prism two times consecutively in one face with ATR mode in each measurement period per minute. Calculating the rms of the difference between two consecutive
observations (including the distance and angle) per minute, the rms of $X, Y$ and $Z$ coordinate components is 0.1 mm , derived by the law of propagation of uncertainty.

Based on the above accuracy evaluation, it can be concluded that the accuracy of the monitoring system can reach 0.1 mm and this system can monitor the VGOS antenna reference point within 0.3 mm rms .

The ambient temperature of the Tianma VGOS telescope is obtained from temperature sensors in the park of the Tianma 65 m telescope, which represents the air temperature around the VGOS telescope.

### 2.2. Observations of Thermal Deformation

To determine the horizontal position of the ARP, a 2D transformation model modified from the 3D transformation model (Lösler et al. 2013) is introduced to determine the relative relationship between the prism and ARP. There are two different coordinate systems introduced into the 2D transformation model. The first is the monitoring observation coordinate system of the terrestrial survey instrument (the total station), and the second is the telescope coordinate system. The definition of the observation coordinate system is that the origin of the coordinate system is the center of the total station, the $x$ axis is the horizontal axis of the total station, the $z$-axis corresponds to the local plumb line, and the $y$-axis is perpendicular to the $x$-axis and the $z$-axis. The definition of the telescope coordinate system is that the origin of the coordinate system is the ARP, the $x$-axis is parallel to the elevation axis, the $z$-axis corresponds to the azimuth axis of the telescope, and the $y$-axis is perpendicular to the $x$-axis and the $z$-axis. Both the fixed observation system of the terrestrial survey instrument and the telescope system are right-handed systems (Lösler 2008). The 2D transformation model can be described by Equation (1)

$$
\begin{equation*}
\boldsymbol{P}_{\mathrm{Obs}}=\boldsymbol{R}(\mathrm{A}+\mathrm{OA}) \cdot \boldsymbol{P}_{\mathrm{Tel}}+\boldsymbol{P}_{R} \tag{1}
\end{equation*}
$$

where $\boldsymbol{R}$ denotes a rotation matrix for a rotation with a specific angle on a horizontal plane, the azimuth angle of the telescope is denoted by $A$ with the associated orientation angles OA between the two x-axes of the observation system and the telescope system, $\boldsymbol{P}_{\mathrm{Tel}}$ and $\boldsymbol{P}_{\mathrm{Obs}}$ are the horizontal positions of the prism in the telescope coordinate system and the observation coordinate system respectively, and $\boldsymbol{P}_{R}$ is the translation vector from the telescope system to the observation coordinate system.

Since there is a complete correlation between the azimuth angles of the telescope $A$ and the orientation angle OA, in advance we make a rotation by OA from the telescope system to the observation system. Therefore, the 2D point $\boldsymbol{P}_{\mathrm{Tel}}$ is transformed into $\boldsymbol{E}$, and the 2D model is modified by

$$
\begin{equation*}
\boldsymbol{P}_{\mathrm{Obs}}=\boldsymbol{R}(A) \cdot \boldsymbol{E}+\boldsymbol{P}_{R} \tag{2}
\end{equation*}
$$



Figure 4. The top three represent three coordinate components of the prism in coordinate system of the total station respectively. The bottom shows the corresponding azimuth angles of the VGOS telescope in the telescope coordinate system.

Once the 2D transformation model is determined, we can monitor the displacement change of the prism with respect to ARP (see Equation (3)).

$$
\begin{equation*}
\Delta \boldsymbol{E}=\boldsymbol{R}(-A) \cdot\left(\boldsymbol{P}_{\mathrm{Obs}}-\boldsymbol{P}_{R}\right)-\boldsymbol{E} \tag{3}
\end{equation*}
$$

If the azimuth angles of the telescope $A$ are not recorded, time series $r$ of the radius from the prism to ARP can reflect variations of the prism with temperature, which can be described by Equation (4).

$$
\begin{equation*}
r=\left\|\boldsymbol{P}_{\text {Obs }}-\boldsymbol{P}_{R}\right\|_{2} \tag{4}
\end{equation*}
$$

The Tianma 13.2 m VGOS telescope rotated one circle at each of three elevation angles on 2020 November 10. Figure 4 shows the 3D positions of the prism with the azimuth of the VGOS antenna. There was no obvious correlation between the prism positions and the telescope elevation positions according to observations, and thus the telescope elevation positions were not shown in Figure 4.

The observed data were substituted into the 2D transformation model, and the results of the least square adjustment were shown in Table 1.

Substituting the estimates in Table 1 and observations from the total station into Equation (4), we can obtain time series of $\mathbf{r}$

Table 1
$\left(p_{x}, p_{y}\right)$ is the Horizontal Position of the ARP in the Monitoring Observation Coordinate System, and $\left(e_{x}, e_{y}\right)$ is the Horizontal Position of the Prism T1 in Telescope Coordinate System with an Orientation Compensation OA

| Parameters | $p_{x}$ | $p_{y}$ | $e_{x}$ | $e_{y}$ |
| :--- | :---: | :---: | :---: | :---: |
| Value (mm) | 3.61 | -9.81 | 0.12 | 0.02 |
| rms (mm) | 0.01 | 0.02 | 0.01 | 0.01 |

that reflects the horizontal distance change from the prism to the ARP (see Figure 5).

Figure 5 shows that the horizontal distance from the prism to ARP is not significantly correlated with ambient temperature, and variations of the horizontal distance are less than 0.1 mm .

To clearly show the correlation between temperature and the height of ARP, we present only a portion of the observed data, and hourly average values are taken in Figure 6 in such a way that high-frequency signals of less than an hour are filtered.

By fast Fourier transform (FFT), we obtained the periodic signals of the ambient temperature in Tianma Park and ARP height respectively. Before FFT, we removed the constant term in the observed data by a weighted moving average filter with the window length more than half of the observation time span.


Figure 5. Variations of the radius from prism to ARP with temperature from 2021 May 21 (MJD 59 355.0) to 2021 July 18 (MJD 59413.0 ). The red line represents the long-term trend obtained by eliminating short-period signals with a weighted moving average of 30-day window.


Figure 6. Variations of ARP heights with temperature from 2021.1.1 (MJD 59 215.0) to 2021.3.17 (MJD 59 290.0). For clarity of the correlation between height changes and temperature, an hourly average is taken.

The results showed that the periods of the signals from these two observed data (ambient temperature and ARP height) are exactly the same (see Figure 7), which means that the influence of other factors (such as the instrument error) on ARP height is
constant except for the ambient temperature. In other words, the instrument error from the monitoring system is a constant in the observed height of the ARP. In fact, because the indoor temperature in the VGOS antenna tower does not change more


Figure 7. The periods of signals decomposed by the observed data by FFT. The horizontal axis represents the periods of the periodic signals derived from temperature. The vertical axis represents the periods of the periodic signals from the ARP height. The unit is day. The periods of the signals from these two observed data (ambient temperature and ARP height) are exactly the same.
than $1^{\circ} \mathrm{C}$, the survey pillar of the total station does not undergo significant thermal deformation. Moreover, because the Tianma VGOS telescope is stationary most of the time during the monitoring, the instrumental error of the total station caused in one face measurement should be a constant. It can be concluded that the instrument error of the monitoring system is a constant, and therefore thermal deformation as a relative value is not affected by the systematic error in our work.

## 3. Data Analysis and Results

According to the calculation in Section 2.2, we can conclude that the horizontal displacement of the reference point with temperature is negligible because the reference point is on the symmetrical axis of the telescope. In the following, we only discuss the modeling of the thermal height for the Tianma VGOS telescope in the following.

### 3.1. The Correlation between Ambient Temperature and Thermal Height

The IERS thermal expansion model defined by Nothnagel (Nothnagel 2009) considers a time lag for the variations in temperature to affect the antenna. Based on a 7 days time series of the thermal height of the reference point and ambient temperature at the Hartebeesthoek radio telescopes, Nothnagel used a constant offset and a sine function with a period of 24 h
(see Equation (5)) to fit the observed data (including thermal height and ambient temperature), with results showing that the time lag was of 2 h for a steel telescope structure (Nothnagel et al. 1995).

$$
\begin{equation*}
x(t)=A \cdot \sin \left(2 \pi \frac{t}{t_{p}}+\varphi\right)+\text { offset } \tag{5}
\end{equation*}
$$

where $A$ is the amplitude, $t$ the epoch of the observed data, $t_{p}$ the period of 24 h , and $\varphi$ the initial phase angle. When the period $t_{p}$ is fixed at 24 h , the initial phase difference between the daily signals of temperature and thermal height is the time lag.

Considering the daily maximum temperature difference is different, using Equation (5) to fit all the data will cause a large model error.

By a weighted moving average filter with a window length of more than 1 day and less than 2 days, the observed height of the ARP is divided into a high-frequency signal (accumulation of signals with periods less than and equal to 1 day) and a lowfrequency signal (accumulation of signals with periods more than 1 day). Through correlation calculation, the Pearson correlation coefficient between the observed height of the ARP and ambient temperature is 0.83 . The Pearson correlation coefficient between the two low-frequency signals of height and temperature is 0.99 , while that between the two highfrequency signals is 0.68 . By cross-correlation, however, the


Figure 8. The top part is the observed height of the ARP, the middle is the low-frequency signal, and the bottom is the high-frequency signal. The observed data are the sum of the low-frequency and high-frequency signals.
time lag of thermal expansion relative to temperature is 4.27 h . The Pearson correlation coefficient is 0.80 after the time lag is taken into account. According to the above calculation, only the high-frequency signal needs to consider time lag, while the low-frequency signal does not. Figure 8 shows the observed height of the ARP, as well as the low-frequency and highfrequency signals in the observed height.

### 3.2. The Modeling of Thermal Expansion for the Tianma VGOS Telescope

According to the analysis and calculation in Section 3.1, by modeling the high-frequency and low-frequency signals in thermal deformation respectively, the model accuracy can reach 0.1 mm in theory (see Figure 8). Different from Nothnagel's modeling method, we use Equation (5) to fit the observed data including ambient temperature and the height of the ARP within each day. By least square adjustment, we can obtain the time series of the amplitude and phase of the sinusoidal signal and the constant offset (see Equation (5)). Figure 9 shows the relationship between the daily maximum temperature difference and the daily initial phase difference (time lag) of the two sine signals of temperature and thermal height, with the result showing that the
weighted mean of time lag is $\Delta t=3.13 \mathrm{~h}(\Delta \varphi=0.820 \mathrm{rad})$, and the weighted standard deviation of the time series of the time lag is 38 minutes. This result is statistically equivalent to that from cross-correlation (see Section 3.1).

Figure 10 shows the relationship between the daily maximum temperature difference and the ratio of the daily amplitude of thermal height to that of the temperature, with the result showing that the weighted mean of the ratio is $\Gamma_{\text {sin }}=1.42 \times 10^{-5} \mathrm{~m} /{ }^{\circ} \mathrm{C}$. The weighted standard deviation of the ratio time series is $0.45 \times 10^{-5} \mathrm{~m} /{ }^{\circ} \mathrm{C}$.

With the assumption of the IERS expansion model that the telescope structures expand or shrink linearly with temperature, the linear relationship between daily $h_{\text {offset }}$ and $T_{\text {offset }}$ is given as Equation (6):

$$
\begin{equation*}
h_{\text {offset }}=\Gamma_{\text {offset }}\left(T_{\text {offset }}-T_{0}\right)+h_{0} \tag{6}
\end{equation*}
$$

where $\Gamma_{\text {offset }}$ is the ratio of $h_{\text {offset }}-h_{0}$ to $T_{\text {offset }}-T_{0}, T_{0}=$ $19^{\circ} \mathrm{C}$ is the global temperature and pressure model (GPT) of Tianma 13.2 m VGOS telescope (Boehm et al. 2007; Nothnagel 2009), and $h_{0}$ the mean telescope height corresponding to the reference temperature $T_{0}$.


Figure 9. The relationship between the daily maximum temperature difference and the daily initial phase difference (time lag) with the red line as the error bar ( $\sigma$ ). From left to right, with the increase of temperature difference, the accuracy of the parameters to be estimated is higher.


Figure 10. The relationship between the daily maximum temperature difference and the ratio of the daily periodic signal's amplitude (i.e., $A$ in Equation (5)) of thermal height to that of the temperature with the red line as the error bar ( $\sigma$ ). From left to right, with the increase of temperature difference, the accuracy of the parameters to be estimated is higher.

Since there are only two unknowns ( $h_{0}$ and $\Gamma_{\text {offset }}$ ), Equation (6) can be solved by the least square method, with $h_{0}=7.94506 \mathrm{~m} \pm 8 \mu \mathrm{~m}$, and $\Gamma_{\text {offset }}=6.43 \pm$ $0.76\left(10^{-5} \mathrm{~m} /{ }^{\circ} \mathrm{C}\right)$. Figure 11 reflects the linear relationship between daily offsets of thermal heights and temperature,
which verifies the rationality of the thermal expansion assumption that telescope structures expand or shrink linearly with temperature.

Based on the above results, Equation (7) gives the modified thermal expansion model for the vertical deformation of the


Figure 11. The linear relationship between daily offsets of thermal heights and temperature. Data are obtained from the result of least square adjustment from Equation (6).

Tianma VGOS telescope.

$$
\left\{\begin{array}{l}
\Delta_{\text {thermal }}=h(t)-h_{0}  \tag{7}\\
=h_{\sin }(t)+h_{\text {offset }}(t)-h_{0} \\
\left.=\Gamma_{\sin } T_{\sin } t-\Delta t\right)+\Gamma_{\text {offset }}\left(T_{\text {offset }}(t)-T_{0}\right) \\
\Gamma_{\text {sin }}=1.42 \times 10^{-5} \mathrm{~m} /{ }^{\circ} \mathrm{C} \\
\Delta t=3.13 \mathrm{hr} \\
\Gamma_{\text {offset }}=6.43 \times 10^{-5} \mathrm{~m} /{ }^{\circ} \mathrm{C} \\
h_{0}=7.94506 \mathrm{~m} \\
T_{0}=19^{\circ} \mathrm{C}
\end{array}\right.
$$

where $T_{\text {sin }}$ and $T_{\text {offset }}$ are the daily sine signal and constant offset respectively by decomposing daily ambient temperature $T$ with Equation (5), and $\Delta_{\text {thermal }}$ is the thermal height (thermal deformation in vertical). If the ambient temperature is obtained, the model value of the height of the ARP can be derived from Equation (7).

### 3.3. Verification of Thermal Expansion Model

For elevation-azimuth mounts the contribution of the thermal expansion effects on telescope height variations is as follows (Nothnagel 2009):

$$
\begin{align*}
\Delta h_{\text {therm }} & =\gamma_{f} \cdot\left(T\left(t-\Delta t_{f}\right)-T_{0}\right) \cdot h_{f} \\
& +\gamma_{a} \cdot\left(T\left(t-\Delta t_{a}\right)-T_{0}\right) \cdot h_{p} \tag{8}
\end{align*}
$$

where $\Delta h_{\text {therm }}$ the total contribution of the thermal expansion effect on telescope height, the thermal expansion coefficients
are $\gamma_{a}\left(12 \times 10^{-6}\right.$ per $\left.{ }^{\circ} \mathrm{C}\right)$ and $\gamma_{f}\left(10 \times 10^{-6}\right)$ for the antenna steel structure and for the concrete foundation, respectively, $h_{p}$ and $h_{f}$ are the height of antenna steel structure and that of telescope foundation, respectively, $\Delta t_{a}$ of 2 hours and $\Delta t_{f}$ of 6 hours are the time lags between the change in temperature and the expansion of the steel and concrete telescope structure, respectively, $T$ is the measured ambient temperature of the telescope, and $T_{0}$ is the reference (air) temperature.

Considering the height of the concrete foundation $h_{f}$ is 8.32 m and that of the steel structure $h_{p}$ is 1.48 m , the data observed from 2020 November 11 (MJD 59 164) to 2021 August 31 (MJD 59457) are substituted into the IERS model (Equation (8)) and the modified model (Equation (7)) respectively to calculate the thermal height of the Tianma VGOS telescope, with the results being in Figure 12. It should be noted that the modified model is derived from the data observed from 2020 November 11 to 2021 August 31.
By comparing the modified thermal expansion model (see Equation (7)) with the IERS model, the results show that the residuals of the IERS model relative to observations are $\pm 1.4 \mathrm{~mm}$, and the residuals of the modified model are $\pm 0.5 \mathrm{~mm}$, with the rms of IERS model and modified model 0.43 and 0.15 mm , respectively. The results can verify the millimeter accuracy claimed by the IERS model (Nothnagel 2009). For the VGOS telescope, however, the reference point must be stable or modelable to 0.3 mm rms (Petrachenko et al. 2009).


Figure 12. Comparison between the modified thermal expansion model and IERS model. Residuals $=$ Model-Observed.


Figure 13. Verification of the modified thermal expansion model.

Additional data observed from 2021 September 1 to 2021 October 18 are introduced to validate the reliability of the modified model (see Figure 13), with results showing that our modified model can meet the requirement of the accuracy of the VGOS antenna reference point within 0.3 mm rms.

## 4. Discussion and Conclusion

To study variations of the reference point with ambient temperature, we developed a monitoring system to observe the change of the prism's displacement at the Tianma 13.2 m

VGOS telescope. We conducted a thorough error analysis of the monitoring system before it was installed, with results showing that the monitoring system in one face with the ATR mode can determine the three components of the prism position with the accuracy of 0.1 mm . According to the spectral analysis of the observed data including ambient temperature and the height of the ARP, the instrumental error is a constant term that has no influence on the thermal deformation of a relative value. In future work, we will adopt the two-face ATR mode of the total station to reduce the instrumental error in the future.

Since the horizontal displacement of the prism is less than 0.1 mm during our monitoring, and both the prism and the reference point are located on the azimuth axis of the telescope, we believe that variations of the horizontal position of the reference point with temperature can be neglected.

Converting the observed data including the height of the ARP and ambient temperature to the frequency domain by FFT, we find that the two groups of periodic signals have the same frequency, and the amplitude of the periodic signal with the same frequency is different. However, the amplitudes of all periodic signals in the IERS thermal expansion model are considered to be equal (see Equation (8)), which leads to a large model error. After correlation analysis and prediction of model accuracy, we decompose the daily observed data into a 24 h sine signal and a constant offset. Then, the linear relationship between thermal height and temperature is established to determine the modified thermal expansion model for the Tianma VGOS telescope. Compared with the IERS thermal expansion model, the accuracy of our modified model is improved by 1.9 times and achieves the VLBI2010 project goal of 0.3 mm rms position stability of the VGOS telescope reference point.

In future work, we will improve the monitoring system to eliminate instrumental errors and estimate the general form of our modified thermal expansion model to evaluate the model accuracy in VLBI data analysis.

## Acknowledgments

This work was funded by the National Natural Science Foundation of China (NSFC, Grant Nos. 11080922 and
12103077). We would like to thank Naifeng Fu from School of Marine Science and Technology for the advice to this work, and Shuangjing Xu, Zhibin Zhang, and Zhengxiong Sun from Shanghai Astronomical Observatory for their help to this work.

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