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A new method for resolving phase ambiguity in radio interferometry using Earth rotation synthesis

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Abstract As a key technique in deep space navigation, radio interferometry can be used to determine the accurate location of a spacecraft in the plane-of-sky by measuring its signal propagation time delay between two remote stations. To improve the measurement accuracy, differential phase delay without phase ambiguity is usually desired. Aiming at the difficulties of resolving phase ambiguity with few stations and narrowband downlink signals, a new method is proposed in this work by taking advantage of the Earth rotation. The high accurate differential phase delay between the spacecraft and a calibrator can be achieved not only in the in-beam observation mode but also in the out-of-beam observation mode. In this paper we firstly built the model of phase ambiguity resolution. Then, main measurement errors of the model are analyzed, which is followed by tests and validations of the model and method using the tracking data of the Cassini mission and Chang'E-3 mission. The results show that the phase ambiguities can be correctly resolved to generate a 10-picosecond level accuracy differential phase delay. Angular measurement accuracy of the Cassini reaches the milli-arc-second level, and the relative position accuracy between the Chang'E-3 rover and lander reaches the meter level.

Key words: radio interferometry — Earth rotation synthesis — phase ambiguity — differential phase delay — Cassini mission — Chang'E-3 mission

1 INTRODUCTION

China's lunar exploration project has finished the third phase – autonomous lunar surface sampling return, and autonomous Mars exploration "Tianwen-1" has been launched in July 2020 (Ye et al. 2014). As exploration missions become increasingly complex, the demand for navigation and positioning accuracy is also increasing. Radio interferometry is an important technique for deep-space probe positioning by measuring the time delay of the spacecraft signal between two widely separated stations. The angular position of the spacecraft in the sky plane can be obtained by this technique (Lanyi et al. 2007; Li et al. 2013). In order to reduce the influence of space ionosphere, atmospheric disturbance, station location error, and clock error, differential interferometry is usually performed by alternatively observing the spacecraft and a reference radio source to improve the accuracy of delay measurement (Qian & Li 2012). Currently, the main radio interferometry technique used in deep space exploration missions is delta Differential One-way Ranging (delta-DOR), which tries to increase the signal bandwidth by using multi-frequency DOR beacons, and can obtain group delay with accuracy of about 1 ns (Curkendall & Border 2013; Maddè et al. 2006). In the Chang'E-3/4/5 missions, the spacecraft was equipped with an X-band 38 MHz bandwidth DOR beacon, with a delay measurement accuracy of 0.5 ns and an equivalent angular position accuracy of about 15 mas on a 2000 km baseline (Huang et al. 2014). For the delta-DOR technology, the group delay measurement accuracy is limited by the signal bandwidth. The maximum bandwidth allocated by the International Telecommunication Union (ITU) for use in the X-band of deep space measurement and control is only 50 MHz (Date & Date 2012).

To further improve the accuracy of radio interferometry, one way is to calculate the phase delay instead of the group delay. However, phase delay can only be obtained after the phase ambiguities being resolved. A method to resolve the phase ambiguity is using a combination of multiple baselines, which requires a large number of observation stations (Zhou et al. 2015a). With this method, VLBA (Very Long Baseline Array) and EVN (European VLBI Network) has carried out several successful observations to the Mars Exploration Rover-B, Cassini-Huygens, and Venus Express, and accurate positions of those probes has been determined using more than ten stations (Martin-Mur et al. 2006 ; Jones et al. 2011; Duev et al. 2012). But there are only four stations in the CVN (China's VLBI Network), it is not easy to do such observations. Frequency synthesis is another way to resolve the phase ambiguity problem (Hao 2010 ; Chen et al. 2013). This method needs a wideband downlink signal or wideband beacons. For example, in the Japanese SELENE (SELenological and ENgineering Explorer) lunar mission, special designed downlink multi-frequency beacons were used to obtain a phase delay of 10 ps magnitude between two sub-satellites (Kikuchi et al. 2009; Liu et al. 2010). In the Chang'E-3 mission, China's TT&C (Telemetry, Tracking and Command) system carried out the same beam interferometry measurements for the lander and the rover, but the two probes do not emit the multi-frequency beacons or wideband signal required by the frequency synthesis method. To address this situation, Huang and Liu proposed a kinematic statistical positioning-based method, which treats the phase ambiguity as a fixed systematical bias in the lander positioning solution. This method realizes the relative positioning of the two probes with an accuracy of 1m (Huang et al. 2014; Li et al. 2014; Liu et al. 2014).

In order to solve the phase ambiguity problem under the condition of few stations and narrowband signals, a method is proposed in this paper by taking advantage of the change of baseline length and direction caused by the Earth rotation. The method is validated using the data from the Cassini Saturn exploration mission and the Chang'E-3 mission. We arrange this paper in this way: the phase ambiguity solution model is presented in Section 2; the error sources introduced in this solution are described in Section 3; the validation of this method using tracking data of the Cassini and Chang'E-3 missions is given in Section 4; and a conclusion is drawn in Section 5.

2 PHASE AMBIGUITY RESOLUTION

In the geocentric J2000 coordinate system (X, Y, Z), the baseline coordinates formed by the two remote stations are (L_x, L_y, L_z) (length in signal wavelength). The right ascension and declination of the spacecraft are α and δ respectively. A right-hand coordinate system (U, V, W) as shown in Figure 1 is established, in which W pointing in the direction of the spacecraft, V locating in the plane formed by Z and W, and pointing north, U pointing east. Then the baseline (L_x, L_y, L_z) is represented in the

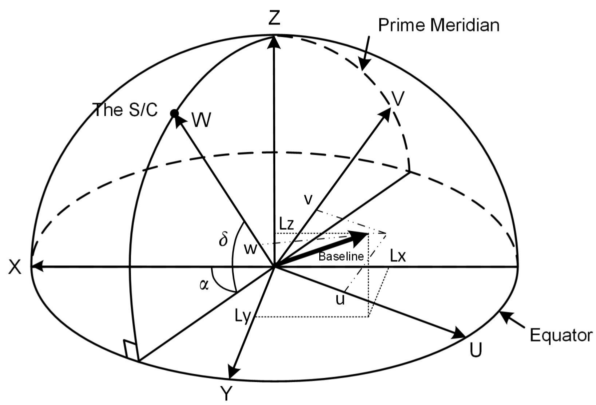

Fig. 1 Schematic diagram of the coordinate system, XYZ is the geocentric J2000 coordinate system and UVW is the new coordinate system.

coordinate system (U, V, W) as (Thompson et al. 2017; Bagri & Majid 2008)

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -\sin\alpha & \cos\alpha & 0 \\ -\sin\delta\cos\alpha & -\sin\delta\sin\alpha & \cos\delta \\ \cos\delta\cos\alpha & \cos\delta\sin\alpha & \sin\delta \end{bmatrix} \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} . \quad (1)$$

From Equation (1) we can obtain

$$\begin{cases} u = -\sin\alpha L_x + \cos\alpha L_y \, , \\ v = -\sin\delta\cos\alpha L_x - \sin\delta\sin\alpha L_y + \cos\delta L_z \, , \\ w = \cos\delta\cos\alpha L_x + \cos\delta\sin\alpha L_y + \sin\delta L_z \, . \end{cases} \quad (2)$$

Assuming that the spacecraft is far enough away from the Earth that its signal can be treated as a parallel wave, the geometric phase difference ϕ_g between the spacecraft signals arriving at the two stations is

$$\begin{aligned} \phi_g &= 2\pi w \\ &= 2\pi(\cos\delta\cos\alpha L_x + \cos\delta\sin\alpha L_y + \sin\delta L_z) \, . \end{aligned} \quad (3)$$

Considering the propagation errors caused by the space ionosphere and the Earth atmosphere, as well as errors from the station location, clock and other factors, the actual measured phase difference ϕ_s is

$$\phi_s = \phi_g + \phi_e \, , \quad (4)$$

where the measurement errors introduced by various factors are indicated by ϕ_e. To reduce the influence of these errors, the differential interferometry is often used, i.e., alternately observing a nearby reference source with well-known positions and using the phase measurement of the reference source to correct the spacecraft signal phase measurement. Assuming the phase measurement of the reference source is

$$\phi_r = \hat{\phi}_g + \hat{\phi}_e \, , \quad (5)$$

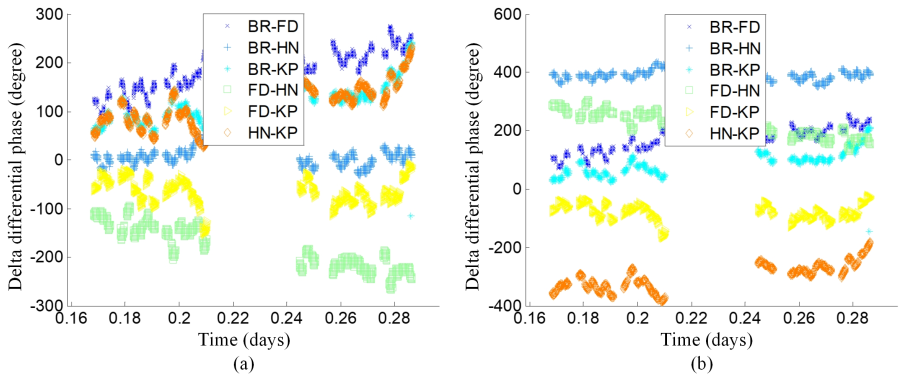

Fig. 2 (a) Delta differential phase with unresolved phase ambiguities of the Cassini orbiter; (b) Delta differential phase with resolved phase ambiguity of the Cassini orbiter.

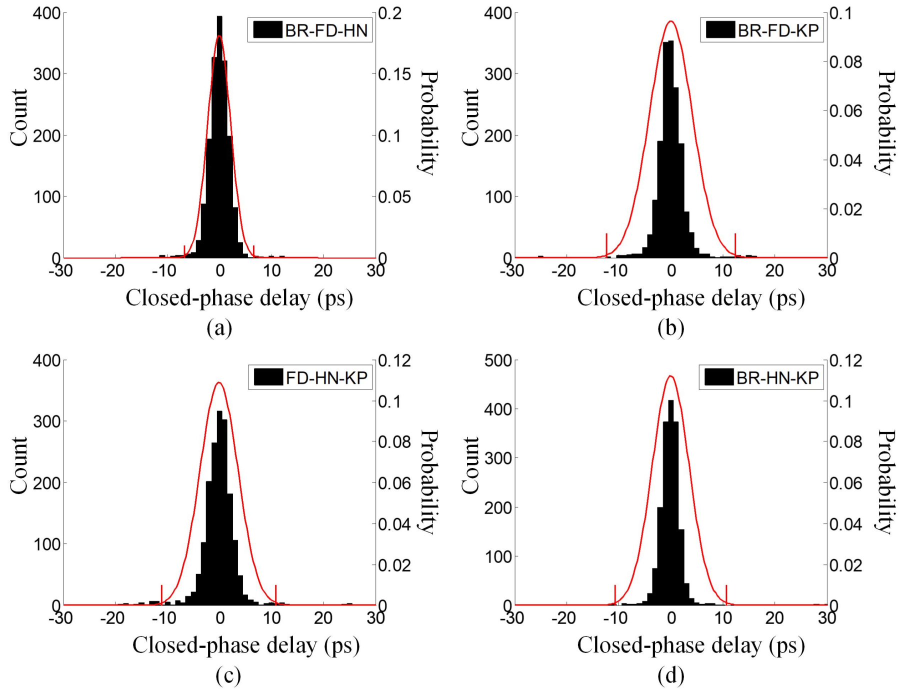

Fig. 3 Close-phase delays of four sets of baselines of the Cassini observation experiments.

where $\hat{\phi}_g$ is the theoretical geometric phase difference between the reference source signals arriving at the two stations, and $\hat{\phi}_e$ is the measurement errors of the reference source introduced by various factors. The differential phase measurements ϕ_{s-r} between the spacecraft and reference source can be represented as

$$\phi_{s-r} = \phi_s - \phi_r = \phi_g - \hat{\phi}_g + \phi_e - \hat{\phi}_e \,. \qquad (6)$$

Due to the small angular separation between the spacecraft and the reference source, the error factors are almost identical for both, namely $\phi_e - \hat{\phi}_e \approx 0$.

Because the reference source position is already well known, we can get an accurate $\hat{\delta}_g$. We assume that the difference between the actual angular position of the spacecraft and its priori angular position is $(\Delta\alpha, \Delta\delta)$, where $\Delta\alpha$ is the difference of the right ascension and $\Delta\delta$ is the difference of the declination. Differentiating Equation (6) and then we can get the delta differential phase between the spacecraft and the reference source $\Delta\hat{\phi}_{s-r}$ from Equation (2) and Equation (3) as

$$\Delta\phi_{s-r} = \Delta\phi_g = 2\pi(u\cos\delta\Delta\alpha + v\Delta\delta)\,. \qquad (7)$$

In which $\Delta\phi_g$ is the delta differential phase between the spacecraft signals arriving at the two stations. Considering there are phase ambiguities in $\Delta\phi_g$, Equation (7) can be expressed as

$$\Delta\phi = \Delta\phi_g - 2N\pi = 2\pi(u\cos\delta\Delta\alpha + v\Delta\delta) - 2N\pi\,, \quad (8)$$

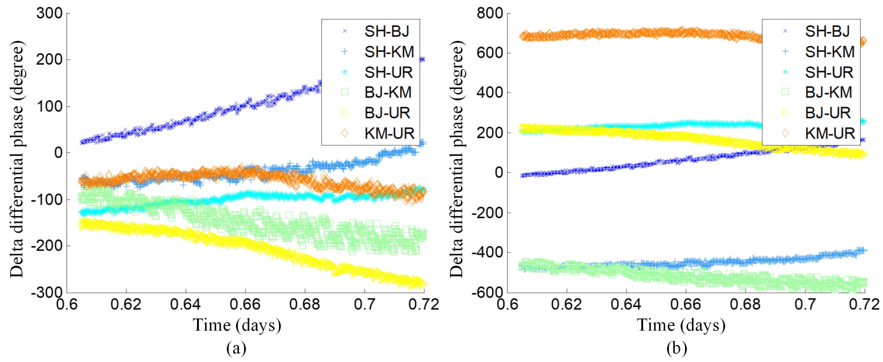

Fig. 4 (a) Delta differential phase with unresolved phase ambiguities of the Chang'E-3 rover; (b) Delta differential phase with resolved phase ambiguity of the Chang'E-3 rover.

where $\Delta\phi \in (0, 2\pi]$ and it is the delta differential phase obtained from the radio interferometry observations. N is the phase ambiguity. Therefore, if N can be resolved, it is possible to get $(\Delta\alpha, \Delta\delta)$, and then we can determine the accurate position $(\alpha + \Delta\alpha, \delta + \Delta\delta)$ of the spacecraft using its priori angular position (α, δ).

The phase ambiguity of each baseline can be assumed to be constant if there is no maneuver of the spacecraft over several hours. Thus, we can establish the relationship between the spacecraft position uncertainty $(\Delta\alpha, \Delta\delta)$ and the delta differential phase measurements for an observation as

$$\boldsymbol{AX} = \boldsymbol{B} + \boldsymbol{\epsilon}\,, \tag{9}$$

where $\boldsymbol{A} = \begin{bmatrix} u_1 & v_1 & 1 & 0 & \dots & 0 \\ u_2 & v_2 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ u_k & v_k & 0 & 0 & \dots & 1 \end{bmatrix}$, $\boldsymbol{X} = \begin{bmatrix} \cos\delta\Delta\alpha \\ \Delta\delta \\ N_1 \\ \dots \\ N_k \end{bmatrix}$, $\boldsymbol{B} = \frac{1}{2\pi}\begin{bmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \dots \\ \Delta\phi_K \end{bmatrix}$, $\boldsymbol{\epsilon}$ is the measurement error matrix with a mean of 0 and a covariance matrix of C_ϵ. K is the total number of the baselines formed by all stations, N_i is the phase ambiguity of the ith baseline, and $\Delta\phi_i$ is the delta differential phase measurement of the ith baseline.

Using the least-square method (LSM), we can get $\boldsymbol{X}$ from Equation (9) as:

$$\boldsymbol{X} = (\boldsymbol{A^T W A})^{-1}\boldsymbol{A^T W B}\,, \tag{10}$$

where $\boldsymbol{W}$ is the weight matrix, and we usually set it to $\mathrm{diag}(1, 1, 1..., 1)$. The mean square error of the parameter estimates is

$$\boldsymbol{M} = (\boldsymbol{A^T W A})^{-1}\boldsymbol{A^T W C_\epsilon W A}(\boldsymbol{A^T W A})^{-1}\,. \tag{11}$$

From Equation (9), we can find that during the observation, due to the effect of the Earth rotation, the projection (u, v) of each baseline will vary with time, which effectively extends the dimensionality of the $\boldsymbol{A}$ matrix and thus generate a good result using the LSM from Equation (10).

Note that to obtain the equatorial deviation $\Delta\alpha$, the first parameter of the solution $\boldsymbol{X}$ in Equation (9) need to be divided by $\cos\delta$, and that the phase ambiguity is a real solution generated from Equation (10) that should be rounded to the nearest integer.

3 ERROR ANALYSIS

3.1 Parallel and Spherical Wave Propagation Model Errors

In Section 2, it is assumed that the spacecraft is far away from the ground-based stations and its signal is approximately parallel waves. While in fact for spacecraft in the solar system, the spherical wave propagation of the signal needs to be considered when performing the correlation processing to calculate the theoretical time delay (Sekido & Fukushima 2006). However, the error of the parallel wave approximation is greatly reduced by the double differential of the spacecraft and reference source signal. Assuming that the baseline vector is $\boldsymbol{B}$, the priori position direction vector of the spacecraft is $\boldsymbol{s_0}$, and that the deviation between the vector of the actual spacecraft position direction and the priori position direction vector is $\boldsymbol{\Delta s_0}$. Then, the delta differential phase $\Delta\phi_p$ generated with parallel wave model is

$$\Delta\phi_p = 2\pi\boldsymbol{\Delta s_0 B}\,. \tag{12}$$

The delta differential phase $\Delta\phi_s$ generated with spherical wave model is

$$\begin{aligned}\Delta\phi_s = & 2\pi(|r_1\boldsymbol{s_0} - \boldsymbol{B}| - r_1 \\ & - |r_2(\boldsymbol{s_0} + \boldsymbol{\Delta s_0}) - \boldsymbol{B}| + r_2)\,,\end{aligned} \tag{13}$$

where r_1 and r_2 are the priori ranges of the spacecraft and the actual range (both in the wavelength of the signal).

Table 1 Results of the Phase Ambiguity of the Six Baselines and the Spacecraft Angular Position Deviation of the Cassini Orbiter

	Phase ambiguity of each baseline						S/C angular position deviation	
	BR-FD	BR-HN	BR-KP	FD-HN	FD-KP	HN-KP	RA (mas)	Dec (mas)
Real solution	0.19	–1.38	–0.07	–1.19	0.11	1.31	–2.16	0.72
Integer solution	0	–1	0	–1	0	1	–1.97	1.03

Table 2 Results of the Phase Ambiguity of Another Six Baselines and the Spacecraft Angular Position Deviation of the Cassini Orbiter

	Phase ambiguity of each baseline						S/C angular position deviation	
	LA-PT	LA-NL	LA-OV	PT-NL	PT-OV	NL-OV	RA (mas)	Dec (mas)
Real solution	–0.06	–0.04	0.01	–0.03	–0.07	1.04	–2.25	1.51
Integer solution	0	0	0	0	0	1	–2.04	1.06

Nowadays, the range measurement accuracy can reach the meter level. Therefore, if $\boldsymbol{\Delta s_0}$ is very small, the difference between $\Delta\phi_p$ and $\Delta\phi_s$ can be ignored. For example, for a moon satellite, when the range measurement error is 2 m, and the deviation of the target actual position from the priori position is 5 mas, then the time delay difference between the two signal propagation models is about 1 ps. As for a Mars satellite, the time delay difference is about 0.04 ps. Therefore, we can replace the spherical wave model with the parallel wave model without introducing positioning noticeably errors.

3.2 Errors Introduced by Spacecraft Motion

The spacecraft moves fast in the J2000 coordinate system, resulting in continuous variation of its right ascension α and declination δ. This will lead to variation of the baseline projection. In practice, however, we usually use the mean right ascension $\bar{\alpha}$ and declination $\bar{\delta}$ to replace the right ascension α and declination δ, which introduces model errors and we will analyze it in the following.

Assuming $\nabla\alpha = \alpha - \bar{\alpha}, \nabla\delta = \delta - \bar{\delta}$, and after derivation from Equation (2) we can get

$$\begin{cases} \Delta u = (-\cos\alpha L_x - \sin\alpha L_y)\nabla\alpha\,, \\ \Delta v = (\sin\delta\sin\alpha L_x - \sin\delta\cos\alpha L_y)\nabla\alpha\,, \\ \qquad -(\cos\delta\cos\alpha L_x + \cos\delta\sin\alpha L_y + \sin\delta L_z)\nabla\delta\,. \end{cases} \tag{14}$$

Assuming that the phase ambiguity has been resolved, and the observational equation generated from Equation (7) is

$$\boldsymbol{J}\boldsymbol{X} = \boldsymbol{\varphi} + \boldsymbol{\epsilon}\,, \tag{15}$$

in which

$$\boldsymbol{J} = \begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \\ \dots & \dots \\ u_K & v_K \end{bmatrix},\ \boldsymbol{X} = \begin{bmatrix} \cos\delta\Delta\alpha \\ \Delta\delta \end{bmatrix},\ \boldsymbol{\varphi} = \frac{1}{2\pi}\begin{bmatrix} \Delta\phi_{g1} \\ \Delta\phi_{g2} \\ \dots \\ \Delta\phi_{gK} \end{bmatrix}.$$

When the variation of the baseline projection (u, v) introduced by spacecraft motion is considered, the observational equation can be given as

$$\hat{\boldsymbol{J}}\hat{\boldsymbol{X}} = (\boldsymbol{J} + \boldsymbol{\Delta J})(\boldsymbol{X} + \boldsymbol{\Delta X}) = \boldsymbol{\varphi} + \boldsymbol{\epsilon}\,, \tag{16}$$

where $\hat{\boldsymbol{J}} = (\boldsymbol{J} + \boldsymbol{\Delta J})$, $\hat{\boldsymbol{X}} = (\boldsymbol{X} + \boldsymbol{\Delta X})$, $\Delta J = \begin{bmatrix} \Delta u_1 & \Delta v_1 \\ \Delta u_2 & \Delta v_2 \\ \dots & \dots \\ \Delta u_K & \Delta v_K \end{bmatrix}$, $\boldsymbol{\Delta X}$ is the deviation of the angular positioning result that we have to analyze. From Equation (15) and Equation (16) we can get

$$(\boldsymbol{J} + \boldsymbol{\Delta J})\boldsymbol{\Delta X} = -\boldsymbol{\Delta J}\boldsymbol{X}\,. \tag{17}$$

Thus, we can determine $\boldsymbol{\Delta X}$ as below

$$\begin{aligned} \boldsymbol{\Delta X} =& -[(\boldsymbol{J} + \boldsymbol{\Delta J})^{\boldsymbol{T}}(\boldsymbol{J} + \boldsymbol{\Delta J})]^{-1} \\ & \times(\boldsymbol{J} + \boldsymbol{\Delta J})^{\boldsymbol{T}}\boldsymbol{\Delta J}\boldsymbol{X}\,. \end{aligned} \tag{18}$$

For a moon satellite, such as a lunar lander, its angular position varies by about 1 degree of its right ascension and declination in the J2000 coordinate system over 1 hour, which results in a 1.3% variation of the (u, v) values. The variation will be less than 0.7% if the mean right ascension and declination of the pass is chosen as the value for the entire observation. Therefore, the deviation of the spacecraft positioning result caused by the spacecraft motion should be less than 0.7%, which is acceptable for practical missions. Moreover, this error will be further decreased as the spacecraft-Earth distance increases.

4 EXPERIMENTAL VALIDATION AND DISCUSSION

4.1 Positioning of the Cassini Orbiter

The Cassini-Huygens mission was launched on 1997 October 15 for an integrated exploration of the Saturn system. It consists of the Cassini orbiter and the Huygens

Table 3 Results of the Phase Ambiguity of the Six Baselines and the Relative Angular Separation between the Chang'E-3 Rover and Lander on December 15

	Phase ambiguity of each baseline						Relative angular separation between the rover and lander	
	SH-BJ	SH-KM	SH-UR	BJ-KM	BJ-UR	KM-UR	RA (mas)	Dec (mas)
Real solution	0.10	1.14	–0.93	1.03	–1.04	–2.07	–3.68	3.04
Integer solution	0	1	–1	1	–1	–2	–3.19	3.22

Table 4 Results of the Phase Ambiguity of the Six Baselines and the Relative Angular Separation between Chang'E-3 Rover and Lander on December 20

	Phase ambiguity of each baseline						Relative angular separation between the rover and lander	
	SH-BJ	SH-KM	SH-UR	BJ-KM	BJ-UR	KM-UR	RA (mas)	Dec (mas)
Real solution	–0.18	1.09	–0.19	1.27	–0.02	–1.28	–4.73	–2.13
Integer solution	0	1	0	1	0	–1	–4.32	1.15

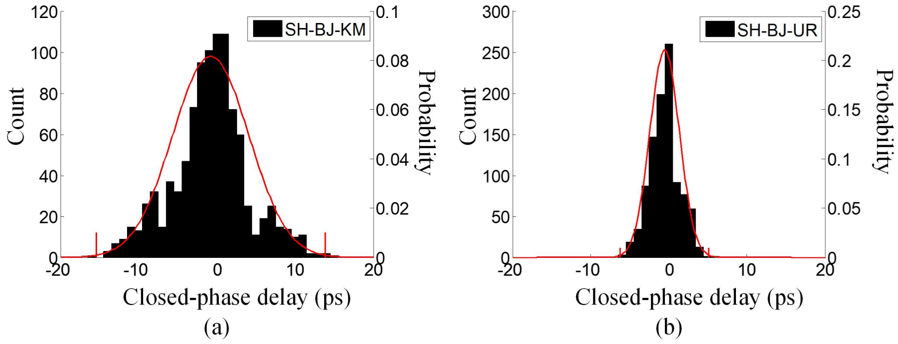

Fig. 5 Close-phase delays of four sets of baselines of the Chang'E-3.

lander. This mission entered orbit around Saturn in July 2004 and successfully terminated in July 2017. On 2007 March 1, the National Radio Astronomy Observatory (NRAO), USA, organized 10 antennas of its VLBA to track Cassini for a total of about 2 h (Jones et al. 2011), with observation intervals from 4:02 to 5:02 and from 5:52 to 6:52 UTC. A phase-referenced fast alternating observation mode with a reference radio source of J0931+1414 was employed. This radio source has an angular distance of less than 2 degrees from the orbiter, and its radio source flux density is 0.15 Jy. An alternative cycle between the orbiter and the radio source is about 3min20s, in which the radio source was observed for about 1min, the orbiter was observed for about 2min, and the remaining 20s was used for antenna rotation. The recorded data from each station were shipped to Socorro, New Mexico for correlation processing, and the correlation results were stored in Flexible Image Transport System (FITS) format (Pence et al. 2010). To test the methods in this work, the observations from four of the stations (Brewster-BR, Fort Davis-FD, Hancock-HN, and Kitt Peak-KP) were selected for processing. We presented the results in Figure 2.

Figure 2(a) shows the delta differential phase of each baseline with unresolved phase ambiguities, and (b) shows the delta differential phase after the phase ambiguity has been resolved. The phase ambiguity results of the baselines in Cassini observations are presented in Table 1.

Table 1 shows the real and integer solutions of the phase ambiguity of the six baselines and their corresponding deviations of the spacecraft angular position. This deviation is the difference between actual and priori angular positions of the spacecraft. From the result of the VLBA observation experiments, the value of the deviation determined from the phase-referenced images is (-2.1, 1.3) mas, which is consistent with our results (Jones et al. 2011). The difference between the phase-referenced images and our results is mainly due to that only four stations were used here in this paper, while 10 stations were involved in the VLBA experiments. But we can see that few stations can also generate a very good result using the method in this paper.

We further present the statistical information of our results in Figure 2.

Figure 2 shows the histogram of the closed-phase delay statistics and the normal probability density distribution of the four groups of baselines (the red vertical line indicates 3σ interval). It can be seen that the closed-phase delay shows a good normal distribution with a mean value of approximately zero, reflecting the correctness of the phase ambiguity solution. The accuracy of the phase delays reaches 10 ps.

Observations from another four stations (Pie Town-PT, Los Alamos-LA, N. Liberty-NL, and Owens Valley-OV) were also processed to verify the method. The results are shown in Table 2. We can get a similar S/C angular position from the data of these four stations. If we use more stations, more observations will be involved to improve the LSM result, and we can get a better angular position with smaller errors.

In order to resolve the phase ambiguity, the method in this paper requires the delta differential phase measurement of the signal to be continuous without cycle jumps during the observation pass. This requires 1) a small angular separation between the spacecraft and the reference source to ensure that the propagation path has as little difference as possible between the signals from the two sources. Generally, the angular distance should to be less than 3 degrees; 2) a short observation interval between the spacecraft and the reference source to ensure that the perturbation changes in the atmosphere and ionosphere between the two adjacent observations will not exceed half a phase full cycle, otherwise the phase measurements of the two adjacent observations cannot be connected without ambiguity. A typical alternating observation period should be less than 2 min (X-band). Once the phase ambiguity can be resolved correctly, the accuracy of the angular position measurements of the spacecraft will reach the mas magnitude (1 mas $\approx$ 4.8 nrad), which is much better than the delta-DOR measurements.

4.2 Chang'E-3 Rover Positioning

The Chang'E-3 mission included a lander and a rover. The rover carried out science explorations of the lunar surface after separating from the lander. During the lunar surface operation of the rover, it needs to be tracked and located in almost real time. Four stations of CVN (Beijing Miyun-BJ, Shanghai Tianma-SH, Yunnan Kunming-KM and Xinjiang Urumqi-UR) have conducted continuous observations of the two probes. Since the rover and the lander are very close to each other (with maximum distance of 150 meters), they can be observed simultaneously in the main beam of the ground station, which greatly reduces the phase error introduced by the signal propagation path. Using the lander as a reference source, the delta differential phase between the rover and the lander can be obtained. Since the same theoretical time delay model was used for both the lander and the rover in the correlation processing, the delta differential phase here directly reflects the relative angular position of the rover and the lander (Zhou et al. 2015b). We present the results in Figure 4.

Figure 4(a) shows the delta differential phase obtained directly generated from the radio interferometry between the two probes from 14:31 to 17:17 on 2013 December 15, which involves phase ambiguities. The delta differential phase after resolving the phase ambiguity is shown in Figure 4(b). During the observation period, the rover stayed at a fixed point and did not move. Using the method proposed in this paper to solve the phase ambiguity in this pass, the real and integer solutions and their corresponding relative angular positions of the rover and lander are shown in Table 3. We also investigate the statistical information of the results in Figure 5.

Figure 5 is a statistical histogram of the closed-phase delay statistics and the normal probability density distribution of four groups of baselines. The closed-phase delay has a mean value approximating zero (the red vertical line indicates 3σ interval). It can be seen that the variance of the closed-phase delay for the baseline combination including the Kunming station is larger because of the poor quality of the measurement data from the Kunming station. In addition, because we use the 12th to 14th side band of the lander's downlink signal as the reference source, the signal-to-noise ratio is not good, resulting in a large phase noise. Thus, the variance of the closed-phase delay is larger than that of the Cassini experiments shown in Figure 2.

Since the terrain near the Chang'E-3 landing area is relatively flat, assuming that the rover and the lander are located on the same plane, the north-east position of the rover relative to the lander can be determined to be (9.21, 2.09) m using the relative angular positions of the two probes. As a comparison, the visual localization results of the relative position of the lander and rover is (9.03, 1.50) m (Liu et al. 2015). The difference between our method and the visual method is less than 1m, which shows the high accuracy of our method.

Furthermore, we processed the data from 15:27 to 15:59 on 2013 December 20. The rover moved to a new site at this moment. The result is shown in Table 4. Similarly, we can get the relative position of the rover to the lander, which is (8.23, −3.68) m, and the visual localization result is (8.36, −5.65) m. It shows a good consistency between the two methods.

5 CONCLUSIONS

In this paper, a method for solving the phase ambiguities using Earth rotation is presented. It provides a new solution for differential phase delay acquisition under the condition of few stations and narrowband signals. The method can support not only the fast-alternating radio interferometry between the spacecraft and the reference radio source, but also the relative positioning of multiple targets in the same beam. Model error analysis shows that the method can meet the requirements for a mission around the Moon or other deep-space bodies. The results of the Cassini and Chang'E-3 experiments effectively verify the validity and correctness of the method, and the closed-phase delay statistics of the two experiments have good normal distribution with the mean value approximated to zero. The phase delay measurement accuracy reaches 10 ps, and the angular separation measurement accuracy reaches the mas level. The method is possible to be employed for China's future deep space missions, especially in the Mars mission "Tianwen-1" and the China's first asteroid explorations about two years later. It can also provide reference for international deep space missions.

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