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Viscous holographic $f(Q)$ cosmology with some versions of holographic dark energy with generalized cut-offs

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Abstract The work reported in this paper demonstrates the cosmology of $f(Q)$ gravity and the reconstruction of various associated parameters with different versions of holographic dark energy with generalized cut-offs, where $Q = 6H^2$. The Universe is considered to be filled with viscous fluid characterized by a viscous pressure $\Pi = -3H\xi$, where $\xi = \xi_0 + \xi_1 H + \xi_2(\dot{H} + H^2)$ and H is the Hubble parameter. Considering the power law form of expansion, we have derived the expression of $f(Q)$ under a non-viscous holographic framework and it is then extended to viscous cosmological settings with extended generalized holographic Ricci dark energy. The forms of $f(Q)$ for both the cases are found to be monotonically increasing functions of Q. In the viscous holographic framework, $f(Q)$ is reconstructed as a function of cosmic time t and is found to stay at a positive level with Nojiri-Odintsov cut-off. In these cosmological settings, the slow roll parameters are computed and a scope of exit from inflation and quasi-exponential expansion are found to be available. Finally, it is observed that warm inflationary expansion can be obtained from this model.

Key words: Holographic dark energy — Ricci dark energy — $f(Q)$ gravity — bulk-viscosity — equation of state parameter — slow roll parameter.

1 INTRODUCTION

Accelerated expansion of the Universe in the late 90s was observationally reported by Riess et al. (1998) and Perlmutter et al. (1999). Immediately after the Big Bang, there was another phase of accelerated expansion, which is known as the inflationary epoch. In between these two epochs, there were phases of radiation and matter domination. Prior to the late 90s, it was believed that the Universe was expanding with deceleration. However in 1998, two independent research groups led by Riess (Riess et al. 1998) and Perlmutter (Perlmutter et al. 1999) respectively observed that the Universe is expanding with acceleration rather than deceleration. This was a breakthrough in modern cosmology and got support from subsequent observational studies with type Ia supernovae (SNeIa), cosmic microwave background (CMB) radiation anisotropies, large scale structure (LSS) and X-ray experiments (de Bernardis et al. 2000; Seljak et al. 2005; Abazajian et al. 2005; Planck Collaboration et al. 2014; Tegmark et al. 2004; Allen et al. 2004). It is thought that this acceleration is driven by some exotic matter characterized by negative pressure. This exotic matter is coined as "dark energy" (DE) (Bamba et al. 2012a; Copeland et al. 2006; Brevik et al. 2017b; Chattopadhyay & Debnath 2009). The equation of state (EoS) parameter of DE is given by $w = \frac{p}{\rho}$, where p is the pressure and ρ is the density of the DE. From the Friedmann's equations, it is verified that the necessary condition for the accelerated expansion of the Universe is $w < -\frac{1}{3}$. The simplest candidate of DE is Λ having EoS parameter $w = -1$ (Visser 2004). Λ is consistent with observations. There are also different candidates of DE that have been proposed in the literature to get rid of the limitations of a cosmological constant (Abazajian et al. 2004). These are the candidates having time varying EoS parameter. They are (Bamba et al. 2012a; Copeland et al. 2006):

(i) Scalar field models,
(ii) Holographic models of DE and
(iii) Chaplygin gas models.

About 68.3% of the total density is due to DE and 26.8%

is due to dark matter (DM). Baryonic matter contributes about 4.9% of the total energy density. Practically, the contribution of radiation is negligible.

Modifications of Einstein's gravity give rise to new degrees of freedom in the gravitational sector. In the models of DE, the infrared cutoff is modified, which is also an example of the reason for modified gravity. The concept of modified gravity is of utmost importance in studying the late time acceleration of the Universe. The models of modified gravity are represented by
(i) Braneworld models (Nojiri & Odintsov 2000),
(ii) $f(G)$ gravity, where $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$, G represents the Gauss-Bonnet invariant, R is the Ricci scalar curvature, $R_{\mu\nu}$ represents the Ricci curvature tensor and $R_{\mu\nu\lambda\sigma}$ is the Reimann curvature tensor (García et al. 2011; Bamba et al. 2017),
(iii) $f(T)$ gravity where T represents the torsion (Bamba et al. 2012b),
(iv) $f(R)$ gravity, R indicates the Ricci scalar curvature (Nojiri & Odintsov 2011),
(v) $f(Q)$ gravity, where $Q = 6H^2$ (Lazkoz et al. 2019),
(vi) DBI models (Chakraborty & Chattopadhyay 2020b), etc.

At this juncture, we would like to mention the exhaustive review works by Nojiri et al. (2017); Nojiri & Odintsov (2011), where an extensive study has been demonstrated in the various aspects of modified gravity theory and its consequences. To be accepted as a cosmological theory, the constraints on modified gravity not only have to satisfy local astrophysical data but also global constraints. Hence, we can say that there are many challenges to make it viable (Nojiri et al. 2017). The main motive of modified gravity is to unify early inflation with the late time acceleration of the Universe. It can be noted that in $F(R)$ gravity, ΛCDM was unified with inflation (Nojiri & Odintsov 2011, 2007b; Appleby & Battye 2007; Nojiri & Odintsov 2008; Cognola et al. 2008; Artymowski & Lalak 2014; Fay et al. 2007). The R in $f(R)$ is the fundamental term. Apart from Einstein's gravity, we can consider teleparallelism with the Weitzenbck connection. Here the fundamental term is the torsion T. Another type of modified gravity which is very useful in studying the inflationary era and transition of acceleration from deceleration regimes of the Universe is $f(G)$ gravity. This is Gauss-Bonnet gravity. G is the topological invariant in 4-dimensions of spacetime. It is also observed that $f(G)$ gravity is less constrained than $f(R)$ gravity (Bamba et al. 2017).

There are many components in the Universe. Each component has different cooling rates. Hence, the term bulk viscosity arises. In general, viscosity means resistance to flow. There are couplings among the different components of the cosmic substratum due to which the bulk viscous pressure in cosmic media emerges. The DE and DM models developed by Nojiri and Odintsov (Nojiri & Odintsov 2005, 2006a) were treated as imperfect fluids. A plethora of literatures has suggested to us that the bulk viscous pressure plays a major role in the late time acceleration of the Universe (Brevik et al. 2004, 2010, 2015; Chakraborty & Chattopadhyay 2020a,c,b, 2021; Chattopadhyay & Chakraborty 2021; Chakraborty et al. 2021). The late time acceleration is not the only accelerated phase of the Universe. There was a very early phase of evolution of the Universe named the "Inflationary Scenario" (Brevik et al. 2015). In this very early phase, both bulk and shear viscosity were thought to play a very significant role (Pun et al. 2008). Chimento et al. (2000) reported that in the combination of a cosmic fluid characterized by bulk dissipative pressure and a quintessence matter, it is possible to have an accelerated expansion of the Universe. The entropy for a coupled fluid and a relationship between the entropy of closed FRW Universe and the energy contained in it was established by Brevik et al. (2010). Brevik et al. (2015), by considering the bulk viscous pressure as a function of the Hubble parameter, demonstrated the Little Rip, Pseudo Rip and Bounce cosmologies in a bulk-viscosity framework.

The paper is organized as follows: In Section 2, we discuss the holographic dark energy (HDE) and holographic principle (HP). In Section 3, we discuss the mathematical background of $f(Q)$ gravity. In Section 4, we examine non-viscous holographic $f(Q)$ gravity. Here we find the expression of $f(Q)$ and examine its behavior. In the Section 5, we study the viscous extended generalized holographic Ricci $f(Q)$ gravity. Here, we derive the expression for $f(Q)$ and investigate its behavior. We reconstruct the pressure $p_{\rm rec,R}$, density $\rho_{\rm rec,R}$ and EoS parameter $w_{\rm rec,R}$. In Section 6, we discuss the case of viscous holographic $f(Q)$ gravity with Nojiri-Odintsov cut-off. We reconstruct the infrared cut-off L and density $\rho_{\rm NO,rec}$. We examine the behavior of $f(Q)$ in this case. In Section 7, we aim to study the holographic Ricci $f(Q)$ gravity as scalar field in the bulk-viscosity framework. Here, we also reconstruct the Hubble slow roll parameters $\epsilon_{\rm H}$ and $\eta_{\rm H}$. We reconstruct the dissipative coefficient Γ and $2V - \dot{\phi}^2$. Lastly, we conclude in Section 8.

2 HOLOGRAPHIC DARK ENERGY

The HDE model is one of the mostly studied models of DE. It is extensively discussed in Li (2004); Myung & Seo (2009); Li et al. (2009); Nojiri et al. (2019b); Salako et al. (2015). The HDE model is based on the HP. HP is a tenet of string theories. It states that the description of a volume of space can be thought of as encoded on a lower dimensional boundary to the region. The energy density of HDE is given by $\rho_\Lambda = 3c^2{M_\mathrm{p}}^2L^{-2}$, where c is the numerical constant, L is the infrared cut-off and M_p is the reduced Planck mass. $M_\mathrm{p} = \frac{1}{\sqrt{8\pi G}} \approx 1$. Therefore, $\rho_\Lambda = 3c^2L^{-2}$ (Li et al. 2009; Pasqua et al. 2019; Nojiri et al. 2019a, 2021; Sarkar et al. 2021). If we assume L to be the size of the current Universe, for instance, the Hubble radius H^{-1}, then the DE density will be very close to the observational value (Nojiri et al. 2019b). Therefore, density of HDE becomes

$$\rho_\Lambda = 3c^2H^2\,. \tag{1}$$

In this context, we would like to mention that the HDE with Nojiri - Odintsov (NO) cut-off, or in other words NO generalized HDE, is the most general version of HDE. Ricci HDE (RHDE), generalized Ricci HDE, etc. are some particular cases of NO HDE with corresponding choice of NO cut-off. Such generalized NO cut-offs have been discussed in Nojiri & Odintsov (2006b); Chakraborty et al. (2021); Nojiri et al. (2021); Sarkar et al. (2021); Khurshudyan (2016a,b); Sarkar & Chattopadhyay (2021); Elizalde & Timoshkin (2019).

3 $F(Q)$ GRAVITY-A BRIEF OVERVIEW

In this section, we give an overview of $f(Q)$ gravity. Earlier, $f(Q)$ gravity was discussed in Mandal et al. (2020, 2021) and Lazkoz et al. (2019). As a purpose of the current work is to present a holographic viscous $f(Q)$ cosmology, we first present a mathematical background of $f(Q)$ gravity. The action for symmetric teleparallel gravity is given by (Jiménez et al. 2018)

$$S = \int \frac{1}{2} f(Q)\sqrt{-g}d^4x + \int \mathcal{L}_\mathrm{m}\sqrt{-g}d^4x \tag{2}$$

where f is the function of nonmetricity Q, g is the determinant of the metric $g_{\mu\nu}$ and $\mathcal{L}_\mathrm{m}$ is the matter Lagrangian density. The nonmetricity tensor is defined by

$$Q_{\lambda\mu\nu} = \nabla_\lambda g_{\mu\nu}. \tag{3}$$

Its traces are

$$Q_\alpha = {Q_\alpha}^\mu{}_\mu \tag{4}$$

and

$$\tilde{Q}_\alpha = Q^\mu{}_{\alpha\mu}. \tag{5}$$

Therefore, superpotential will be

$$P^\alpha{}_{\mu\nu} = \frac{1}{4}\left[-Q^\alpha{}_{\mu\nu} + 2Q_{(\mu}{}^\alpha{}_{\nu)} + Q^\alpha g_{\mu\nu} - \tilde{Q}^\alpha g_{\mu\nu} - \delta^\alpha_{(\mu}Q_{\nu)}\right] \tag{6}$$

where $Q = -Q_{\alpha\mu\nu}P^{\alpha\mu\nu}$. Now, the energy momentum tensor will be

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_\mathrm{m})}{\delta g^{\mu\nu}}. \tag{7}$$

Varying Equation (2) with respect to metric tensor $g_{\mu\nu}$, the equations of motion will be

$$\begin{aligned} -T_{\mu\nu} = &\frac{2}{\sqrt{-g}}\nabla_\gamma(\sqrt{-g}f_\mathrm{Q}P^\gamma{}_{\mu\nu}) + \frac{1}{2}g_{\mu\nu}f \\ &+ f_\mathrm{Q}(P_{\mu\gamma i}Q_\nu{}^{\gamma i} - 2Q_{\gamma i\mu}P^{\gamma i}{}_\nu) \end{aligned} \tag{8}$$

where $f_\mathrm{Q} = \frac{df}{dQ}$. Also, varying Equation (2) with respect to the connection, one obtains

$$\nabla_\mu\nabla_\nu(\sqrt{-g}f_\mathrm{Q}P^\gamma{}_{\mu\nu}) = 0. \tag{9}$$

We consider an FLRW Universe represented by an isotropic, homogeneous and spatially flat line element

$$ds^2 = -dt^2 + a^2(t)\delta_{\mu\nu}dx^\mu dx^\nu \tag{10}$$

where $a(t)$ is the scale factor. For the line element defined in Equation (10), the trace of nonmetricity tensor will be $Q = 6H^2$. The energy momentum tensor of the cosmological fluid is given by

$$T_{\mu\nu} = (\rho + p_\mathrm{eff})u_\mu u_\nu + p_\mathrm{eff}g_{\mu\nu} \tag{11}$$

where p_eff is effective pressure and is given by $p_\mathrm{eff} = thermodynamic\ pressure + \Pi$, viscous pressure $\Pi = -3H\xi$ and $\xi = \xi_0 + \xi_1 H + \xi_2(\dot{H} + H^2)$; ξ_0, ξ_1 and ξ_2 are positive constants and $\xi > 0$. Utilizing Equations (10) and (11) in Equation (8), modified field equations will be (Jiménez et al. 2018; Mandal et al. 2021; Lazkoz et al. 2019)

$$3H^2 = \frac{1}{2f_\mathrm{Q}}\left(-\rho + \frac{f}{2}\right) \tag{12}$$

and

$$\dot{H} + 3H^2 + \frac{\dot{f}_\mathrm{Q}}{f_\mathrm{Q}}H = \frac{1}{2f_\mathrm{Q}}(p_\mathrm{eff} + \frac{f}{2}). \tag{13}$$

Conservation equation in this case will be

$$\dot{\rho} + 3H(\rho + p_\mathrm{eff}) = 0, \tag{14}$$

where ρ and p are the density and pressure respectively under $f(Q)$ gravity.

4 NON-VISCOUS HOLOGRAPHIC $F(Q)$ GRAVITY

In this section, our aim is to study the behavior of "f" in non-viscous holographic $f(Q)$ gravity. To proceed, we consider a power law form of scale factor $a(t)$.

$$a(t) = a_0t^n, \tag{15}$$

$n > 0$. Then

$$\dot{a}(t) = a_0 n t^{-1+n}. \tag{16}$$

We know Hubble parameter $H = \frac{\dot{a}(t)}{a(t)}$. Using the expression of $a(t)$ and $\dot{a}(t)$ from Equations (15) and (16) respectively, we get H as

$$H = \frac{n}{t}. \tag{17}$$

Differentiating Equation (17) with respect to t, we get

$$\dot{H} = -\frac{n}{t^2}. \tag{18}$$

Now, applying the expression of H from Equation (17) in Equation (1), we get density of HDE.

$$\rho_\Lambda = \frac{3c^2 n^2}{t^2}. \tag{19}$$

Differentiating Equation (19) with respect to t, we get $\dot{\rho}_\Lambda$.

$$\dot{\rho}_\Lambda = -\frac{6c^2 n^2}{t^3}. \tag{20}$$

Now, relying on $\dot{\rho}_\Lambda$ from Equation (20), ρ_Λ from Equation (19) and H from Equation (17) in the conservation Equation (14), we get thermodynamic pressure of non-viscous HDE p_Λ as

$$p_\Lambda = -\frac{n\left(-2c^2 + 3c^2 n\right)}{t^2}. \tag{21}$$

Now, we will find the solution of $f(Q)$. As $Q = 6H^2$, using H from Equation (17), we get Q as

$$Q = \frac{6n^2}{t^2}. \tag{22}$$

Differentiating Equation (22) with respect to t, we get $\dot{Q}$.

$$\dot{Q} = -\frac{12n^2}{t^3}. \tag{23}$$

Again, differentiating Equation (23) with respect to t, we get $\ddot{Q}$.

$$\ddot{Q} = \frac{36n^2}{t^4}. \tag{24}$$

Using the expression of H from Equation (17), $\dot{H}$ from Equation (18) and p_Λ from Equation (21) (in place of $p_{\rm eff}$) in the second field Equation (13), we obtain $f(t)$ as

$$f(t) = -\frac{6c^2 n^2}{t^2} + t^{-3n} C_1 + \frac{C_2}{t}. \tag{25}$$

From expression (22), we get t in terms of Q as

$$t = \sqrt{\frac{6n^2}{Q}}. \tag{26}$$

Utilizing the expression of t from Equation (26) in Equation (25), we get $f(Q)$ as

$$f(Q) = \frac{C_2}{\sqrt{6}\sqrt{\frac{n^2}{Q}}} + 6^{-3n/2} C_1 \left(\frac{n^2}{Q}\right)^{-3n/2} - c^2 Q. \tag{27}$$

We have plotted the evolution of $f(Q)$ versus Q for non-viscous holographic $f(Q)$ gravity in Figure 1. Figure 1 suggests to us that $f(Q)$ stays at a positive level. Figure 1 also suggests to us that with the increase of Q, $f(Q)$ increases. We can interpret that f is a monotonically increasing function of Q. As $f(Q) \to 0$, $Q \to 0$. This indicates that one of the sufficient conditions for a realistic reconstruction model is satisfied under the reconstruction in absence of bulk viscosity.

5 VISCOUS EXTENDED GENERALIZED HOLOGRAPHIC RICCI $F(Q)$ GRAVITY

HDE unifies phantom inflation with late time acceleration of the Universe. It is suggested by Nojiri & Odintsov (2006b). Extended holographic Ricci Dark Energy (EHRDE) is the generalized form of HDE (Zhang 2009; George et al. 2019; Pankunni & Mathew 2014; Rao et al. 2018). The Ricci scalar is defined by

$$R = -6(\dot{H} + 2H^2 + \frac{\kappa}{a^2}) \tag{28}$$

where κ is the curvature of the Universe. The density of RHDE is given by $\rho_{\rm R} = 3c^2\left(\dot{H} + 2H^2 + \frac{\kappa}{a^2}\right)$, where c is a numerical parameter which tells us the characteristic of holographic Ricci DE and at $c^2 < 1/2$, the quintom behavior of EoS parameter is observed. In our study, we will consider a flat Universe. So, $\kappa = 0$. Therefore, $\rho_{\rm R} = 3c^2\left(\dot{H} + 2H^2\right)$. The density of RHDE is generalized to EHRDE by Granda and Oliveros and is expressed as

$$\rho_{\rm R} = 3c^2\left(\alpha\dot{H} + \beta H^2\right). \tag{29}$$

Here, also we have assumed the power law form of scale factor as in Equation (15). Inserting the expression of H and $\dot{H}$ from Equations (17) and (18) respectively in Equation (29), we get the density of EHRDE as

$$\rho_{\rm R} = 3c^2\left(-\frac{n\alpha}{t^2} + \frac{n^2\beta}{t^2}\right). \tag{30}$$

Differentiating Equation (30) with respect to t, we get

$$\dot{\rho}_{\rm R} = 3c^2\left(\frac{2n\alpha}{t^3} - \frac{2n^2\beta}{t^3}\right). \tag{31}$$

Now, considering the expression for H and $\dot{H}$ from Equations (17) and (18) respectively in the viscous coefficient $\xi = \xi_0 + \xi_1 H + \xi_2(\dot{H} + H^2)$, we get

$$\xi = \xi_0 + \frac{n\xi_1}{t} + \left(-\frac{n}{t^2} + \frac{n^2}{t^2}\right)\xi_2. \tag{32}$$

Utilizing the expression of ξ from Equation (32) and H from Equation (17) in the viscous pressure $\Pi = -3H\xi$, we get

$$\Pi = -\frac{3n\left(\xi_0 + \frac{n\xi_1}{t} + \left(-\frac{n}{t^2} + \frac{n^2}{t^2}\right)\xi_2\right)}{t}. \tag{33}$$

Using $\dot{\rho}_{\rm R}$ from Equation (31), $\rho_{\rm R}$ from Equation (30), H from Equation (17) and Π from Equation (33) in the conservation Equation (14), we arrive at thermodynamic pressure $p_{\rm R}$ of viscous EHRDE.

$$p_{\rm R} = \frac{-2c^2t\alpha + 3c^2nt\alpha + 2c^2nt\beta - 3c^2n^2t\beta}{t^3} + \frac{3nt^2\xi_0 + 3n^2t\xi_1 - 3n^2\xi_2 + 3n^3\xi_2}{t^3}. \tag{34}$$

Now, using $p_{\rm R}$ from Equation (34), H from Equation (17) and $\dot{H}$ from Equation (18) in the second field Equation (13), we obtain $f(t)$ as

$$\begin{aligned}
f(t) &= \frac{t^{-3(1+n)}}{(1-3n)^2(-2+3n)}\left(A_1 + t^{3n}A_2 - A_3\right)\\
A_1 &= C_3(1-3n)^2(-2+3n)t^3\\
A_2 &= (-2+3n)t(1-3n)^2\left[C_4t + 6c^2n(\alpha - n\beta)\right]\\
&\quad +18(-2+3n)n^2t^2\xi_0 + 18(1-3n)^2n^3t\xi_1\\
&\quad +3(1-3n)^2n^3(-2+3n)\xi_2\\
A_3 &= 18n^2\left[2+9(-1+n)n\right]t^{2+3n}\xi_0\log t.
\end{aligned} \tag{35}$$

Inserting the expression of t from Equation (26) in Equation (35), we find $f(Q)$

$$\begin{aligned}
f(Q) = &\frac{C_4}{\sqrt{6}\sqrt{\frac{n^2}{Q}}} + 6^{-3n/2}C_3\left(\frac{n^2}{Q}\right)^{-3n/2} + \frac{c^2Q(\alpha-n\beta)}{n}\\
&+\frac{3\sqrt{6}\sqrt{\frac{n^2}{Q}}Q\xi_0}{(1-3n)^2} + \frac{3nQ\xi_1}{-2+3n} + \frac{nQ\xi_2}{2\sqrt{6}\sqrt{\frac{n^2}{Q}}} - \frac{3\sqrt{\frac{3}{2}}\sqrt{\frac{n^2}{Q}}Q\xi_0\log\left[\frac{6n^2}{Q}\right]}{-1+3n}.
\end{aligned} \tag{36}$$

We have plotted the evolution of $f(Q)$ versus Q for viscous extended generalized holographic Ricci $f(Q)$ gravity in Figure 2. Figure 2 indicates that $f(Q)$ stays at a positive level. Figure 2 suggests to us that with the increase of Q, there is also an increase of $f(Q)$. We can conclude that f is a monotonically increasing function of Q. As $f(Q) \to 0$, $Q \to 0$. This indicates that one of the sufficient conditions for a realistic reconstruction model is satisfied under the reconstruction in the presence of bulk viscosity.

Differentiating Equation (36) with respect to Q, we get

$$\begin{aligned}
f_{\rm Q} &= -\frac{2^{\frac{1}{2}(-5-3n)}3^{-\frac{3}{2}(1+n)}}{(1-3n)^2n^2(-2+3n)}\left(\frac{n^2}{Q}\right)^{\frac{1}{2}(-1-3n)}\\
&\quad\times\left(-A_1 + A_2 6^{3n/2}\left(\frac{n^2}{Q}\right)^{3n/2} + A_3\right)\\
A_1 &= 18\sqrt{6}C_3(1-3n)^2n(-2+3n)\left(\frac{n^2}{Q}\right)^{3/2}\\
A_2 &= -6C_4(-2+3n)(1-3n)^2\left(\frac{n^2}{Q}\right)\\
&\quad +12\sqrt{6}c^2n(-2+3n)(1-3n)^2(-\alpha+n\beta)\sqrt{\frac{n^2}{Q}}\\
&\quad -324n(-2+3n)Q\xi_0\left(\frac{n^2}{Q}\right)^2\\
&\quad -36\sqrt{6}(1-3n)^2n\left(\frac{n^2}{Q}\right)^{3/2}Q\xi_1\\
&\quad -9(1-3n)^2n^3(-2+3n)\xi_2\\
A_3 &= 2^{1+\frac{3n}{2}}3^{3+\frac{3n}{2}}\left[2+9(-1+n)n\right]\left(\frac{n^2}{Q}\right)^{2+\frac{3n}{2}}\\
&\quad \times Q\xi_0\log\left(\frac{6n^2}{Q}\right).
\end{aligned} \tag{37}$$

Differentiating Equation (37) with respect to time t, we get

$$\begin{aligned}
\dot{f}_{\rm Q} &= \frac{2^{-\frac{3}{2}(2+n)}3^{-1-\frac{3n}{2}}\left(\frac{n^2}{Q}\right)^{-3n/2}Q}{(1-3n)^2n^3}\\
&\quad\times\left(A_1 - \frac{6^{3n/2}n\left(\frac{n^2}{Q}\right)^{3n/2}}{Q}A_2\right)\\
A_1 &= -6\sqrt{6}C_3(1-3n)^2(-2+3n)\left(\frac{n^2}{Q}\right)^{3/2}\\
A_2 &= -2C_4(1-3n)^2 + 3n\left[-12n\xi_0 + (1-3n)^2Q\xi_2\right]\\
&\quad +18n^2(-1+3n)\xi_0\log\left(\frac{6n^2}{Q}\right).
\end{aligned} \tag{38}$$

Relying on these expressions for $f(t)$, $f(Q)$, $f_{\rm Q}$ and $\dot{f}_{\rm Q}$ in the first and second field Equations (12) and (13), we obtain reconstructed pressure $p_{\rm rec,R}$ and density $\rho_{\rm rec,R}$ of viscous extended generalized holographic Ricci $f(Q)$ gravity.

$$\begin{aligned}
p_{\rm rec,R} &= \frac{1}{t^3}(A_1 + A_2)\\
A_1 &= c^2(-2+3n)t(\alpha - n\beta)\\
A_2 &= 6n\left[t^2\xi_0 + nt\xi_1 + (-1+n)n\xi_2\right]
\end{aligned} \tag{39}$$

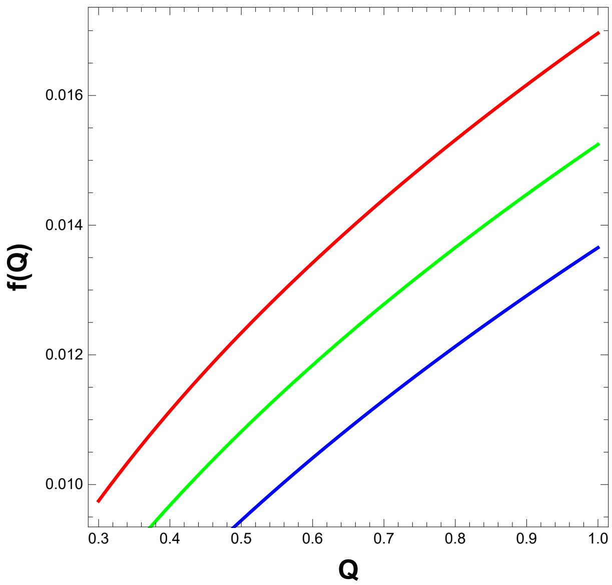

Fig. 1 Evolution of reconstructed $f(Q)$ (Eq. (27)) versus Q for non-viscous holographic $f(Q)$ gravity. We consider $C_1 = 0.008$, $C_2 = 0.005$ and $c = 0.02$. The red, green and blue lines correspond to $n = 0.30, 0.34$ and 0.38 respectively.

and

$$\begin{aligned}
\rho_{\rm rec,R} &= \frac{t^{-3(1+n)}}{2\left[2+9(-1+n)n\right]}\left(A_1 - 6nt^{3n}A_2\right)\\
A_1 &= -C_3(1-3n)^2(-2+3n)t^3\\
A_2 &= c^2\left[2+9(-1+n)n\right]t(\alpha-n\beta)\\
&\quad +3tn\left[(-2+3n)t\xi_0+n(-1+3n)\xi_1\right]\\
&\quad +n^2\left[2+9(-1+n)n\right]\xi_2\,. \qquad (40)
\end{aligned}$$

The reconstructed EoS parameter of viscous extended generalized holographic Ricci $f(Q)$ gravity is given by $w_{\rm rec,R} = \frac{p_{\rm rec,R}}{\rho_{\rm rec,R}}$. Therefore, relying on the expressions for $p_{\rm rec,R}$ and $\rho_{\rm rec,R}$ from Equations (39) and (40) respectively, we derive $w_{\rm rec,R}$ as

$$\begin{aligned}
w_{\rm rec,R} &= -2\left[2+9(-1+n)n\right]t^{3n}\frac{A_1}{A_2+6nt^{3n}A_3}\\
A_1 &= c^2(-2+3n)t(\alpha-n\beta)\\
&\quad +6n\left[t^2\xi_0+nt\xi_1+(-1+n)n\xi_2\right]\\
A_2 &= C_3(1-3n)^2(-2+3n)t^3\\
A_3 &= c^2\left[2+9(-1+n)n\right]t(\alpha-n\beta)\\
&\quad +3tn\left[(-2+3n)t\xi_0+n(-1+3n)\xi_1\right]\\
&\quad +n^2\left[2+9(-1+n)n\right]\xi_2\,. \qquad (41)
\end{aligned}$$

We have plotted evolution of reconstructed EoS parameter $w_{\rm rec,R}$ (Eq. (41)) versus cosmic time t of viscous extended generalized holographic Ricci $f(Q)$ gravity in Figure 3. We can see from the figure that with the evolution of the Universe, i.e. with the increase in cosmic time t, $w_{\rm rec,R}$ is increasingly going from phantom to quintessence. This indicates that there may be an avoidance of the Big Rip.

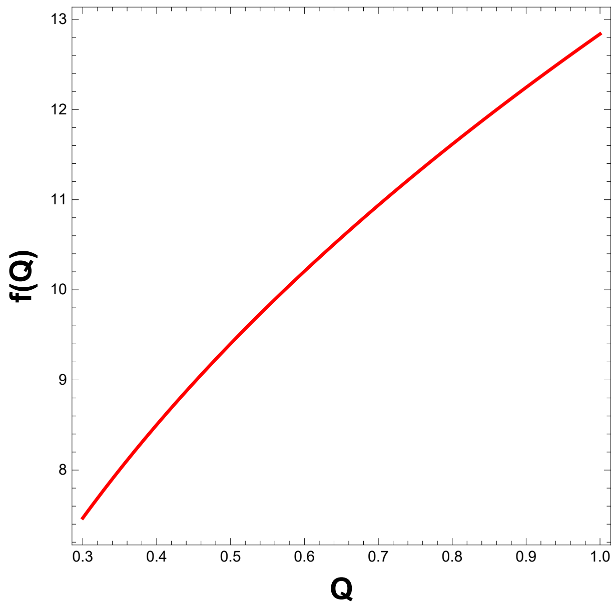

Fig. 2 Evolution of reconstructed $f(Q)$ (Eq. (36)) versus Q for viscous extended generalized holographic Ricci $f(Q)$ gravity. We consider $C_3 = 0.008$, $C_4 = 0.005$, $c = 0.02$, $\xi_0 = 0.06$, $\xi_1 = 0.001$, $\xi_2 = 0.009$, $\alpha = 0.05$ and $\beta = 0.04$. The red, green and blue lines correspond to $n = 0.30,\ 0.31$ and 0.32 respectively.

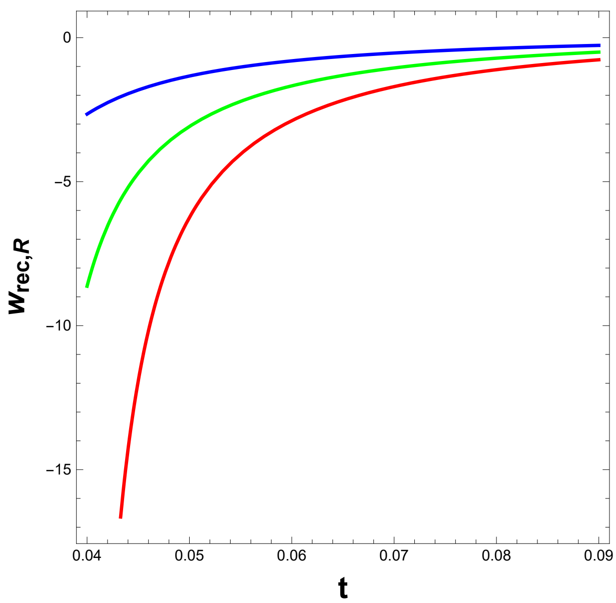

Fig. 3 Evolution of reconstructed EoS parameter $w_{\rm rec,R}$ (Eq. (41)) versus cosmic time t of viscous extended generalized holographic Ricci $f(Q)$ gravity. We consider $C_3 = 0.008, c = 0.02, \xi_0 = 0.06, \xi_1 = 0.001, \xi_2 = 0.009$, $\alpha = 0.05$ and $\beta = 0.04$. The red, green and blue lines correspond to $n = 0.30,\ 0.31$ and 0.32 respectively.

6 VISCOUS HOLOGRAPHIC $F(Q)$ GRAVITY WITH NOJIRI-ODINTSOV CUT-OFF

Nojiri and Odintsov demonstrated an approach to unify any inflationary cosmology with the DE cosmology of the late time Universe (Nojiri & Odintsov 2006c; Nojiri et al. 2020; Nojiri & Odintsov 2017). They deduced that there is a possibility of transition of phantom to non-phantom in such a manner that the Universe could have the phantom EoS in the early as well as in the late time. Nojiri et al. 2014 and Nojiri and Odintsov 2017 demonstrated unification of inflation with ΛCDM under the purview of $f(R)$ gravity. They call this the generalized Nojiri-Odintsov (NO) model and found the corresponding cut-off. The density for NO HDE is defined as

$$\rho_{\rm NO} = \frac{3c^2}{L^2} \tag{42}$$

where c is a numerical constant and L is the infrared cut-off. Let $R_{\rm h}$ be the event horizon; α_0, α_1 and α_2 are numerical constants. Then, $\frac{c}{L}$ is given by

$$\frac{c}{L} = \frac{1}{R_{\rm h}}(\alpha_0 + \alpha_1 R_{\rm h} + \alpha_2 {R_{\rm h}}^2)\,. \tag{43}$$

The event horizon is defined by

$$\dot{R}_{\rm h} = HR_{\rm h} - 1\,. \tag{44}$$

Solving the differential Equation (44) using H from Equation (17), we get

$$R_{\rm h} = -\frac{t}{1-n} + t^n C_5\,. \tag{45}$$

Considering the expression of $R_{\rm h}$ from Equation (45) in Equation (43), we obtain the infrared cut off L as

$$\begin{aligned} L &= \frac{c(-1+n)A_1}{(-1+n)^2\alpha_0 + A_1\left[(-1+n)\alpha_1 + A_1\alpha_2\right]} \\ A_1 &= t + C_5(-1+n)t^n\,. \end{aligned} \tag{46}$$

Using the expression of L from Equation (46) in expression (42), we get reconstructed density $\rho_{\rm NO,rec}$ of NO HDE as

$$\begin{aligned} \rho_{\rm NO,rec} &= \frac{3\left\{(-1+n)^2\alpha_0 + A_1\left[(-1+n)\alpha_1 + A_1\alpha_2\right]\right\}^2}{(-1+n)^2{A_1}^2} \\ A_1 &= t + C_5(-1+n)t^n \end{aligned} \tag{47}$$

Taking the correspondence between the density of NO HDE (Eq. (47)) and density of HDE (Eq. (19)), we arrive at two expressions for c. Let them be defined as c_1 and c_2.

$$\begin{aligned} c_1 &= -\frac{t\left\{(-1+n)^2\alpha_0 + A_1\left[(-1+n)\alpha_1 + A_1\alpha_2\right]\right\}}{(-1+n)nA_1} \\ A_1 &= t + C_5(-1+n)t^n\,. \end{aligned} \tag{48}$$

and

$$c_2 = \frac{t\left(\frac{(-1+n)\alpha_0}{t+C_5(-1+n)t^n} + \alpha_1 + \frac{t\alpha_2}{-1+n} + C_5 t^n\alpha_2\right)}{n}\,. \tag{49}$$

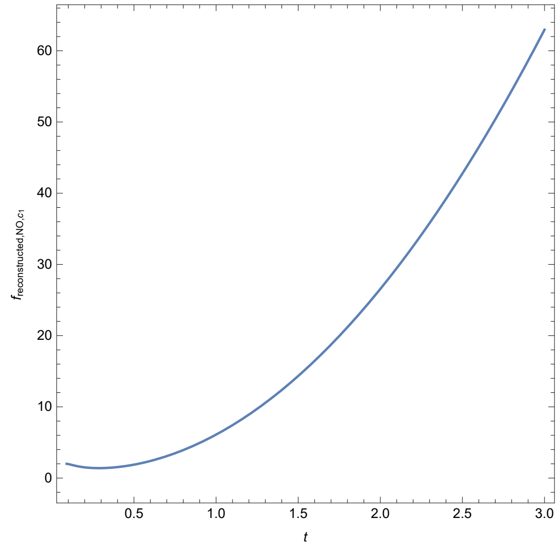

Fig. 4 Evolution of $f_{\rm reconstructed,NO,c_1}$ versus t of HDE in terms of viscous NO cut-off for $c = c_1$ (Eq. (48)) in $f(Q)$ gravity.

Case I: Behavior of $f(t)$ when $c = c_1$. Inserting $c = c_1$ from Equation (48) in Equation (19), we get the density $\rho_{\Lambda,\rm NO,c1}$ of HDE in terms of NO cut-off.

$$\begin{aligned} \rho_{\Lambda,\rm NO,c1} &= \frac{3\left\{(-1+n)^2\alpha_0 + A_1\left[(-1+n)\alpha_1 + A_1\alpha_2\right]\right\}^2}{(-1+n)^2{A_1}^2} \\ A_1 &= t + C_5(-1+n)t^n\,. \end{aligned} \tag{50}$$

From the first field Equation (Eq. (12)) of $f(Q)$ gravity and using H from Equation (17) and $\dot{Q}$ from Equation (23), we get the differential equation

$$tf'[t] = 2\rho - f[t]\,. \tag{51}$$

Substituting $\rho_{\Lambda,\rm NO,c1}$ from Equation (50) in place of ρ in Equation (51), we solve it numerically by taking initial condition as $f(0.09) = 2$ and the numerical value of parameters as $n = 0.9$, $\alpha_0 = 0.6$, $\alpha_1 = 0.3$, $\alpha_2 = 0.2$ and $C_5 = 0.004$. Considering the cosmic time t interval $[0.09, 3]$, we plot the evolution $f_{\rm reconstructed,NO,c_1}$ versus t of HDE in terms of viscous NO cut-off for $c = c_1$ (Eq. (48)) in Figure 4. Figure 4 suggests to us that with the evolution of the Universe, i.e., with the increase in cosmic time t, $f_{\rm reconstructed,NO,c_1}$ increases. The figure indicates that $f_{\rm reconstructed,NO,c_1}$ is found to stay positive.

Case II: Behavior of $f(t)$ when $c = c_2$.

Using $c = c_2$ from Equation (49) in Equation (19), we get the density $\rho_{\Lambda,\rm NO,c2}$ of HDE in terms of NO cut-off.

$$\rho_{\Lambda,\rm NO,c2} = 3 \times \left[\frac{(-1+n)\alpha_0}{t+C_5(-1+n)t^n} + \alpha_1 + \frac{t\alpha_2}{-1+n} + C_5 t^n\alpha_2\right]^2\,. \tag{52}$$

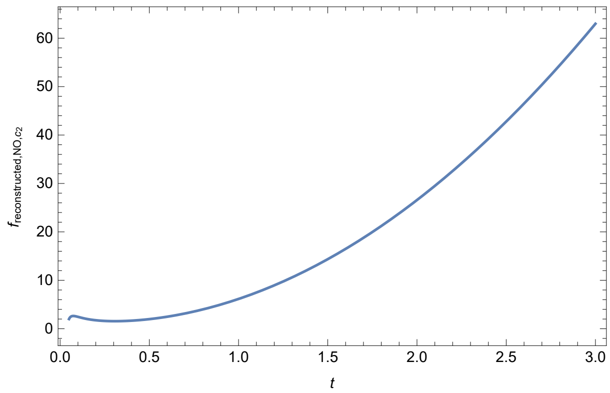

Fig. 5 Evolution of $f_{\rm reconstructed,NO,c_2}$ versus t of HDE in terms of viscous NO cut-off for $c = c_2$ (Eq. (49)) in $f(Q)$ gravity.

Utilizing this $\rho_{\Lambda,\rm NO,c2}$ from Equation (52) in place of ρ in Equation (51), we solve it numerically by taking initial condition as $f(0.05) = 2$ and the numerical values of parameters as $n = 0.9$, $\alpha_0 = 0.6$, $\alpha_1 = 0.3$, $\alpha_2 = 0.2$ and $C_5 = 0.004$. Considering the cosmic time t interval $[0.05, 3]$, we plot the evolution of $f_{\rm reconstructed,NO,c_2}$ versus t of HDE in terms of viscous NO cut-off for $c = c_2$ (Eq. (49)) in Figure 5. The figure suggests to us that $f_{\rm reconstructed,NO,c_2}$ is found to stay positive. Figure 4 indicates that with the evolution of the Universe, i.e. with the increase in cosmic time t, $f_{\rm reconstructed,NO,c_2}$ increases.

7 VISCOUS EXTENDED GENERALIZED HOLOGRAPHIC RICCI $F(Q)$ GRAVITY AS A SCALAR FIELD IN THE BULK VISCOSITY FRAMEWORK

The equation of motion for single-field inflation can be rewritten by the Hamiltonian-Jacobi formulation. Here, scalar field is a time variable and varies monotonically with time during any epoch. Hence, slow roll parameters can be written as

$$\epsilon_{\rm H} = 3\frac{\frac{\dot{\phi}^2}{2}}{V + \frac{\dot{\phi}^2}{2}} \tag{53}$$

and

$$\eta_{\rm H} = \epsilon_{\rm H} - \frac{1}{2}\frac{\dot{\epsilon}_{\rm H}}{H\epsilon_{\rm H}}\,. \tag{54}$$

During the inflationary phase, warm inflation invokes a significant component of radiation. Let us assume the inflation decay rate or dissipative coefficient is Γ. It is responsible for decay of the scalar field into radiation during the inflationary expansion of the Universe. It is defined by (Lyth & Liddle 2009)

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + \frac{dV}{d\phi} = 0\,. \tag{55}$$

To study inflation, we assume the quintessence scalar field model of DE as it is very effective in theory. The equation of energy density and pressure of quintessence scalar field are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{56}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{57}$$

ϕ is a scalar field and $V(\phi)$ is its potential. From Equations (56) and (57), we have

$$\dot{\phi} = \sqrt{\rho_\phi + p_\phi} \tag{58}$$

and

$$V(\phi) = \frac{\rho_\phi - p_\phi}{2}\,. \tag{59}$$

Let us consider the correspondence between viscous extended generalized holographic Ricci $f(Q)$ gravity and quintessence scalar field model in a bulk viscous framework. So, we have $p_\phi = p_{\rm rec,R}$ ($p_{\rm rec,R}$ from Equation (39) and $\rho_\phi = \rho_{\rm rec,R}$ ($\rho_{\rm rec,R}$ from Equation (40). Hence, from Equations (39), (40), (58) and (59), we have expressions for scalar field and potential as

$$\begin{aligned}
\dot{\phi} &= \frac{1}{\sqrt{2}}\left[\frac{t^{-3(1+n)}}{2+9(-1+n)n}\left(A_1 + 2t^{3n}A_2\right)\right]^{1/2}\\
A_1 &= -C_3(1-3n)^2(-2+3n)t^3\\
A_2 &= 2c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta)\\
&\quad +3n(2-3n)^2t^2\xi_0 + 3n^2\left[4+3n(-5+3n)\right]t\xi_1\\
&\quad +3n^2(-2+n)(-2+3n)(-1+3n)\xi_2
\end{aligned} \tag{60}$$

and

$$\begin{aligned}
V(t) &= \frac{1}{4t^3}\left[-C_3(-1+3n)t^{3-3n} + \frac{A_1}{2+9(-1+n)n}\right]\\
A_1 &= 4tc^2(-2+3n)(1-3n)^2(-\alpha+n\beta)\\
&\quad +6nt^2(-2+3n)(2-9n)\xi_0\\
&\quad -6n^2t(-1+3n)(-4+9n)\xi_1\\
&\quad -6n^2(2-3n)^2(-1+3n)\xi_2\,.
\end{aligned} \tag{61}$$

Using the expression of $\dot{\phi}$ from Equation (60) and $V(t)$ from Equation (61) in Equations (53) and (54), we get the Hubble slow roll parameters as

$$\begin{aligned}
\epsilon_{\rm H} &= -\frac{3}{2}\frac{A_1 + 2t^{3n}A_2}{-A_1 + 6nt^{3n}A_3}\\
A_1 &= -C_3(1-3n)^2(-2+3n)t^3\\
A_2 &= 2c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta)\\
&\quad +3n(2-3n)^2t^2\xi_0 + 3n^2\left[4+3n(-5+3n)\right]t\xi_1\\
&\quad +3n^2(-2+n)(-2+3n)(-1+3n)\xi_2\\
A_3 &= c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta)\\
&\quad +3nt\left[(-2+3n)t\xi_0 + n(-1+3n)\xi_1\right]\\
&\quad +n^2\left[2+9(-1+n)n\right]\xi_2
\end{aligned} \tag{62}$$

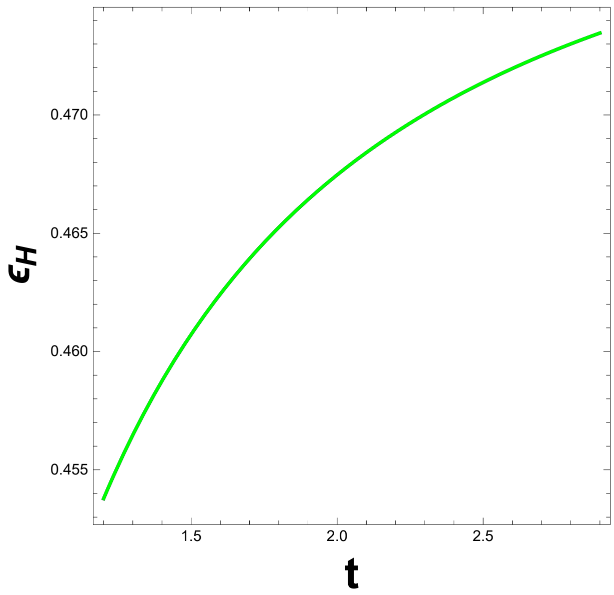

Fig. 6 Evolution of Hubble slow roll parameter $\epsilon_{\rm H}$ (Eq. (62)) versus the cosmic time t of viscous extended generalized holographic Ricci $f(Q)$ gravity. We consider $C_3 = -0.32$, $c = 0.009$, $\alpha = 0.5$, $\xi_0 = 0.0007$, $\xi_1 = 0.92$, $\xi_2 = 0.00009$, $\beta = 0.4$ and $n = 1.006$. The red, green and blue lines correspond to $\alpha = -0.5, 0.6$ and 0.7 respectively.

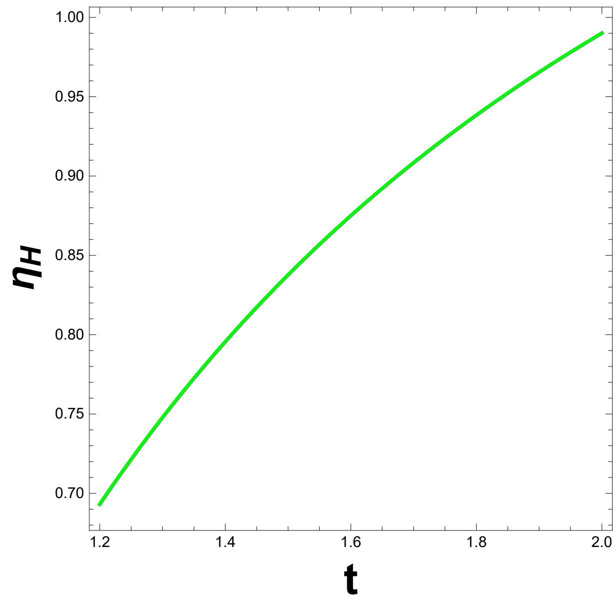

Fig. 7 Evolution of Hubble slow roll parameter $\eta_{\rm H}$ (Eq. (63)) versus the cosmic time t of viscous extended generalized holographic Ricci $f(Q)$ gravity. We consider $c = 0.009$, $\xi_0 = 0.07$, $\xi_1 = 0.0001$, $\xi_2 = 0.0023$, $\beta = 0.0004$, $n = 0.006$ and $C_3 = -0.4$. The red, green and blue lines correspond to $\alpha = 0.999997, 0.999998$ and 0.999999 respectively.

and

$$\begin{aligned}
\eta_{\rm H} &= \frac{-3n\left(A_1 + 2t^{3n}A_2\right)^2 + 2t^{1+3n}\left(A_4A_5 - A_6A_7\right)}{2n\left(A_1 + 2t^{3n}A_2\right)\left(-A_1 + 6nt^{3n}A_3\right)} \\
A_1 &= -C_3(1-3n)^2(-2+3n)t^3 \\
A_2 &= 2c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta) \\
&\quad +3n(2-3n)^2t^2\xi_0 + 3n^2\left[4+3n(-5+3n)\right]t\xi_1 \\
&\quad +3n^2(-2+n)(-2+3n)(-1+3n)\xi_2 \\
A_3 &= c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta) \\
&\quad +3nt\left[(-2+3n)t\xi_0 + n(-1+3n)\xi_1\right] \\
&\quad +n^2\left[2+9(-1+n)n\right]\xi_2 \\
A_4 &= c^2(2-3n)^2(-1+3n)(-\alpha+n\beta) \\
A_5 &= C_3\left[2+9(-1+n)n\right]^2t^3 \\
&\quad -6n^3t^{3n}\left\{-9t^2\xi_0 + \left[4+3n(-5+3n)\right]\xi_2\right\} \\
A_6 &= 6n\left[2+9(-1+n)n\right] \\
A_7 &= C_3(-2+3n)(t-3nt)^2B_1 + 6n^3t^{3n}B_2 \\
B_1 &= (-1+3n)t^2\xi_0 + n(-2+3n)t\xi_1 + 3(-1+n)^2n\xi_2 \\
B_2 &= 3t^2\xi_0\xi_1 + 4(-2+3n)t\xi_0\xi_2 + n(-1+3n)\xi_1\xi_2
\end{aligned} \quad (63)$$

We plot the evolution of Hubble slow roll parameters ϵ_H and η_H against the cosmic time t in Figure 6 and Figure 7 respectively. We observe from the figures that the Hubble slow roll parameters are less than 1 and are increasing with cosmic time t. This indicates that the model has a scope of exit from inflation. In the very early stages of inflation, the scalar field ϕ is overdamped. Therefore, the condition of inflationary period is constrained by $\frac{1}{2}\dot{\phi}^2 = V$ (Kumar & Xu 2014). In this section, viscous extended generalized holographic Ricci $f(Q)$ gravity is assumed to provide the

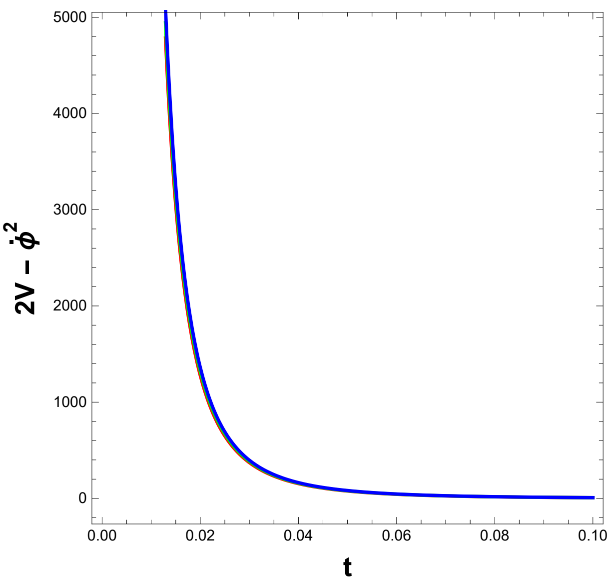

Fig. 8 Evolution of $2V - \dot{\phi}^2$ (Eq. (64)) versus cosmic time t of viscous extended generalized holographic Ricci $f(Q)$ gravity. We consider $c = 0.02$, $\xi_0 = 0.06$, $\xi_1 = 0.001$, $\xi_2 = 0.009$, $\alpha = 0.05$ and $\beta = 0.04$. The red, green and blue lines correspond to $n = 0.40, 0.41$ and 0.42 respectively.

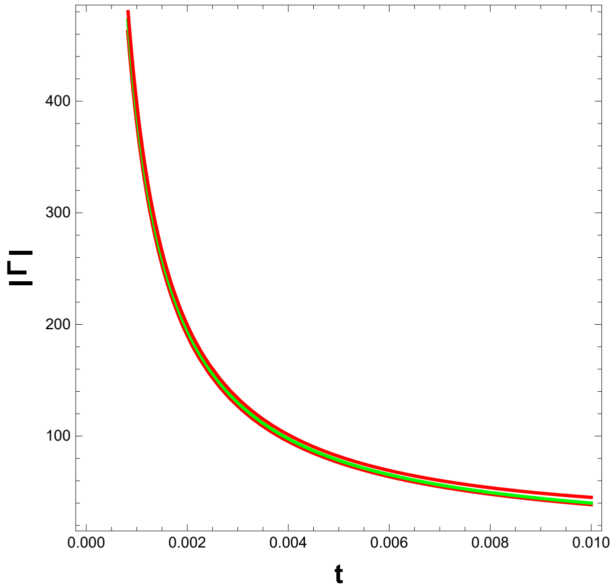

Fig. 9 Evolution of dissipative coefficient Γ (Eq. (68)) versus cosmic time t of viscous extended generalized holographic Ricci $f(Q)$ gravity. We consider $C_3 = 0.8$, $c = 0.02$, $\xi_0 = 0.06$, $\xi_1 = 0.001$, $\xi_2 = 0.009$, $\alpha = 0.4$ and $\beta = 0.04$. The red, green and blue lines correspond to $n = 0.31, 0.32$ and 0.33 respectively.

inflationary scenario. Hence, for the inflationary expansion to happen we should have $2V - \dot{\phi}^2 > 0$ (Brevik et al. 2017a; Odintsov et al. 2018). Using $\dot{\phi}$ and $V(t)$ from Equations (60) and (61) respectively, we have

$$\begin{aligned} 2V - \dot{\phi}^2 &= -\frac{2}{t^3}\left(A_1 + A_2\right) \\ A_1 &= c^2(-2+3n)t(\alpha - n\beta) \\ A_2 &= 6n\left[t^2\xi_0 + nt\xi_1 + (-1+n)n\xi_2\right] \,. \end{aligned} \quad (64)$$

We plot $2V - \dot{\phi}^2$ against cosmic time t in Figure 8. From the figure, we see that $2V - \dot{\phi}^2 > 0$ for the range $0.00001 < t < 0.1$. Hence, quasi-exponential expansion is available for $0.00001 < t < 0.1$ of viscous extended generalized holographic Ricci $f(Q)$ gravity. Now we will study the behavior of dissipative coefficient Γ. Differentiating Equations (60) and (61) with respect to t, we get

$$\begin{aligned} \ddot{\phi} &= \frac{-t^{-4-3n}\left(3nA_1 + 2t^{3n}A_3\right)}{2\left[2t^{-3(1+n)}\left(A_1 + 2t^{3n}A_2\right)A_4\right]^{1/2}} \\ A_1 &= -C_3(1-3n)^2(-2+3n)t^3 \\ A_2 &= 2c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta) \\ &\quad +3n(2-3n)^2t^2\xi_0 + 3n^2\left[4+3n(-5+3n)\right]t\xi_1 \\ &\quad +3n^2(-2+n)(-2+3n)(-1+3n)\xi_2 \\ A_3 &= 4c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta) \\ &\quad +3n(2-3n)^2t^2\xi_0 + 6n^2\left[4+3n(-5+3n)\right]t\xi_1 \\ &\quad +9n^2(-2+n)(-2+3n)(-1+3n)\xi_2 \\ A_4 &= 2+9(-1+n)n \end{aligned} \quad (65)$$

and

$$\begin{aligned} \dot{V}(t) &= \frac{3nA_1 + 2t^{3n}A_2}{4t^{4+3n}\left[2+9(-1+n)n\right]} \\ A_1 &= C_3(1-3n)^2(-2+3n)t^3 \\ A_2 &= -4c^2(-2+3n)t(1-3n)^2(-\alpha+n\beta) \\ &\quad +3nt(-2+3n)(-2+9n)t\xi_0 \\ &\quad +6n^2(-1+3n)(-4+9n)t\xi_1 \\ &\quad +9(2-3n)^2n^2(-1+3n)\xi_2 \,. \end{aligned} \quad (66)$$

Hence,

$$\begin{aligned} \frac{dV}{d\phi} &= \frac{t^{-4-3n}\left(-3nA_1 + 2t^{3n}A_3\right)}{2\left[2t^{-3(1+n)}\left(A_1 + 2t^{3n}A_2\right)A_4\right]^{1/2}} \\ A_1 &= -C_3(1-3n)^2(-2+3n)t^3 \\ A_2 &= 2c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta) \\ &\quad +3n(2-3n)^2t^2\xi_0 + 3n^2\left[4+3n(-5+3n)\right]t\xi_1 \\ &\quad +3n^2(-2+n)(-2+3n)(-1+3n)\xi_2 \\ A_3 &= -4c^2(-2+3n)t(1-3n)^2(-\alpha+n\beta) \\ &\quad +3nt(-2+3n)(-2+9n)t\xi_0 \\ &\quad +6n^2(-1+3n)(-4+9n)t\xi_1 \\ &\quad +9(2-3n)^2n^2(-1+3n)\xi_2 \\ A_4 &= 2+9(-1+n)n \,. \end{aligned} \quad (67)$$

Using the expression of $\ddot{\phi}$ from Equation (65), $\frac{dV}{d\phi}$ from Equation (67), H from Equation (17) and $\dot{\phi}$ from Equation (60) in Equation (55), we get Γ as

$$\begin{aligned} \Gamma &= \frac{18n^2t^{-1+3n}A_3A_4}{A_1 - 2t^{3n}A_2} \\ A_1 &= C_3(1-3n)^2(-2+3n)t^3 \\ A_2 &= 2c^2\left[2+9(-1+n)n\right]t(-\alpha+n\beta) \\ &\quad +3n(2-3n)^2t^2\xi_0 + 3n^2\left[4+3n(-5+3n)\right]t\xi_1 \\ &\quad +3n^2(-2+n)(-2+3n)(-1+3n)\xi_2 \\ A_3 &= t^2\xi_0 + nt\xi_1 + (-1+n)n\xi_2 \\ A_4 &= 2+9(-1+n)n \,. \end{aligned} \quad (68)$$

We have plotted $|\Gamma|$ against cosmic time t in Figure 9. From the figure, we observe that $|\Gamma| > 1$. This implies warm inflation which means the decay of scalar field into radiation during the inflationary phase.

8 CONCLUSIONS

The study described in this paper has been aimed at investigating the behavior of $f(Q)$ gravity with different cut-offs. We have reconstructed thermodynamic pressure and density of viscous extended generalized holographic Ricci $f(Q)$ gravity. We then reconstructed its EoS parameter and examined it accordingly. For the viscous case, we adopted an Eckart approach. This approach was followed because it is consistent with the observational data. The bulk viscous pressure, $\Pi = -3H\xi$, where $\xi = \xi_0 + \xi_1 H + \xi_2(\dot{H} + H^2)$ (Jiménez et al. 2018; Ren & Meng 2006). We also investigated $f(Q)$ gravity with different cut-offs. From the history of the expansion of the Universe, the reconstruction of modified gravity was demonstrated (Nojiri & Odintsov 2007a).

In the Section 2, we assumed the size of the Universe to be Hubble radius H^{-1}. It can be noted that, by considering this, the density of HDE becomes $\rho_\Lambda = 3c^2H^2$. This density is very close to the observational value.

In Section 3, we demonstrated the field Equations (12) and (13) of $f(Q)$ gravity. For this we expressed action of symmetric teleparallel gravity in Equation (2).

In Section 4, we examined the case of non-viscous holographic $f(Q)$ gravity. For this, we considered the power law form of scale factor $a(t) = a_0 t^n$, $n > 0$. From the conservation Equation (14), we deduced thermodynamic pressure of non-viscous HDE as p_Λ in Equation (21). Inserting p_Λ in the second field Equation (13), we derived $f(t)$ in Equation (25). Hence, we got $f(Q)$ in Equation (27). We then plotted $f(Q)$ versus Q of non-viscous holographic $f(Q)$ gravity in Figure 1. We can see that $f(Q)$ stays at a positive level and with the increase of Q, $f(Q)$ increases. We also note that as $f(Q) \to 0, Q \to 0$.

In Section 5, we demonstrated the case "viscous extended generalized holographic Ricci $f(Q)$ gravity". Here, we reconstructed $f(Q)$ gravity in Equation (36) and plotted it in Figure 2 versus Q. The figure also indicates the same as what is in Figure 1. This affirms that $f(Q)$ is positive and as Q increases, $f(Q)$ increases. As $f(Q) \to 0$, $Q \to 0$. In Equations (39) and (40), we reconstructed the pressure $p_{\mathrm{rec,R}}$ and density $\rho_{\mathrm{rec,R}}$ of viscous extended generalized holographic Ricci $f(Q)$ gravity respectively. Hence, reconstructed EoS parameter $w_{\mathrm{rec,R}}$ of Equation (41) for viscous extended generalized holographic Ricci $f(Q)$ gravity is plotted against cosmic time t in Figure 3. The figure suggests that this model indicates the avoidance of a Big Rip singularity.

In Section 6, the case "viscous holographic $f(Q)$ gravity with Nojiri-Odintsov cut-off" was examined. In this case, we reconstructed infrared cut-off L in Equation (46) and reconstructed density in Equation (47) of NO HDE. We derived two expressions for c and named them c_1 and c_2 in Equations (48) and (49) respectively. We then proceeded in two cases i.e., case I for expression c_1 (Eq. (48)) and case II for expression c_2 (Eq. (49)). In case I, we deduced the density $\rho_{\Lambda,\mathrm{NO},c_1}$ (Eq. (50)) of HDE in terms of NO cut-off. Hence, we plotted $f_{\mathrm{reconstructed,NO},c_1}$ versus cosmic time t in Figure 4 of this case. In case II, we derived the density $\rho_{\Lambda,\mathrm{NO},c_2}$ (Eq. (52)) of HDE in terms of NO cut-off. Hence, we plotted $f_{\mathrm{reconstructed,NO},c_2}$ versus cosmic time t in Figure 5 for this case. Both figures, i.e., Figs. 4 and 5, suggest that with cosmic time t, $f_{\mathrm{reconstructed,NO},c_1}$ and $f_{\mathrm{reconstructed,NO},c_2}$ increase and are found to stay at a positive level.

In Section 7, we examined the contribution of slow roll parameters in $f(Q)$ gravity. Here we assumed the background evolution to be viscous extended generalized holographic Ricci $f(Q)$ gravity. The Hubble slow roll parameters are less than 1 and ϵ_{H} (Eq. (62)) and η_{H} (Eq. (63)) are increasing (Figs.6 and 7). This indicates that there is a scope of exit from inflation. It is observed from Figure 8 that $2V - \dot{\phi}^2 > 0$. This indicates that quasi-exponential expansion is available for this model. In Figure 9, we can see that $|\Gamma| > 1$, i.e. the warm inflationary expansion can be obtained from this model.

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