RAA 2021 Vol. 21 No. 10, 242(12pp) doi: 10.1088/1674-4527/21/10/242 © 2021 National Astronomical Observatories, CAS and IOP Publishing Ltd. http://www.raa-journal.org http://iopscience.iop.org/raa

Research in Astronomy and Astrophysics

# Gaussianization of peculiar velocities and bulk flow measurement

Fei Qin

Korea Astronomy and Space Science Institute, Yuseong-gu, Daedeok-daero 776, Daejeon 34055, Korea; *feiqin@kasi.re.kr* 

Received 2021 February 18; accepted 2021 April 27

Abstract The line-of-sight peculiar velocities are good indicators of the gravitational fluctuation of the density field. Techniques have been developed to extract cosmological information from the peculiar velocities in order to test cosmological models. These techniques include measuring cosmic flow, measuring two-point correlation and power spectrum of the peculiar velocity fields, and reconstructing the density field using peculiar velocities. However, some measurements from these techniques are biased due to the non-Gaussianity of the estimated peculiar velocities. Therefore, we rely on the 2MTF survey to explore a power transform that can Gaussianize the estimated peculiar velocities. We find a tight linear relation between the transformation parameters and the measurement errors of log-distance ratio. To show an example for the implementation of Gaussianized peculiar velocities in cosmology, we develop a bulk flow estimator and estimate bulk flow from the Gaussianized peculiar velocities. We use 2MTF mocks to test the algorithm, and we find the algorithm yields unbiased measurements. We also find this technique gives smaller measurement errors compared to other techniques. In Galactic coordinates, at the depth of 30  $h^{-1}$  Mpc, we measure a bulk flow of  $332 \pm 27$  km s<sup>-1</sup> in the direction  $(l, b) = (293^{\circ} \pm 5^{\circ}, 13^{\circ} \pm 4^{\circ})$ . The measurement is consistent with the  $\Lambda$ CDM prediction.

Key words: cosmology: large-scale structure of universe

# **1 INTRODUCTION**

Driven by the expansion of the Universe, galaxies move further apart from us. This motion is called recessional velocity and is described by Hubble's Law which is a linear relation between the redshift and distance of galaxies. On small scales, the mass density field of the Universe is not ideally homogenous and isotropic, which results from gravitational fluctuation. On top of the Hubble recessional velocities, galaxies will have peculiar motions which arise from these gravitational perturbations of the mass density field. The line-of-sight peculiar velocities of galaxies enable us to test the cosmological models through three main techniques.

One technique directly measures the cosmic flow field utilizing peculiar velocities, then compares the cosmological models' prediction to test whether the models accurately describe the motion of galaxies. Some examples of previous work related to this method are Kaiser (1988); Staveley-Smith & Davies (1989); Jaffe & Kaiser (1995); Nusser & Davis (1995); Parnovsky et al. (2001); Nusser & Davis (2011); Turnbull et al. (2012); Ma et al. (2012); Ma & Scott (2013); Ma & Pan (2014); Hong et al. (2014); Scrimgeour et al. (2016); Qin et al. (2018, 2019a); Boruah et al. (2020). The measurements agree with the  $\Lambda$  cold dark matter ( $\Lambda$ CDM) model prediction.

The second technique measures the two-point correlation and/or power spectrum of the peculiar velocity field and fits the cosmological parameters, then compares to the cosmological models' prediction. Some examples of previous work related to this method are Gorski et al. (1989); Kolatt & Dekel (1997); Zaroubi et al. (1997); Juszkiewicz et al. (2000); Silberman et al. (2001); Feldman et al. (2003); Gordon et al. (2007); Johnson et al. (2014); Howlett et al. (2017); Huterer et al. (2017); Dupuy et al. (2019); Howlett (2019); Qin et al. (2019b).

The third technique is the reconstruction of the density/velocity field of the local Universe using peculiar velocities. Some examples of previous work related to this method are Nusser & Davis (1994), Erdoğdu et al. (2006), Lavaux et al. (2010), Springob et al. (2014), Carrick et al. (2015), Pomarède et al. (2017), Springob et al. (2016).

In most of the past literature, the measurement errors of peculiar velocities are assumed to be Gaussian which is not true for the usual peculiar velocity estimator. This can bias the measurements, and many researches have been done to deal with the non-Gaussianity of the estimated peculiar velocities. For example, in terms of cosmic flow measurements, to avoid non-Gaussianity of the estimated peculiar velocities, Nusser & Davis (1995, 2011); Qin et al. (2018, 2019a) rely on the so-called  $\eta$ MLE to measure cosmic flow in the logarithmic distance ratio-space. Watkins & Feldman (2015) developed a peculiar velocity estimator which has Gaussian errors but is biased in some circumstances. In terms of power spectrum measurements, Qin et al. (2019b) apply a power transformation to offset the non-Gaussianity of the momentum power spectrum (Howlett 2019) to fit the growth rate of the large scale structure.

In this paper, we will explore a technique that Gaussianizes the estimated line-of-sight peculiar velocities. In addition, we take the bulk flow measurement as an example to illustrate the implementation of the Gaussianized peculiar velocities in terms of testing cosmology. The survey data considered in this paper are the full-sky 2MASS Tully-Fisher (2MTF) survey (Hong et al. 2019).

The paper is structured as follows: in Section 2 we introduce the 2MTF data and mocks. The mocks are used to test the algorithm. In Section 3 we introduce the peculiar velocity estimators and discuss the Gaussianity of the estimated peculiar velocities. In Section 4 we introduce the algorithm employed to Gaussianize the peculiar velocities. In Section 5 we introduce the bulk flow estimator, which estimates bulk flows from Gaussianized peculiar velocities and test the estimator utilizing mock surveys. In Section 6 we present the bulk flow measured from 2MTF. A conclusion in presented in Section 7.

This paper assumes spatially flat cosmology. The cosmological parameters used in this paper are from the Planck Collaboration et al. (2016):  $\Omega_m = 0.307$ ,  $\Omega_{\Lambda} = 0.693$ ,  $\sigma_8 = 0.823$ ,  $H_0 = 100 \ h \ \mathrm{km \ s^{-1} \ Mpc^{-1}}$  and h = 0.678. These parameters are applied to the calculation of the comoving distances and the bulk flow predicted in  $\Lambda$ CDM.

# 2 DATA AND MOCKS

2MTF (Hong et al. 2019) is a full sky Tully-Fisher survey derived from the Two Micron All-Sky Survey (2MASS, Masters et al. 2008; Huchra et al. 2012; Hong et al. 2014). The redshift of 2MTF galaxies reaches a maximum of  $1.2 \times 10^4$  km s<sup>-1</sup> and is no smaller than 600 km s<sup>-1</sup>. 2MTF is a full-sky survey, but excluding the Galactic plane region where Galactic latitude  $|b| < 5^{\circ}$ . Figure 1 depicts the survey geometry (redshift distribution and sky coverage) of the 2MTF galaxies. The logarithmic distance (log-distance) ratio for a galaxy is defined as

$$\eta \equiv \log_{10} \frac{d_z}{d_h} \,, \tag{1}$$

where  $d_z$  is the apparent comoving distance of a galaxy inferred from its observed redshift z, and  $d_h$  is the true comoving distance of the galaxy. In the 2MTF survey,  $\eta$ is estimated from the Tully-Fisher relation (Masters et al. 2008; Hong et al. 2014). The distribution of the logdistance ratio of the 2MTF galaxies is displayed in the top panel of Figure 2, and the measurement error of logdistance ratio,  $\epsilon$ , is shown in the bottom panel of Figure 2.

We rely on 16 mock 2MTF catalogs to test the algorithm used in this paper. These mocks are not included in the real data analysis and the comparison of  $\Lambda$ CDM. The mock sampling algorithm is clearly presented in Howlett et al. (2017). 2MTF has a well defined selection function (Hong et al. 2014), enabling us to generate high quality mocks which can accurately realize the survey geometry and selection function of 2MTF (Howlett et al. 2017; Qin et al. 2018, 2019a,b). In this paper, the 16 mocks are generated from two different simulations: the Gigaparsec WiggleZ (GiggleZ) (Poole et al. 2015) and Synthetic UniveRses For Surveys (SURFS) (Elahi et al. 2018) simulations. The cosmological parameters applied in the GiggleZ simulation are  $\Omega_m = 0.273$  and h =0.705, while for the SURFS simulation,  $\Omega_m = 0.3121$ and h = 0.6751. Two different simulations are used to produce the mocks, enabling us to ensure that the algorithm presented in this paper gives consistent answers for different cosmologies (Qin et al. 2018).

#### **3 PECULIAR VELOCITY ESTIMATORS**

If we neglect the relativistic motions and gravitational lensing effects, the line-of-sight peculiar velocity of a galaxy, V, can be estimated from its log-distance ratio  $\eta$ and observed redshift z through (Colless et al. 2001; Hui & Greene 2006; Davis & Scrimgeour 2014; Scrimgeour et al. 2016; Qin et al. 2018, 2019a)

$$V = c \left(\frac{z - z_h}{1 + z_h}\right) \,, \tag{2}$$

where c is the speed of light. The Hubble recessional redshift  $z_h$  is numerically calculated from the true comoving distance,  $d_h$  of the galaxy via

$$d_h(z_h) = \frac{c}{H_0} \int_0^{z_h} \frac{dz'}{E(z')} , \qquad (3)$$

where

$$E(z) = \frac{H(z)}{H_0} = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$$
. (4)



**Fig. 1** The survey geometry of 2MTF. The *left panel* displays the distribution of redshift of 2MTF galaxies. The *right panel* features the sky coverage of the 2MTF galaxies and the color of the points indicates the galaxy redshift, according to the color bar.



**Fig. 2** The *top panel* showcases the distribution of logdistance ratio  $\eta$  of 2MTF. The *bottom panel* depicts the distribution of measurement errors of log-distance ratio  $\epsilon$ .

Here  $H_0$ ,  $\Omega_m$  and  $\Omega_\Lambda$  are the Hubble constant, matter and dark energy densities of the present day Universe, respectively. The true comoving distance

$$d_h = d_z 10^{-\eta},\tag{5}$$

where the apparent comoving distance  $d_z$  is calculated from the observed redshift z directly through a similar expression as Equation (3).

For a galaxy, Equation (2) converts z and  $\eta$  to V nonlinearly. Therefore, the measurement error of V, which is propagated from the measurement error of  $\eta$ , is not Gaussian, even if we assume  $\eta$  has Gaussian error. To see this clearly, utilizing Equations (2), (3) and (4), one can calculate the probability density function (PDF) of an estimated line-of-sight peculiar velocity, given by (Scrimgeour et al. 2016)

$$P(V) = P(\eta) \frac{d\eta}{dV} = P(\eta) \times \frac{(1+z_h)^2}{d_h H_0 E(z_h)(1+z)\ln(10)},$$
(6)

where  $P(\eta)$  denotes the PDF of  $\eta$ . Usually (but not necessarily),  $P(\eta)$  is assumed to be a Gaussian function. However, due to the non-linear term behind  $P(\eta)$ , the resultant P(V) is not Gaussian. Therefore, the peculiar velocity estimated from Equation (3) for a galaxy does not have Gaussian error (see Sect. 4.2 and Fig. 3 for more discussions).

Due to the non-Gaussianity of the peculiar velocities, the cosmological parameters and cosmic flows estimated from the peculiar velocity fields are biased to some extent. Qin et al. (2019a) find that this non-Gaussianity will bias the momentum power spectrum measurements and then bias the estimation of the growth rate of largescale-structure. Employing simulations, Qin et al. (2018) demonstrate that this non-Gaussianity will bias the cosmic flow measurements compared to the true values.

To preserve the Gaussianity of the estimated peculiar velocities, Watkins & Feldman (2015) developed the following estimator to calculate the peculiar velocity for a galaxy

$$V = \frac{cz_{\text{mod}}}{1 + z_{\text{mod}}} \ln \frac{cz_{\text{mod}}}{H_0 d_l} , \quad (V_t \ll cz), \tag{7}$$

where  $d_l$  denotes the luminosity distance and  $z_{mod}$  is given by

$$z_{\text{mod}} = z \left[ 1 + \frac{1}{2} (1 - q_0) z - \frac{1}{6} (1 - q_0 - 3q_0^2 + 1) z^2 \right].$$
(8)

where  $q_0 = 0.5(\Omega_m - 2\Omega_\Lambda)$  is the so called acceleration parameter. At low redshift,  $\ln \frac{cz_{mod}}{H_0 d_l} \approx \ln(10)\eta$ , then the PDF for a peculiar velocity estimated from Equation (7) is expressed as

$$P(V) = P(\eta) \frac{d\eta}{dV} = P(\eta) \times \frac{1 + z_{\text{mod}}}{cz_{\text{mod}} \ln(10)} .$$
(9)

Assuming  $P(\eta)$  is Gaussian, for a given galaxy,  $z_{mod}$  is a certain number, and P(V) is linearly related to  $P(\eta)$ and therefore is Gaussian. Therefore, the peculiar velocity estimated from Equation (7) for a galaxy does have Gaussian error. However, one caveat is that Equation (7) only strictly returns an unbiased estimated peculiar velocity under the assumption that the galaxy's *true* peculiar velocity (not necessarily the measured peculiar velocity) is much smaller than cz for that galaxy. Using the mock 2MTF surveys, Howlett et al. (2017) explore to what extent Equation (7) biases the measured peculiar velocities of 2MTF. As illustrated in figure 9 of Howlett et al. (2017), Equation (7) overestimates large positive peculiar velocities, but underestimates large negative peculiar velocities.

# 4 GAUSSIANIZING THE PECULIAR VELOCITIES

To preserve the Gaussianity and avoid any assumption on the unknown true velocity of the galaxy compared to its redshift, we can instead Gaussianize a peculiar velocity of Equation (2). In this section, we will introduce the algorithm utilized to perform the Gaussianization.

#### 4.1 Box-Cox Transformation

Box & Cox (1964) developed a power transform technique that makes non-Gaussian distributed data more normal distribution-like (Sakia 1992). The Box-Cox (BC) transformation also has been introduced into cosmology in order to offset the non-Gaussianity of measurements. For example, Wang et al. (2019) study the BC transformation of the density power spectrum, and Qin et al. (2019b) use the BC transformation to Gaussianize the momentum power spectrum to fit the growth rate of the large scale structure. In this paper, we will apply the BC transformation to the peculiar velocities to obtain Gaussianized peculiar velocities.

For a set of non-Gaussian distributed data  $\{v_i | i = 1, 2, ..., M\}$  (i.e., the normalized histogram of the data set is not Gaussian), the BC transformation of  $v_i$  is defined as (Box & Cox 1964)

$$Y_i \equiv \begin{cases} \frac{(v_i + \delta)^{\lambda} - 1}{\lambda}, & \lambda \neq 0\\ \ln(v_i + \delta), & \lambda = 0 \end{cases}$$
(10)

where  $\delta$  is a shift of the whole data set in order to keep all the data being positive. Such shift will not change the analysis of variance (Box & Cox 1964; Sakia 1992).  $\lambda$  is the transformation parameter for the whole data set, which can be estimated by maximizing the following logarithmic likelihood function (Box & Cox 1964; Sakia 1992; Qin et al. 2019b)

$$L(\lambda) \sim (\lambda - 1) \sum_{i}^{M} \ln(v_i + \delta) - \frac{M}{2} \ln\left(\frac{\sum_{i} (Y_i(\lambda) - \bar{Y}(\lambda))^2}{M}\right),$$
(11)

following the steps presented in Box & Cox (1964) and Qin et al. (2019b). Using Equation (10), and applying the estimated  $(\lambda, \delta)$  to each  $v_i$  in the data set, we can obtain a set of  $Y_i$ , then  $\{Y_i | i = 1, 2, ..., M\}$  is the corresponding Gaussianized data set.

# 4.2 Methodology

The starting point is the non-Gaussian PDF of peculiar velocity, Equation (6). We will present an algorithm for applying BC transformation to Equation (6). For convenience, we assume the  $P(\eta)$  term in Equation (6) is a Gaussian function, i.e. we assume the measured log-distance ratio of the *n*-th galaxy,  $\eta_n$ , has Gaussian error  $\epsilon_n$ , then the PDF of log-distance ratio of this galaxy is given by the Gaussian equation

$$P(\eta) = \frac{1}{\sqrt{2\pi\epsilon_n^2}} \exp\left(-\frac{(\eta - \eta_n)^2}{2\epsilon_n^2}\right) .$$
(12)

To clarify, this assumption is not necessary for the following presented algorithm, but we make this assumption to clearly and conveniently present our algorithm. Although  $P(\eta)$  is assumed to be a Gaussian function here, due to the non-linear relation between  $P(\eta)$  and P(V) in Equation (6), the resultant P(V) is not Gaussian. Therefore the Gaussian assumption of peculiar velocity in the past literature is not true and should be abandoned.

To clearly present the algorithm, we randomly choose one galaxy from the 2MTF catalog. The 2MASS ID of this galaxy is '2MASX09582105+3222119'. The log-distance ratio (and error), redshift and peculiar velocity (estimated using Eq. (2)) of this galaxy are listed in Table 1, and we also list its Galactic longitude  $\ell$  and latitude b in the table.

The BC transformation parameter  $\lambda$  is estimated from a set of samples utilizing Equation (11). For a galaxy, we first need to generate a set of samples which has PDF of Equation (6), then estimate  $\lambda$  for this galaxy using these samples. The details of the algorithm are presented

Table 1The Properties of the 2MTF Galaxy'2MASX09582105+3222119'

2MASX09582105+3222119				
$\eta$	0.133183			
$\epsilon$	0.108191			
cz	1748 km s <sup>-1</sup>			
V	460.15 km s <sup>-1</sup>			
l	194.22°			
b	52.32°			

as follows <sup>1</sup>: (i) Generate a spline function  $P_{\rm spl}(V)$  for Equation (6):

Generate a set of  $\eta \in [-1, 1]^2$  and calculate  $P(\eta)$ from Equation (12), where  $\eta_n = 0.133183$  and  $\epsilon_n = 0.108191$ . Compute the corresponding  $z_h$  and  $d_h$  using the  $\eta$  values and cz = 1748 km s<sup>-1</sup>. Then use Equation (2) and Equation (6) to calculate a set of velocities v and the corresponding P(v). Thus we obtain a set of interpolated points q = [v, P(v)] and the corresponding spline function  $P_{\rm spl}(V)$ , as represented by the dash-dotted green curve in Figure 3. The curve deviates significantly from a Gaussian, i.e. the peculiar velocity of this galaxy does not have Gaussian error.

# (ii) Generate a spline function for the inverse cumulative function corresponding to Equation (6):

Using the above interpolated points q and the spline function  $P_{\rm spl}(V)$ , one can numerically estimate the cumulative distribution function (CDF) with

$$CDF = \int_{\min(v)}^{v} P_{\rm spl}(v') dv' \,. \tag{13}$$

Then we can obtain a spline function  $V = f_{spl}(CDF)$ , which is the inverse function of CDF, as featured in Figure 4.

# (iii) Generate velocity samples which have a PDF of Equation (6):

Generate  $M = 150\,000$  uniformly distributed random points in the interval of [0,1] as the input to  $V = f_{spl}(CDF)$  to obtain velocity samples  $\{v_i | i =$  $1, 2, ..., M\}$ . In Figure 3, the blue bars depict the normalized histogram of these samples, which matches the  $P_{spl}(V)$  curve.

# (iv) BC transform the velocity samples:

Set  $\delta = 25r$ , where r corresponds to the width such that  $P_{\rm spl}(V) = 0.1 \times \max[P_{\rm spl}(V)]$ , as signified by the yellow arrow in Figure 3. (See Sect. 4.3 and Appendix B for more discussion about the choice of  $\delta$ .)

Choose a  $\lambda$  in  $(-\infty, +\infty)^3$ . Substitute  $\lambda$ ,  $\delta$  and  $v_i$  into Equation (10), then into Equation (11) to calculate  $L(\lambda)$ . Repeat this step to find the value of  $\lambda$  that maximizes  $L(\lambda)$ . As displayed in Figure 5,  $\lambda = 15.646$  is the best estimated BC transformation parameter for this galaxy.

# (v) Gaussianize the peculiar velocity:

Plugging the galaxy's peculiar velocity of V = 460.15 km s<sup>-1</sup> and  $\lambda = 15.646$  as well as the above  $\delta$  into Equation (10), we obtain  $Y = 7.3 \times 10^{66}$ . This is the BC transformed 'velocity' for this galaxy. Substitute  $\lambda = 15.646$ ,  $\delta$  and  $\{v_i | i = 1, 2, ..., M\}$  into Equation (10) to obtain a data set  $\{Y_i | i = 1, 2, ..., M\}$ . The standard deviation (std) of this data set,  $\sigma = 1.77 \times 10^{65}$ , is measurement error of Y. In fact, the mean value of  $\{Y_i | i = 1, 2, ..., M\}$ ,  $\langle Y_i \rangle = 7.3 \times 10^{66}$ , is equal to the Y transformed directly from V = 460.15 km s<sup>-1</sup>. As shown in Figure 5, the distribution of  $\{Y_i | i = 1, 2, ..., M\}$  (yellow bars) matches the Gaussian curve (red curve) very well.

Applying the above algorithm to each of the 2MTF galaxies, we finally obtain the  $(\lambda, \delta, Y, \sigma)$  for each galaxy. The data set of  $(Y, \sigma)$  is the Gaussianized 'velocities' which we can use to measure the power spectrum, two-point correlation and cosmic flows. When fitting the measurements to the model, we also need to apply the same  $(\lambda, \delta)$  to the modeled velocity value for each galaxy.

To reiterate, the above algorithm is independent of  $P(\eta)$ . Although  $P(\eta)$  is assumed to be Gaussian in Equation (12), this assumption is not necessary in the above algorithm.  $P(\eta)$ , in principle, can be chosen as any distribution. We even do not need to know the analytic expression of  $P(\eta)$ . As long as we can obtain a numeral function for  $P(\eta)$ , we can still calculate  $P_{\rm spl}(V)$  in step (i), and then Gaussianize the peculiar velocity applying the above algorithm. This demonstrates the flexibility of the above algorithm.

## **4.3** $\lambda$ as a Function of $\epsilon$

In Figure 7, we plot the BC transformation parameter  $\lambda$  against the measurement error of log-distance ratio  $\epsilon$  for all the 2MTF galaxies. The yellow line is the best fit to the dots, and the fit equation is given by

$$\lambda = k\epsilon + b . \tag{14}$$

For  $\delta = 25r$ , the fit result is

$$k = 135.1, b = 0.3247.$$
 (15)

The relation Equation (14) is also applicable for the 2MTF mocks, see Appendix A and Figure A.2 for more discussion. Does this relation also exist for other surveys? From the details of the algorithm presented in Section 4.2,

<sup>&</sup>lt;sup>1</sup> The code for the algorithm can be downloaded from https://github.com/FeiQin-cosmologist/GaussPv

 $<sup>^2\,</sup>$  From Figure 2 we know the interval  $\eta\in [-1,1]$  is large enough to cover all the 2MTF galaxies.

<sup>&</sup>lt;sup>3</sup> In practice, the estimated  $\lambda$  is all in [5, 30] for 2MTF.



**Fig. 3** The dash-dotted green curve is the spline function of Eq. (6) for '2MASX09582105+3222119'. The blue bars display the normalized histogram of the velocity samples  $\{v_i | i = 1, 2, ..., M\}$  generated from step (iii) according to Eq. (6). For comparison, the red curve indicates the position of Gaussian PDF, centered at the mean value of these velocities samples, with width calculated from the std of these velocities samples. The yellow arrow indicates the width where  $P_{spl}(V) = 0.1 \times \max[P_{spl}(V)]$ .



**Fig. 4** The black dots are the interpolated points generated in step (i). The blue curve traces the inverse spline function of CDF for '2MASX09582105+3222119'.

we find that the BC transformation parameter  $\lambda$  does not depend on any particular survey. In Appendix A, we find this relation also exists in the 6dFGSv survey. 6dFGSv (Springob et al. 2014) is a Fundamental Plane survey. Therefore, the linear relation Equation (14) is applicable for both Tully-Fisher and Fundamental Plane surveys (at least, for 2MTF and 6dFGSv). In future work, we also need to explore whether this relation is true for the peculiar velocity samples measured from Type Ia supernovae (as well as any other distance measurement techniques). If this relation widely exists, then one can compute  $\lambda$  directly from Equation (14), rather than performing the whole process incorporated in the algorithm. This will be very time saving for plenty



Fig. 5 The pink curve signifies the logarithmic likelihoods  $L(\lambda)$  for '2MASX09582105+3222119'. The yellow dashed vertical line indicates the position of  $\lambda$  which maximizes  $L(\lambda)$ .



**Fig. 6** The yellow bars display the normalized histogram of the BC transformed 'velocity' samples  $\{Y_i | i = 1, 2, ..., M\}$  from step (v). For comparison, the red curve indicates the position of Gaussian PDF, centered on the mean value of these samples, with width calculated from the std of these samples. The blue dot-dashed vertical line indicates the position of the mean of these samples. The mean is  $7.324 \times 10^{66}$ .

of mocks and the upcoming larger surveys, such as SkyMapper (Wolf et al. 2018), Dark Energy Spectroscopic Instrument (DESI) (DESI Collaboration et al. 2016), Large Synoptic Survey Telescope (LSST) (Ivezić et al. 2019), Widefield ASKAP L-band Legacy All-sky Blind surveY (WALLABY) (Koribalski 2012; Koribalski et al. 2020) and Taipan Galaxy Survey (da Cunha et al. 2017).

The choice of  $\delta$  can change the fit parameter k, but will not change the linear relation or b. Choosing smaller  $\delta$  will result in smaller k, but may not keep all the samples positive in Equation (10). Too large  $\delta$  will result in overflow in the memory of the computer. See Appendix B for more discussion.



**Fig.7**  $\lambda$  as a function of  $\epsilon$  for all 2MTF galaxies. The yellow line is the best fit to the blue data points.

#### **5 BULK FLOW ESTIMATION TECHNIQUES**

In this paper, we take the bulk flow measurement as an example to demonstrate the implementation of the Gaussianized peculiar velocities. We will introduce a new bulk flow estimator that estimates bulk flow from the Gaussianized peculiar velocities. We also rely on mocks to test the technique and compare it to the techniques used in other literature. In future work, we intend to explore new techniques for the power spectrum and twopoint correlation measurements as well as the density field reconstruction utilizing Gaussianized peculiar velocities.

# 5.1 Bulk Flow Estimation Techniques in Previous Work

Galaxies' peculiar motions form the cosmic flow field in the nearby Universe. The bulk flow velocity is the dipole component of the cosmic flow field (Staveley-Smith & Davies 1989; Jaffe & Kaiser 1995; Parnovsky et al. 2001; Feldman et al. 2010; Qin et al. 2019a). Measuring the bulk flow velocity and comparing to the cosmological model prediction enables us to test whether the model accurately describes the motion of galaxies in the nearby Universe. In the past literature, bulk flow is usually measured in velocity space and log-distance ratio space.

Firstly, we consider measuring the bulk flow in velocity space (v-space). The two main v-space measurement techniques are maximum likelihood estimation (MLE, Kaiser 1988) and minimum variance (MV) estimation (Watkins et al. 2009; Feldman et al. 2010). In this paper we only focus on the MLE technique. In Kaiser (1988), under the assumption that peculiar velocities have Gaussian errors, the likelihood of N line-of-sight peculiar velocities can be written as

$$S(\mathbf{B}, \alpha_{\star}) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi \left(\alpha_{n}^{2} + \alpha_{\star}^{2}\right)}} \exp\left(-\frac{1}{2} \frac{(V_{n} - \mathbf{B} \cdot \hat{\mathbf{r}}_{n})^{2}}{\alpha_{n}^{2} + \alpha_{\star}^{2}}\right),$$
(16)

where the vector  $\mathbf{B} = [B_x, B_y, B_z]$  is the bulk flow velocity to be estimated,  $\hat{\mathbf{r}}_n$  is the unit vector pointing to the *n*-th galaxy and  $\alpha_n$  is the measurement error of  $V_n$ . Finally,  $\sigma_{\star}$  is introduced to account for the intrinsic scatter of the peculiar velocities, and is usually assumed to be 300 km s<sup>-1</sup> (Sarkar et al. 2007; Scrimgeour et al. 2016) (thoughout this paper we vary it as a free parameter). In this technique, the peculiar velocities estimated from Equation (7) can be the input to Equation (16). For convenience, we call this technique *w*MLE.

Secondly, to avoid the non-Gaussianity of the estimated peculiar velocities, the measurement of bulk flow can be performed in the log-distance ratio-space, or  $\eta$ -space. Nusser & Davis (1995, 2011); Qin et al. (2018, 2019a) relied on the so-called  $\eta$ MLE to measure the bulk flow. Boruah et al. (2020) measured the bulk flow in distance modulus space. Since  $\eta$  is simply linearly converted from the distance modulus, their method and any other similar method also be classified as  $\eta$ MLE. In this paper we employ the  $\eta$ MLE of Qin et al. (2018), and the algorithm is clearly presented in section 4.2 of that previous work.

#### 5.2 Bulk Flow Estimator

In this section, we will introduce the bulk flow estimator that estimates bulk flow from the BC-transformed peculiar velocities.

After we obtain the  $(\lambda, \delta, Y, \sigma)$  for each galaxy, we can write the likelihood of N galaxies, each with  $(Y_n, \sigma_n)$ , as

$$L(\mathbf{B}, \sigma_{\star}) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi \left(\sigma_{n}^{2} + \sigma_{\star}^{2}\right)}} \exp\left(-\frac{1}{2} \frac{(Y_{n} - F_{n}(\mathbf{B}))^{2}}{\sigma_{n}^{2} + \sigma_{\star}^{2}}\right),$$
(17)

where

$$F_n(\mathbf{B}) = \begin{cases} \frac{(\mathbf{B} \cdot \hat{\mathbf{r}}_n + \delta_n)^{\lambda_n} - 1}{\lambda_n}, & \lambda_n \neq 0\\ \ln(\mathbf{B} \cdot \hat{\mathbf{r}}_n + \delta_n), & \lambda_n = 0 \end{cases}$$
(18)

the vector **B** is the bulk flow velocity to be estimated and  $\hat{\mathbf{r}}_n$  is the unit vector pointing to the *n*-th galaxy.  $\sigma_{\star}$ is introduced to account for the intrinsic scatter in the velocities. In this paper, we set it as a free parameter.  $F_n(\mathbf{B})$  is the BC transformation of the model peculiar velocity for the *n*-th galaxy, transformed using  $(\lambda_n, \delta_n)$  of that galaxy.

The maximum likelihood **B** cannot be obtained analytically due to the non-linear relationship between the

model **B** and  $F_n(\mathbf{B})$ . Instead, combining uniform priors on the  $\sigma_{\star}$  and **B** with the likelihood in Equation (17) to obtain the posterior probability of these four independent parameters, we can estimate the bulk flow employing the Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm. The flat priors of the four parameters are in the interval  $B_i \in [-1200, +1200]$  km s<sup>-1</sup> and  $\sigma_{\star} \in [-1000, +1000] h$  km s<sup>-1</sup> Mpc<sup>-1</sup>.

The measurement error of the bulk flow component,  $e_{B_i}$  (i = x, y, z) is the std of the MCMC samples of the corresponding MCMC chain<sup>4</sup>. The measurement error of the bulk flow amplitude,  $e_B$ , is calculated utilizing (Scrimgeour et al. 2016; Qin et al. 2018)

$$e_B^2 = JC_{ij}J^T$$
,  $(i = 1, 2, 3)$ , (19)

where J is the Jacobian of the bulk flow and  $\partial B/\partial B_i$ .  $C_{ij}$  is the covariance of the bulk flow components calculated using the MCMC samples.

In the following section, we will test and compare the above bulk flow estimation technique to wMLE and  $\eta$ MLE using mock 2MTF surveys.

## 5.3 Testing Using Mocks

In order to compare and test how well the bulk flow estimators are expected to recover the true bulk flow from the 2MTF survey, we applied the three estimators to 16 mock 2MTF catalogs.

The 'true' bulk flow velocity,  $\mathbf{B}_t$ , within each mock is defined by averaging over the true galaxy velocities  $\mathbf{v}_t$ along orthogonal axes (Qin et al. 2018, 2019a)

$$B_{t,i} = \frac{1}{N} \sum_{n=1}^{N} v_{t,in} , \quad (i = x, y, z) , \qquad (20)$$

where  $\mathbf{v}_t$  is known from the simulations. Only the mock 2MTF galaxies in the simulation are considered to compute  $\mathbf{B}_t$ .

Figure 8 depicts the measured bulk flow against the true bulk flow in equatorial coordinates. All three bulk flow estimators can recover the true bulk flow. The top panel is the  $\eta$ MLE measurement. The middle panel is the *w*MLE measurement. The bottom panel is the measurement implementing the technique of Section 5.2. The scatters of the points in the three panels are most likely due to the intrinsic scatter  $\sigma_{\star}$  (or  $\alpha_{\star}$  of *w*MLE) of the true velocities in the mocks (Qin et al. 2018, 2019a).  $\sigma_{\star}$  (or  $\alpha_{\star}$ ) accounts for the non-linear peculiar motions of galaxies. In the future, a more accurate peculiar velocity estimator, which can more accurately predict and model non-linear motion, needs to be developed. Such new estimators will result in more non-Gaussianity than Equation (2), indicating the importance and usefulness of our method for Gaussianizing the estimated peculiar velocities.

If only having a look at the scatters of the symbols in the three panels (the difference between the measured values and the true values), one can find that the scatters are almost the same for the three estimators. However, the technique of Section 5.2 gives smaller measurement errors. We plot the measurement errors of the bulk flows generated by this technique against those of  $\eta$ MLE and wMLE, as depicted in Figure 9, and we find this technique gives smaller measurement errors compared to both  $\eta$ MLE and wMLE.

Using the Gaussianized peculiar velocity fields to measure the bulk flow can reduce the measurement errors. In the future, we will also explore whether relying on the Gaussianized peculiar velocities can reduce the measurement errors of the power spectrum and two-point correlation, and then reduce the measurement errors of the cosmological parameters.

The value of  $\delta$  will not change the bulk flow measurements. See Appendix B for more discussion.

#### 6 BULK FLOW RESULT AND DISCUSSION

#### 6.1 Results

Figure 10 features the bulk flow velocity measurement in Galactic coordinates using the Gaussianized 2MTF peculiar velocities along with the bulk flow estimator in Section 5.2. The vertical dashed line in each panel indicates the best estimated bulk flow velocity component  $B_i$  (i = x, y, z). The histograms show the distribution of MCMC samples for each  $B_i$  and the shaded regions are the  $1\sigma$  measurement errors of  $B_i$ .

For comparison, in Table 2, we list the measured bulk flow velocity and its direction using the three estimators. Our new estimator gives the smallest error compared to  $\eta$ MLE and wMLE.

#### 6.2 Comparison with ACDM Theory

In this section, we compare the estimated bulk flow amplitude,  $|\mathbf{B}|$ , to the predictions from  $\Lambda$ CDM. At redshift zero, assuming the  $\Lambda$ CDM model, the growth rate  $f = \Omega_m^{0.55}$  (Linder & Cahn 2007). The variance of the bulk flow velocity is (Gorski 1988; Li et al. 2012; Hong et al. 2014; Andersen et al. 2016; Qin et al. 2018, 2019a)

$$\sigma_B^2 = \frac{H_0^2 f^2}{2\pi^2} \int \mathcal{W}^2(k) \mathcal{P}(k) dk , \qquad (21)$$

<sup>&</sup>lt;sup>4</sup> We use the PYTHON package emcee (Foreman-Mackey et al. 2013) to perform the MCMC. For each of the four parameters, we use 24 walkers, and for each walker, we generated 100 000 MCMC samples. Therefore, there are 2 400 000 samples in each of the four MCMC chains. This is smooth enough to estimate the measurement errors.

	B	$B_x$	$B_{y}$	$B_z$	l	b	Depth
	$km s^{-1}$	$\rm km~s^{-1}$	km s <sup>-1</sup>	$\rm km~s^{-1}$	deg	deg	$Mpch^{-1}$
This paper	$332.41 \pm 27.45$	$126.77 \pm 30.18$	$-298.66 \pm 27.68$	$72.31 \pm 20.27$	$292.99 \pm 5.30$	$12.56 \pm 3.54$	30
$\eta$ MLE	$333.99 \pm 30.31$	$116.66 \pm 33.81$	$-304.29 \pm 30.84$	$73.16 \pm 23.33$	$290.98 \pm 5.90$	$12.65\pm3.98$	30
wMLE	$338.91 \pm 31.18$	$107.67 \pm 34.45$	$-313.15 \pm 32.23$	$72.15 \pm 24.47$	$288.97 \pm 6.01$	$12.29 \pm 4.11$	30

Table 2 Comparing the Bulk Flow Velocities of 2MTF Survey Measured from the Three Estimators



**Fig. 8** Comparing the measured bulk flow for the 16 2MTF mocks in equatorial coordinates. The *top* and *middle* panels are for the  $\eta$ MLE and *w*MLE, respectively. The *bottom panel* is measured from the Gaussianized velocities along with the estimator in Sect. 5.2.

where  $\mathcal{P}(k)$  is the linear matter density power spectrum generated with the CAMB}package (Lewis et al. 2000; Howlett et al. 2012) and  $\mathcal{W}(k)$  is the Fourier transform of the survey window function. The computation of the



Fig. 9 Comparing the measurement errors of bulk flows measured from the Gaussianized peculiar velocities (of 16 2MTF mocks in equatorial coordinates) to those of  $\eta$ MLE (*top panel*) and *w*MLE (*bottom panel*).

accurate W(k) of 2MTF is clearly presented in section 6.2 of Qin et al. (2018).

The PDF of the bulk flow amplitude is given by (Li et al. 2012; Hong et al. 2014; Scrimgeour et al. 2016; Qin et al. 2018, 2019a; Boruah et al. 2020)

$$p(|\mathbf{B}|) = \sqrt{\frac{2}{\pi}} \left(\frac{3}{\sigma_B^2}\right)^{1.5} |\mathbf{B}|^2 \exp\left(-\frac{3|\mathbf{B}|^2}{2\sigma_B^2}\right) , \quad (22)$$

where the most likely  $|\mathbf{B}|$  is expressed as  $B_p = \sqrt{2/3}\sigma_B$ , and the cosmic variance of  $|\mathbf{B}|$  is given by  $B_{p \ -0.356\sigma_B}^{+0.419\sigma_B}$  $(1\sigma, \text{Scrimgeour et al. (2016)}; \text{Qin et al. (2018)})$ . The upper and lower limits mean that the integral of Equation (22) in the interval  $[B_p - 0.356\sigma_B, B_p + 0.419\sigma_B]$  is 0.68.

The  $\Lambda$ CDM model predicted bulk flow amplitude for the 2MTF is quoted from Qin et al. (2018) and is listed in Table 3. The measurement is consistent with the  $\Lambda$ CDM prediction.

# 7 CONCLUSIONS

We developed an algorithm that can Gaussianize the lineof-sight peculiar velocities estimated from Equation (2). We also find that the BC transformation parameter  $\lambda$ 



**Fig. 10** The bulk flow velocity measurements of the 2MTF survey. The marginalized histograms display the distribution of the MCMC samples of  $B_x$ ,  $B_y$  and  $B_z$ . The shaded region in the histograms is  $1\sigma$ . The filled 2D contours indicate the 1, 1.5, 2 and  $2.5\sigma$  regions. The vertical dashed line indicates the best estimated value of  $B_x$ ,  $B_y$  and  $B_z$ .

**Table 3** Comparing the measured bulk flow to the  $\Lambda$ CDM prediction. Errors on the  $\Lambda$ CDM prediction denote the cosmic variance.

Data set	$\eta$ MLE	ΛCDM
	$\rm km~s^{-1}$	$\rm km~s^{-1}$
2MTF	$332\pm27$	$315^{+161}_{-137}$

is a linear function of the measurement error of logdistance ratio  $\epsilon$ . This relation exists for the Tully-Fisher survey, 2MTF and the Fundamental Plane survey, 6dFGSv. However, more works need to be done in the future to further examine this relation by utilizing different surveys.

We developed a bulk flow estimation technique to measure the bulk flow from the Gaussianized peculiar velocities. We also test the estimator using 2MTF mocks, and find that measuring bulk flow from the Gaussianized peculiar velocities can reduce the measurement errors compared to wMLE and  $\eta$ MLE. In the future, we will also develop new techniques for the power spectrum and twopoint correlation measurements using the Gaussianized peculiar velocities.

We have measured the bulk flow velocity by relying on the Gaussianized 2MTF survey. The estimated bulk flow is  $332 \pm 27 \text{ km s}^{-1}$  at a depth of  $30 h^{-1}$  Mpc, and the result is consistent with the  $\Lambda$ CDM prediction.



**Fig. A.1** Same as Fig. 7, but for 16 2MTF mocks. The blue line is the best fit to the data points. The different colors of the points indicate the points from 16 different mocks.

Acknowledgements We would like to thank David Parkinson for multiple discussions. FQ is supported by the project "Understanding Dark Universe Using Large Scale Structure of the Universe", funded by the Ministry of Science. This research has made use of the emcee package (Foreman-Mackey et al. 2013), ChainConsumer package (Hinton 2016), SCIPY package (Virtanen et al. 2020) and MATPLOTLIB package (Hunter 2007).

# Appendix A: $\lambda$ AS A FUNCTION OF $\epsilon$ FOR 2MTF MOCKS AND 6dFGSv SURVEY

Figure A.1 shows the relation between  $\lambda$  and  $\epsilon$  for 16 2MTF mocks. The fit is produced by putting all the 16 mocks together. The result is:

$$a = 129.5, b = 0.8089.$$
 (A.1)

Figure A.2 shows the relation between  $\lambda$  and  $\epsilon$  for the 6dFGSv survey (Springob et al. 2014). 6dFGSv is the peculiar velocity survey from the Six-degree-Field Galaxy Survey (6dFGS, Jones et al. 2009, 2004). The survey is only in the southern sky, with Galactic latitude  $|b| > 10^{\circ}$ out to  $cz \approx 16500$  km s<sup>-1</sup>. The log-distance ratio of the 6dFGSv sample is measured using the fundamental plane (Magoulas et al. 2012). Relying on Equation (15), the fit parameters are

$$a = 104.9, \ b = 3.151.$$
 (A.2)

This indicates that the linear relation Equation (15) also exists for a Fundamental Plane survey-6dFGSv.

#### Appendix B: THE EFFECTS OF $\delta$

We choose  $\delta = 20r$  and  $\delta = 15r$  to estimate the bulk flow for the 2MTF mocks. The top panel of Figure B.1 displays the bulk flow measurements for 16 2MTF mocks with  $\delta = 20r$  (blue dots) and  $\delta = 15r$  (red stars) against



**Fig. A.2** Same as Fig. 7, but for 6dFGSv survey.



Fig. B.1 The top panel displays the bulk flow measurements for 16 2MTF mocks with  $\delta = 20r$  (*blue dots*) and  $\delta = 15r$  (*red stars*) against the measurements with  $\delta = 25r$ . The bottom panel is for the measurement errors.

the measurements with  $\delta = 25r$ . The black dashed line is the identity line. Choosing different values of  $\delta$  will not change the bulk flow measurements. The bottom panel is for the measurement errors. Choosing different values of  $\delta$ will also not change the measurement errors.

We also choose different values of  $\delta$  to estimate the parameter  $\lambda$  for the real 2MTF survey. As shown in Figure B.2, the blue, red, yellow, green and pink points are for  $\delta = 25r$ ,  $\delta = 20r$ ,  $\delta = 15r$ ,  $\delta = 10r$  and  $\delta = 5r$ , respectively. The black lines are the best fits to the blue, red, yellow, green and pink points, from top to bottom respectively. As  $\delta$  increases, the slope is increasing, and while the intercept will not change too much, the fit value of (k, b) is presented in Table B.1.



**Fig. B.2** Same as Fig. 7, but for different  $\delta$ . The blue, red, yellow, green and pink points are for  $\delta = 25r$ ,  $\delta = 20r$ ,  $\delta = 15r$ ,  $\delta = 10r$  and  $\delta = 5r$ , respectively. The black lines are the best fit to the blue, red, yellow, green and pink points, from top to bottom respectively.

**Table B.1** Comparing (k, b) with Different Values of  $\delta$ 

$\delta$	k	b	
	$\rm km~s^{-1}$	$\mathrm{km}~\mathrm{s}^{-1}$	
25r	135.1	0.3247	
20r	108.8	0.366	
15r	82.93	0.3939	
10r	55.33	0.5435	
5r	28.70	0.6186	

#### References

- Andersen, P., Davis, T. M., & Howlett, C. 2016, MNRAS, 463, 4083
- Boruah, S. S., Hudson, M. J., & Lavaux, G. 2020, MNRAS, 498, 2703
- Box, G. E. P., & Cox, D. R. 1964, Journal of the Royal Statistical Society., 26, 211
- Carrick, J., Turnbull, S. J., Lavaux, G., & Hudson, M. J. 2015, MNRAS, 450, 317
- Colless, M., Saglia, R. P., Burstein, D., et al. 2001, MNRAS, 321, 277
- da Cunha, E., Hopkins, A. M., Colless, M., et al. 2017, PASA, 34, e047
- Davis, T. M., & Scrimgeour, M. I. 2014, MNRAS, 442, 1117
- DESI Collaboration, Aghamousa, A., Aguilar, J., et al. 2016, arXiv e-prints, arXiv:1611.00036
- Dupuy, A., Courtois, H. M., & Kubik, B. 2019, MNRAS, 486, 440
- Elahi, P. J., Welker, C., Power, C., et al. 2018, MNRAS, 475, 5338
- Erdoğdu, P., Lahav, O., Huchra, J. P., et al. 2006, MNRAS, 373, 45
- Feldman, H. A., Watkins, R., & Hudson, M. J. 2010, MNRAS, 407, 2328
- Feldman, H., Juszkiewicz, R., Ferreira, P., et al. 2003, ApJL, 596, L131

- Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306
- Gordon, C., Land, K., & Slosar, A. 2007, Phys. Rev. Lett., 99, 081301
- Gorski, K. 1988, ApJL, 332, L7
- Gorski, K. M., Davis, M., Strauss, M. A., White, S. D. M., & Yahil, A. 1989, ApJ, 344, 1
- Hinton, S. R. 2016, The Journal of Open Source Software, 1, 00045
- Hong, T., Springob, C. M., Staveley-Smith, L., et al. 2014, MNRAS, 445, 402
- Hong, T., Staveley-Smith, L., Masters, K. L., et al. 2019, MNRAS, 487, 2061
- Howlett, C. 2019, MNRAS, 487, 5209
- Howlett, C., Lewis, A., Hall, A., & Challinor, A. 2012, J. Cosmol. Astropart. Phys., 2012, 027
- Howlett, C., Staveley-Smith, L., Elahi, P. J., et al. 2017, MNRAS, 471, 3135
- Huchra, J. P., Macri, L. M., Masters, K. L., et al. 2012, ApJS, 199, 26
- Hui, L., & Greene, P. B. 2006, Phys. Rev. D, 73, 123526
- Hunter, J. D. 2007, Computing in Science and Engineering, 9, 90
- Huterer, D., Shafer, D. L., Scolnic, D. M., & Schmidt, F. 2017, J. Cosmol. Astropart. Phys., 2017, 015
- Ivezić, Ž., Kahn, S. M., Tyson, J. A., et al. 2019, ApJ, 873, 111
- Jaffe, A. H., & Kaiser, N. 1995, ApJ, 455, 26
- Johnson, A., Blake, C., Koda, J., et al. 2014, MNRAS, 444, 3926
- Jones, D. H., Saunders, W., Colless, M., et al. 2004, MNRAS, 355, 747
- Jones, D. H., Read, M. A., Saunders, W., et al. 2009, MNRAS, 399, 683
- Juszkiewicz, R., Ferreira, P. G., Feldman, H. A., Jaffe, A. H., & Davis, M. 2000, Science, 287, 109
- Kaiser, N. 1988, MNRAS, 231, 149
- Kolatt, T., & Dekel, A. 1997, ApJ, 479, 592
- Koribalski, B. S. 2012, PASA, 29, 359
- Koribalski, B. S., Staveley-Smith, L., Westmeier, T., et al. 2020, Ap&SS, 365, 118
- Lavaux, G., Tully, R. B., Mohayaee, R., & Colombi, S. 2010, ApJ, 709, 483
- Lewis, A., Challinor, A., & Lasenby, A. 2000, ApJ, 538, 473
- Li, M., Pan, J., Gao, L., et al. 2012, ApJ, 761, 151
- Linder, E. V., & Cahn, R. N. 2007, Astroparticle Physics, 28, 481
- Ma, Y.-Z., Branchini, E., & Scott, D. 2012, MNRAS, 425, 2880
- Ma, Y.-Z., & Pan, J. 2014, MNRAS, 437, 1996
- Ma, Y.-Z., & Scott, D. 2013, MNRAS, 428, 2017

Magoulas, C., Springob, C. M., Colless, M., et al. 2012,

MNRAS, 427, 245

- Masters, K. L., Springob, C. M., & Huchra, J. P. 2008, AJ, 135, 1738
- Nusser, A., & Davis, M. 1994, ApJL, 421, L1
- Nusser, A., & Davis, M. 1995, MNRAS, 276, 1391
- Nusser, A., & Davis, M. 2011, ApJ, 736, 93
- Parnovsky, S. L., Kudrya, Y. N., Karachentseva, V. E., & Karachentsev, I. D. 2001, Astronomy Letters, 27, 765
- Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, A&A, 594, A13
- Pomarède, D., Hoffman, Y., Courtois, H. M., & Tully, R. B. 2017, ApJ, 845, 55
- Poole, G. B., Blake, C., Marín, F. A., et al. 2015, MNRAS, 449, 1454
- Qin, F., Howlett, C., & Staveley-Smith, L. 2019b, MNRAS, 487, 5235
- Qin, F., Howlett, C., Staveley-Smith, L., & Hong, T. 2018, MNRAS, 477, 5150
- Qin, F., Howlett, C., Staveley-Smith, L., & Hong, T. 2019a, MNRAS, 482, 1920
- Sakia, R. M. 1992, Journal of the Royal Statistical Society, 41, 169
- Sarkar, D., Feldman, H. A., & Watkins, R. 2007, MNRAS, 375, 691
- Scrimgeour, M. I., Davis, T. M., Blake, C., et al. 2016, MNRAS, 455, 386
- Silberman, L., Dekel, A., Eldar, A., & Zehavi, I. 2001, ApJ, 557, 102
- Springob, C. M., Magoulas, C., Colless, M., et al. 2014, MNRAS, 445, 2677
- Springob, C. M., Hong, T., Staveley-Smith, L., et al. 2016, MNRAS, 456, 1886
- Staveley-Smith, L., & Davies, R. D. 1989, MNRAS, 241, 787
- Turnbull, S. J., Hudson, M. J., Feldman, H. A., et al. 2012, MNRAS, 420, 447
- Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, Nature Methods, 17, 261
- Wang, M. S., Percival, W. J., Avila, S., Crittenden, R., & Bianchi, D. 2019, MNRAS, 486, 951
- Watkins, R., & Feldman, H. A. 2015, MNRAS, 450, 1868
- Watkins, R., Feldman, H. A., & Hudson, M. J. 2009, MNRAS, 392, 743
- Wolf, C., Onken, C. A., Luvaul, L. C., et al. 2018, PASA, 35, e010
- Zaroubi, S., Zehavi, I., Dekel, A., Hoffman, Y., & Kolatt, T. 1997, ApJ, 486, 21