



Evolution of Close Binary System Parameter Distributions

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Abstract

In this paper, we investigate the orbital and stellar parameters of low- and intermediate-mass close binary systems. We use models, presented in the catalog of Han et al. and calculate parameters of accretors. We also construct distributions of systems along luminosity, semimajor axis and angular momentum, and make some conclusions on their evolution with time. We compare the results with observational data and it shows a good agreement. The set of theoretical models published quite adequately describes the observational data and, consequently, can be used to determine the evolutionary path of specific close binary systems, their initial parameter values and final stages.

Key words: (stars:) binaries (including multiple): close – stars: low-mass – stars: mass-loss

1. Introduction

The evolution of binary stars is similar to that of single stars unless there is some form of mass transfer between the two stars. Individual stars are not affected by the presence of a companion if the binary orbit is wide enough, so standard stellar evolution theory is all that is required to describe their evolution. However, if the stars get close enough, they can interact, with consequences for the stars' evolution and orbit. Close binary evolution has been observed in many systems, including cataclysmic variables, X-ray binaries and Algols, as well as in the presence of stars such as blue stragglers, which cannot be explained by single star evolution.

The majority of stars are binary or multiple systems (Duquennoy & Mayor 1991; Raghavan et al. 2010). Those stars in interacting binary systems exchange mass and angular momentum through material transfer between the two stars, and the systems as a whole may lose both mass and angular momentum when the material is ejected from the systems.

In binary systems mass transfer will proceed with different stellar evolution models (Shore 1994). In a close binary system, mass transfer usually occurs via Roche lobe overflow (RLOF), in which the primary star begins to transfer material to its companion star once it fills its Roche lobe (Kopal 1959; Paczynski 1971; Eggleton 1983) and if one of the stars ejects much of its mass in the form of stellar wind. The mass transfer from one star to another affects the angular momentum and orbital period of these systems. There are two mechanisms of mass transfer between the components of close binaries. These are conservative mass transfer, in which both the binary's mass and angular momentum are conserved, and nonconservative mass transfer, in which both the binary's mass and angular momentum decay with time. As a result, there are observational

evidences for both conservative and nonconservative mass transfer in close binaries (Podsiadlowski et al. 2001; Yakut 2006; Manzoori 2011; Pols 2012). More recent discussions on observations of conservative and nonconservative scenarios of mass transfer can be found in Qian et al. (2020); Dervisoglu et al. (2018); Peters et al. (2021); Vidante & Malasan (2022); Miszuda et al. (2022), while these and other problems related to the evolution of close binaries are reviewed in Tutukov & Cherepashchuk (2020; Offner et al. (2022).

If one star loses mass via a stellar wind, the companion may accrete some of the material, affecting the orbit. In these systems, the tidal interaction plays an important role in changing the orbit of a close binary system. The degree of interaction is critically dependent on the stellar radius to the star separation ratio (Zahn 1977; Hut 1981). As the binary approaches an equilibrium state of minimum energy, tides can synchronize the spin of the stars with the orbit and circularize an eccentric orbit. The existence of a companion introduces a tidal effect, which acts to elongate the star along the line between the centers of mass, resulting in tidal bulges. As noted by Hurley et al. (2002), if the rotational period of the stars is shorter than the orbital period, frictional forces on the star's surface will drag the bulge axis ahead of the line of centers.

In this paper, we concentrate on determining the angular momentum and orbital period evolution of close binary systems at various events, such as the starting of RLOF, the minimum luminosity during RLOF and the end of the last episode of RLOF from the catalog of Han et al. (2000). Then, in order to determine the evolution of close binary systems, we examine the orbital and stellar parameters of close binary systems such as semimajor axis, mass, luminosity, effective temperature, and radius of the accretor stars in the range of masses of the donor

between $0.212 M_{\odot}$ and $7.943 M_{\odot}$ and mass of accretor between $0.25 M_{\odot}$ to $14.281 M_{\odot}$.

The goal of this paper is to determine the angular momentum and orbital period evolution of close binary stars from the catalog of Han et al. (2000). We will look at the angular momentum, semimajor axis, mass, luminosity, effective temperature and radius of accretor stars. Finally, we present the statistical analysis as well as the comparison with observations. The paper is structured as follows. The basic mathematical formulations for orbital and stellar parameters of these systems are presented in Section 2, and the analysis of data is presented in Section 3. Finally, in Section 4 we make our conclusions.

2. Evolutions of Close Binary Systems

2.1. Basic Assumptions

In this paper, we look at close binary systems with accretor masses M_a , donor masses M_d , and the total mass of stars which is $M_t = M_a + M_d$, and volume equivalent radii of R_a and R_d . We assume that each star's mass is distributed spherically symmetrically. The binary is assumed to be in a circular Keplerian orbit with semimajor axis a , orbital angular velocity Ω_{orb} and orbital period P_{orb} at initial phase.

2.2. Angular Momentum Evolution

Matter is transferred between the components in the binary systems, and the orbital period is changed as a result of the angular momentum redistribution between the two stars. Orbital angular momentum of the binary system with a circular orbit is given by

$$J_{\text{orb}} = M_a M_d \sqrt{\frac{Ga}{M_t}}, \quad (1)$$

where a is semimajor axis, G is the universal gravitational constant and M_t is the total mass of the stars.

The total mass of the stars in terms of the mass of accretor and mass ratio can be written as

$$M_t = M_a(1 + q), \quad (2)$$

where $q = \frac{M_d}{M_a}$, which is the mass ratio of the system.

Substituting Equation (2) into Equation (1), the orbital angular momentum can be expressed as

$$J_{\text{orb}} = M_a M_d \left(\frac{Ga}{M_a(1 + q)} \right)^{\frac{1}{2}}. \quad (3)$$

Hence, the semimajor axis of these systems will be obtained by

$$a = \left(\frac{GP_{\text{orb}}^2 (M_a(1 + q))}{4\pi^2} \right)^{\frac{1}{3}}. \quad (4)$$

As noted by Negu & Tessema et al. (2015), the angular momentum stored in the rotation of the two stars is negligible in comparison to the orbital angular momentum, so Equation (3) approximates the angular momentum of the binary. We obtain a general expression for orbital evolution by differentiating Equation (3)

$$2 \frac{\dot{J}_{\text{orb}}}{J_{\text{orb}}} = \frac{\dot{a}}{a} + 2 \frac{\dot{M}_a}{M_a} + 2 \frac{\dot{M}_d}{M_d} - \frac{\dot{M}_a + \dot{M}_d}{M_a(1 + q)}, \quad (5)$$

where \dot{J}_{orb} denotes angular momentum loss from the binary, which can be caused by gravitational wave radiation or mass loss from the binary as a whole or from the component stars.

The total mass and angular momentum of the binary systems are conserved in the case of conservative mass transfer. Hence, we can set $\dot{J}_{\text{orb}} = 0$ and $\dot{M}_a = -\dot{M}_d$. Then, Equation (5) reduces to

$$\frac{\dot{a}}{a} = 2 \left(\frac{M_d}{M_a} - 1 \right) \frac{\dot{M}_d}{M_d}. \quad (6)$$

Equation (6) tells us that, when $\dot{M}_d < 0$, the orbit shrinks ($\dot{a} < 0$) as long as $M_d > M_a$, and the orbit expands when $M_d < M_a$ (Paczynski 1971).

2.3. Stellar Parameter Relations for Close Binary Stars

The relationship between a star's mass and luminosity is a fundamental law that is used in many fields of astrophysics. It is particularly important in the construction of the initial mass function from the luminosity function of stars (Malkov et al. 1997).

In these systems, the mass–luminosity relationship of the stars is determined by

$$L \propto M^n, \quad (7)$$

with a different range of n that can be used to determine the stage of close binary stars, which is dependent on the mass of the stars.

In accordance with Duric (2004), the accretor star's luminosity can be calculated with

$$L_a = L_{\odot} \left(\frac{M_a}{M_{\odot}} \right)^4. \quad (8)$$

The mass–radius relation can also be expressed over a wide range of stellar masses as

$$R \propto M^n, \quad (9)$$

with various n ranges that can be used to determine the stage of close binary stars. As noted by Bonnell & Bate (2005), we can express the accretor star's radius in terms of solar mass and radius

$$R_a = R_{\odot} \left(\frac{M_a}{M_{\odot}} \right)^{0.8}. \quad (10)$$

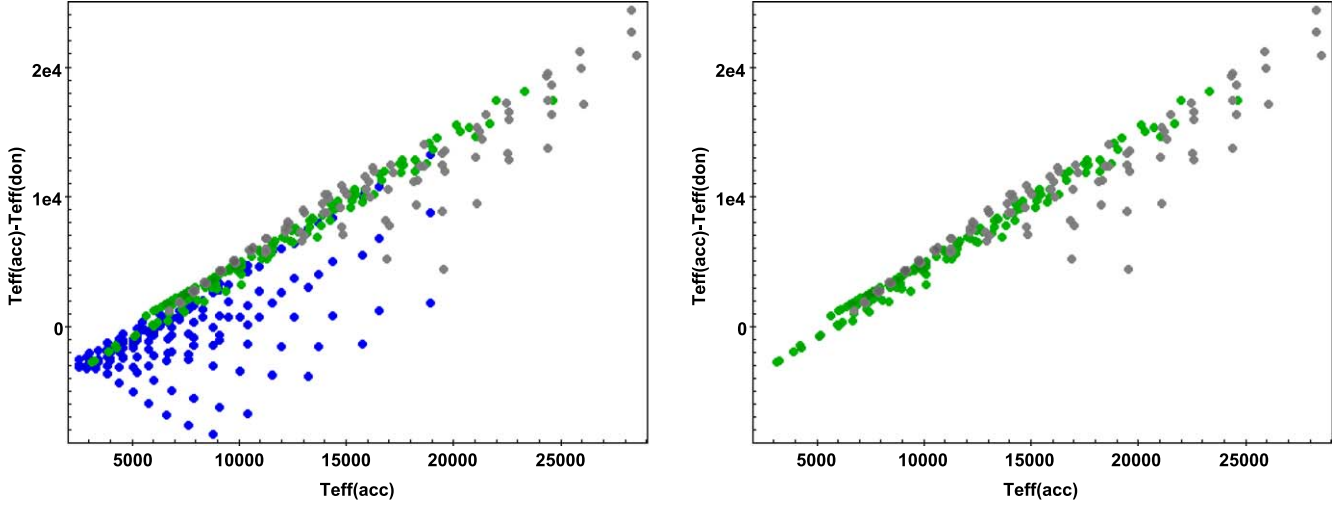


Figure 1. Effective temperatures of accretors and donors of systems from Han et al. (2000). Blue, green and gray colors represent stars at the beginning of RLOF (a-event), at minimum luminosity during RLOF (b-event) and at the end of the last episode of RLOF (c-event). Left panel: all three events, right panel: all events except the first.

A comparison of theoretical and empirical stellar mass–radius relations can be found in Malkov (2007); Eker et al. (2015).

The star’s luminosity can be also expressed as follows

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4, \quad (11)$$

where σ denotes the Stefan–Boltzmann constant and T_{eff} denotes the effective temperature of the stars. Consequently, using Equation (11) the effective temperature of the accretor star can be obtained by

$$T_a = \left(\frac{L_a}{4\pi R_a^2 \sigma} \right)^{\frac{1}{4}}. \quad (12)$$

3. Analysis of Data for Low and Intermediate-mass Close Binaries

3.1. Stellar Parameters and Comparison with Observations

Here we analyze data for donors and accretors of low and intermediate-mass close binaries from the catalog of Han et al. (2000) and compare them with observational data. Astrophysical parameters of donors are given in Han et al. (2000); accretors’ masses were calculated from donors’ masses and mass ratio q . Luminosity and radius of accretors were estimated with Equations (8) and (10), respectively, and temperature was calculated from Equation (12).

We study results of calculations made by Han et al. (2000) for three events, namely, (a) the beginning of RLOF, (b) minimum luminosity during RLOF and (c) the end of the last episode of RLOF, hereafter a-event, b-event and c-event respectively.

First, analyzing T_{eff} of accretors and donors (Figure 1), we should note that, except for a-events, the “temperature—temperature” relation for cool stars with $T_{\text{eff}} < 14,000$ K can be satisfactorily approximated by

$$T_{\text{eff}}(\text{acc}) - T_{\text{eff}}(\text{don}) = 0.99 \times T_{\text{eff}}(\text{acc}) - 5260 \text{ K}. \quad (13)$$

This relation is in excellent agreement with the relation found by Malkov (2021) for 119 systems, included in the comprehensive list of semi-detached double-lined eclipsing binaries

$$T_{\text{eff}}(\text{acc}) - T_{\text{eff}}(\text{don}) = 0.9 \times T_{\text{eff}}(\text{acc}) - 4000 \text{ K}. \quad (14)$$

Stars at the beginning of RLOF (a-event) do not satisfy this relation. This is not surprising, as this stage is very short-lived and therefore is extremely rarely observed. Consequently, such stars are not included in the catalog of Malkov (2021).

Distribution of T_{eff} for donors (left) and accretors (right) is shown in Figure 2. These distributions demonstrate a good agreement with the ones constructed from observational data for semi-detached double-lined eclipsing binaries (Malkov 2020), depicted as pink histograms in Figure 2. The time it takes for a close binary star to go from a-event to b-event is quite short, so almost all known semi-detached systems are observed between b-event and c-event (see also Figure 4, right panel). That is why the observational distributions are qualitatively consistent with theoretical ones, and the positions of the maximum for observed temperatures are close to the theoretical ones.

Figure 3 demonstrates luminosities of donors and accretors for systems from Han et al. (2000). An agreement with observational data (see Figure 5 in Malkov 2020) is less satisfactory, as the majority of observational points lie on or below the $L_{\text{don}} = L_{\text{acc}}$ line. The paucity of observational data

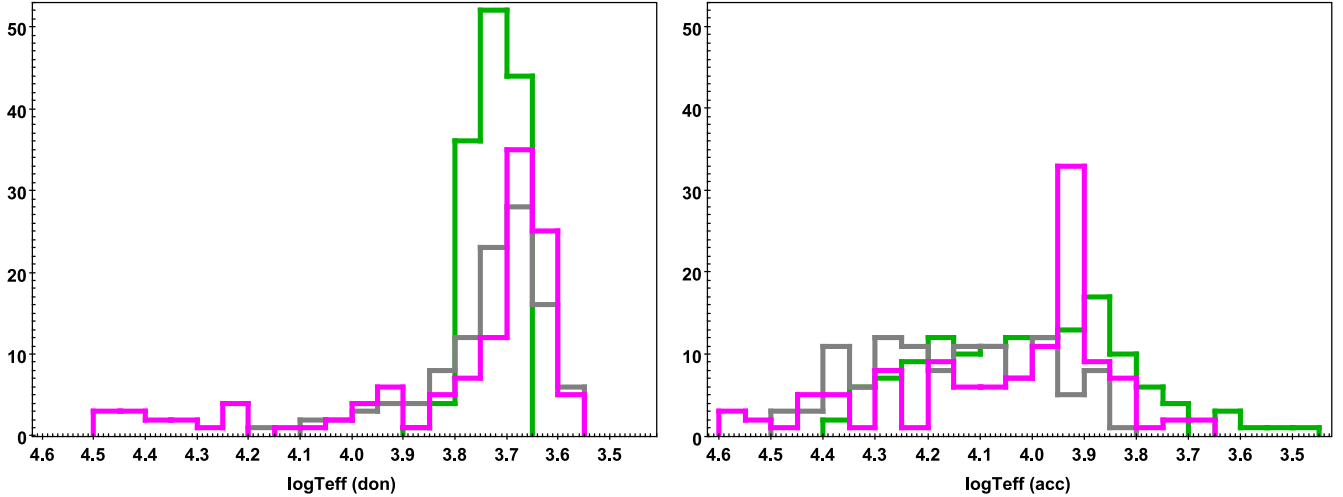


Figure 2. Effective temperatures of donors (left) and accretors (right) of systems from Han et al. (2000). Green and gray colors represent stars at minimum luminosity during RLOF (b-event), and at the end of the last episode of RLOF (c-event), respectively. Observational data for semi-detached double-lined eclipsing binaries from Malkov (2020) are displayed as pink histograms. Note that the X-axes are flipped.

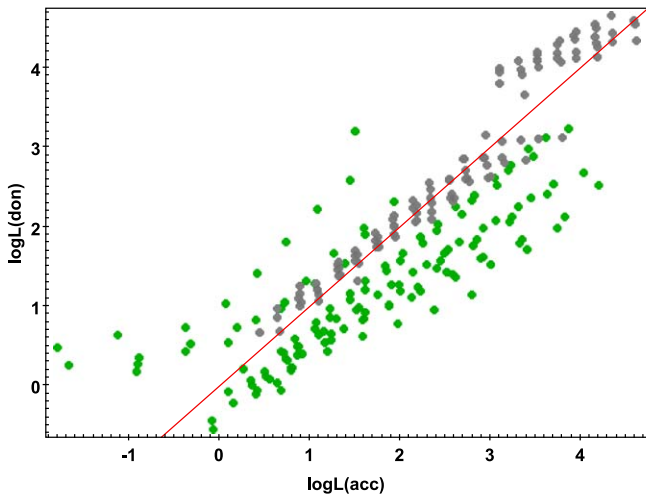


Figure 3. Luminosities of donors and accretors of systems from Han et al. (2000). Colors are as in Figure 2. The one-to-one relation is also shown as a thin red line, for reference.

above that line, however, can be explained. A gray point group in the upper right represents the most luminous (i.e., most massive) stars that are extremely rare in the solar neighborhood. Green points are pairs with relatively large mass ratio q , and, consequently, large magnitude difference prevents such stars from being detected as spectroscopic binaries.

It is interesting also to look at the mass ratio $q = M_{\text{don}}/M_{\text{acc}}$ distribution of systems, included in the catalog of Han et al. (2000) (Figure 4, left panel). Systems at the end of the last episode of RLOF (c-event) demonstrate a sharp maximum at $q \sim 0.1$ while the distribution of b-event systems (minimum

luminosity during RLOF) is bimodal (left panel). There is a sharp maximum at $q \sim 0.7$ and a broader one at $q \sim 2.1$ (note a logarithmic scale for x -axis).

“Large- q ” group comprises initially relatively close systems with a low-mass accretor. During their evolution, donors relatively quickly attain minimum luminosity (the time elapsed between a-event and b-event is minimal for these systems) so that hydrogen abundance at donor’s surface has no time to change from its initial value $H = 0.7$, according to Han et al. (2000) data. Meanwhile their further evolution slows down significantly (c-events for these systems are not reached in the calculations of Han et al. 2000).

A gap between these two maximums is a consequence of the discreteness of the calculation grid. Han et al. (2000) did the calculations for the initial q values of 1, 1.5, 2, 3 and 4. All pairs with initial $q_{\text{ini}} = 4$ are located at the $q \sim 2.1$ maximum, while all pairs with initial $q_{\text{ini}} = 3$ (with two exceptions) or less are located at the $q \sim 0.7$ maximum. The observation points (Figure 4, right panel) are located between the b-event and c-event points, because, as mentioned above, the transition of the star from a-event to b-event is very fleeting.

3.2. Evolution of Stellar Parameters

It is advisable to study the behavior of an ensemble of stars, modeled by Han et al. (2000). Figure 5 depicts evolution of angular momentum J , calculated according to (3). The b-event and c-event distributions look similar, and they are what we should get from observations, as they are relatively long evolutionary stages. On the contrary, the “initial” (a-event) distribution differs significantly from two others. The increase of the angular momentum from the stage when stars are at the

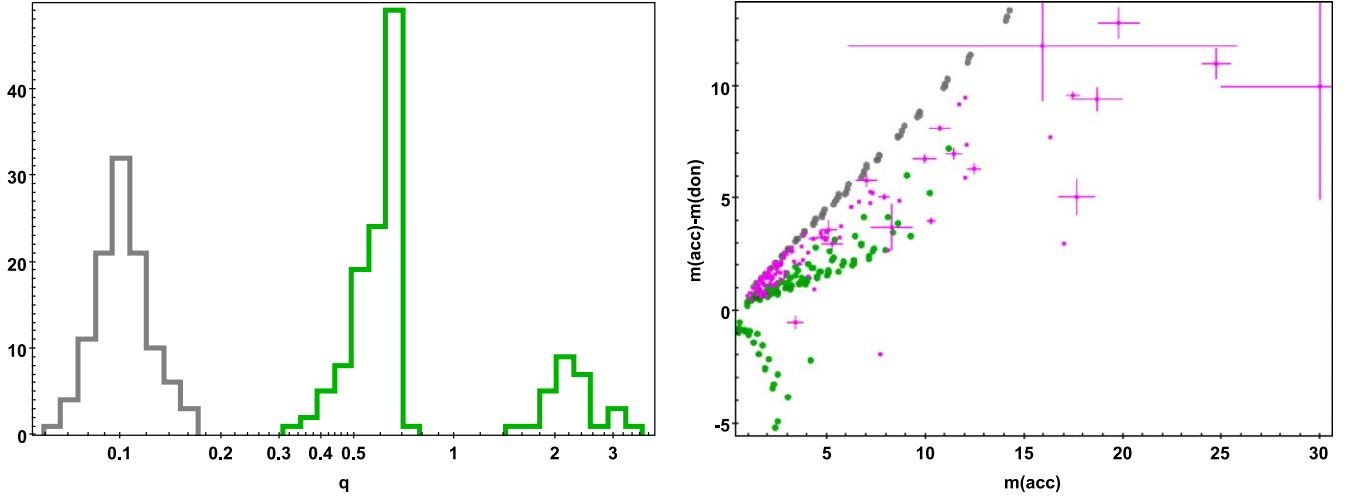


Figure 4. Mass ratio $q = M_{\text{don}}/M_{\text{acc}}$ and masses of systems from Han et al. (2000). Green and gray colors represent stars at minimum luminosity during RLOF (b-event), and at the end of the last episode of RLOF (c-event), respectively. (Left) Distribution of q (note a logarithmic scale for q -axis), (right) donor-accretor mass relation. Observational data for semi-detached double-lined eclipsing binaries from Malkov (2021) are shown as pink dots, with observational errors.

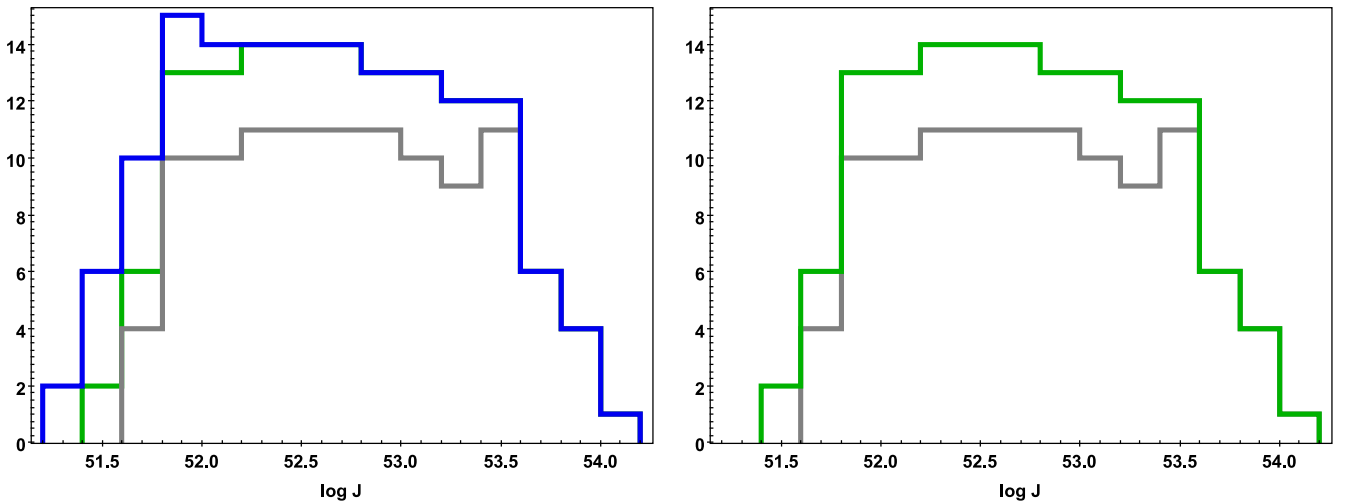


Figure 5. Angular momentum of systems from Han et al. (2000). Blue, green and gray colors represent stars at the beginning of RLOF (a-event), at minimum luminosity during RLOF (b-event) and at the end of the last episode of RLOF (c-event) respectively. (Left) All three events, (right) all events except the first.

beginning of RLOF (a-event) to the subsequent stages can be easily explained. In the process of the binary evolution with mass exchange, the masses of components become comparable, which, with a fixed sum of masses, increases their product (see Equation (2)). This aspect should be taken into account when one restores an initial J -distribution from observations.

Analyzing evolution of total luminosity of the systems $L_{\text{tot}} = L_{\text{don}} + L_{\text{acc}}$ (see Figure 6), one can see that distributions for a-event and b-event look similar, while c-event distribution demonstrates an excess of highly luminous systems. This is because the b-event, by definition, corresponds to the minimum luminosity, and at later stages the luminosity of the system

begins to increase. On the one hand, this makes them easier to detect in this evolutionary stage (c-event), but on the other hand, such systems, being massive, should not be too common in the solar vicinity.

Finally, semimajor axis (a) distributions (see Figure 7) demonstrate a quite understandable excess of wide pairs among well evolved c-event systems. A study of the evolution of semimajor axis (a) distribution allows predictions to be made about the frequency of such systems among resolved spectroscopic binaries (RSB); such systems demonstrated spectral line shift and, at the same time, are observed with interferometric techniques. Minimum a value of a close system to be resolved is

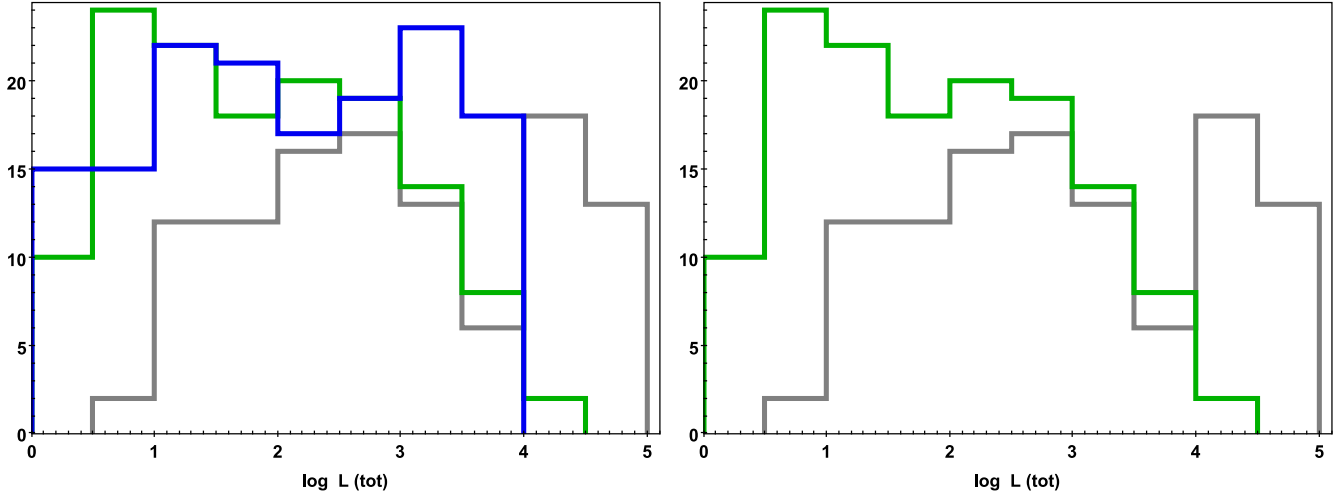


Figure 6. Total luminosity of systems from Han et al. (2000). Colors and panel content are the same as in Figure 5.

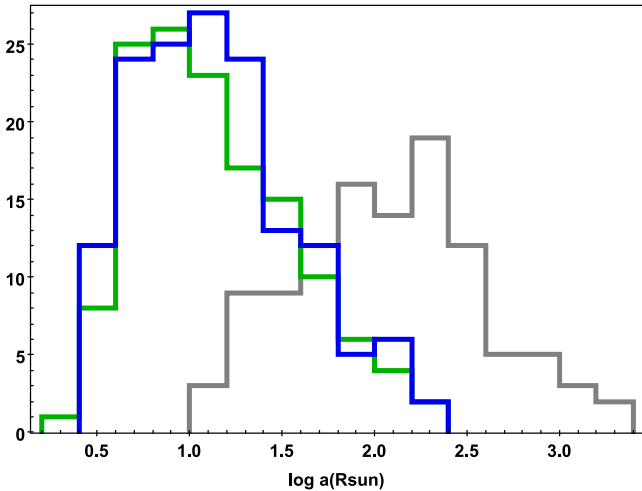


Figure 7. Semimajor axis of systems from Han et al. (2000). Colors are the same as in Figure 5.

about $15 R_{\odot}$, so the majority of systems at c-event and some of the systems at the earlier events can be, in principle, observed as RSB. This means that among the systems detected as RSB, there may be pairs just after or even at the mass exchange stage.

4. Conclusion

For models of low- and intermediate-mass close binary stars, listed in the catalog of Han et al. (2000), we have estimated parameters of accretors, using cataloged data on donors and assuming that accretors satisfy main sequence relations. The results were compared with observational data, collected and discussed in Malkov (2020) and Malkov (2021). We can draw the following conclusions.

The set of theoretical models published in Han et al. (2000) quite adequately (taking into account the observational selection effects) describes the observational data and, consequently, can be used to determine the evolutionary path of specific close binary systems, their initial parameter values and final stages. Theoretical and observational results for effective temperature and mass show a good agreement. Derived relation between effective temperatures of donor and accretor can be used for estimation of the former assuming that the accretor is still a main sequence star. Results for luminosity show a worse agreement, and we have explained this by selection effects. In general, we have demonstrated that observed parameters (in particular, mass ratio q) can tell us a lot about origin and current evolutionary state of the system.

Study of distributions of the systems in terms of luminosity, semimajor axis and angular momentum, and their evolution with time allows us to give some suggestions on the connection between initial and present-day distributions, as well as to make some assumptions on the presence of interacting binary stars among resolved spectroscopic binaries.

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References

- Bonnell, I. A., & Bate, M. R. 2005, *MNRAS*, **362**, 915
 Dervisoglu, A., Pavlovski, K., Lehmann, H., Southworth, J., & Bewsher, D. 2018, *MNRAS*, **481**, 5660
 Duquennoy, A., & Mayor, M. 1991, *A&A*, **248**, 485

- Duric, N. 2004, *Advanced Astrophysics* (Cambridge: Cambridge Univ. Press)
- Eggleton, P. P. 1983, *ApJ*, **268**, 368
- Eker, Z., Soydugan, F., Soydugan, E., et al. 2015, *AJ*, **149**, 131
- Han, Z., Tout, C. A., & Eggleton, P. P. 2000, *MNRAS*, **319**, 215
- Hurley, J. R., Tout, C. A., & Pols, O. R. 2002, *MNRAS*, **329**, 897
- Hut, P. 1981, *A&A*, **99**, 126
- Kopal, Z. 1959, *The International Astrophysics Series* (London: Chapman & Hall), 1959
- Malkov, O. Y. 2007, *MNRAS*, **382**, 1073
- Malkov, O. Y. 2020, *MNRAS*, **491**, 5489
- Malkov, O. Y. 2021, *Astron. Astrophys. Trans.*, **32**, 111
- Malkov, O. Y., Piskunov, A., & Shpil’kina, D. 1997, *A&A*, **320**, 79
- Manzoori, D. 2011, *Advanced Topics in Mass Transfer*, 163
- Miszuda, A., Ko łaczek-Szymanski, P. A., Szewczuk, W., & Daszyska-Daszkiwicz, J. 2022, *MNRAS*, **514**, 622
- Negu, S. H., Tessema, S. B., et al. 2015, *IJAA*, **5**, 222
- Offner, S. S. R., Moe, M., Kratter, K. M., et al. 2022, *The Origin and Evolution of Multiple Star Systems*, arXiv:2203.10066
- Paczynski, B. 1971, *ARA&A*, **9**, 183
- Peters, G. J., Gayley, K., Ignace, R., et al. 2021, *Ultraviolet Spectropolarimetry with Polstar: Conservative and Nonconservative Mass Transfer in OB Interacting Binaries*, arXiv:2111.14047
- Podsiadlowski, P., Rappaport, S., King, A., D’Antona, F., & Burderi, L. 2001, *Evolution of binary and multiple star systems; a meeting in celebration of Peter Eggleton’s LX birthday*, in: *Evolution of Binary and Multiple Star Systems*, **229**, 1
- Pols, O. 2012, *Department of Astrophysics/IMAPP (The Netherlands: Radboud University Nijmegen)*
- Qian, S. B., Zhu, L. Y., Liu, L., et al. 2020, *RAA*, **20**, 163
- Raghavan, D., McAlister, H. A., Henry, T. J., et al. 2010, *ApJS*, **190**, 1
- Shore, S. N. 1994, *Observations and physical processes in binary stars, Interacting Binaries* (Berlin: Springer), 1
- Tutukov, A. V., & Cherepashchuk, A. M. 2020, *PhyU*, **63**, 209
- Vidante, C., & Malasan, H. L. 2022, *JPCS*, 012005
- Yakut, K. 2006, *An observational study of unevolved close binary stars*
- Zahn, J. P. 1977, *A&A*, **57**, 383