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Investigating the relationship between cosmic curvature and dark energy models with the latest supernova sample

Chao Gao (高超)[1], Yun Chen (陈云)[2] and Jie Zheng (郑捷)[1]

[1] Department of Astronomy, Beijing Normal University, Beijing 100089, China; *gaoch@mail.bnu.edu.cn*
[2] Key Laboratory for Computational Astrophysics, National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100101, China; *chenyun@nao.cas.cn*

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Abstract We investigate the relationship between cosmic curvature and model of dark energy (hereafter DE) with recent Type Ia supernovae (hereafter SNe Ia) data, i.e., the Pantheon sample including 1048 SNe Ia with $0.01 < z < 2.3$. We obtain measurements of the dimensionless spatial curvature density today, i.e., $\Omega_{k0} = -0.062^{+0.189}_{-0.169}$, $-0.004^{+0.228}_{-0.134}$, $0.127^{+0.280}_{-0.276}$ and $0.422^{+0.213}_{-0.338}$ at 68% confidence level (CL), respectively, in the scenarios of ΛCDM, ϕCDM (i.e., scalar field DE), ωCDM and $\omega_0\omega_a$CDM models. In the scenario of ΛCDM model, a closed universe is preferred by the Pantheon sample, which is consistent with that from the Planck CMB spectra. However, the uncertainty of Ω_{k0} from the Pantheon SNe sample is about 8 times larger than that from the Planck data, so the former one supports a closed universe at a much lower CL than that from the latter one. An open unverse is supported by the Pantheon sample at $\sim$32% and $\sim$78% CLs, respectively, in the ωCDM and $\omega_0\omega_a$CDM models. Among these models, the ϕCDM model is the one which supports the flat universe most strongly. It shows that Ω_{k0} is significantly dependent on the adopted model of DE, and there is a negative correlation between Ω_{k0} and the equation of state of DE.

Key words: cosmological parameters—dark energy—cosmology: observations

1 INTRODUCTION

As a kind of "standard candles" in the universe, Type Ia supernovae (SNe Ia) supplied the first straightforward evidence for an accelerating universe and for the existence of unknown "dark energy" (DE) driving this acceleration in 1998. At that time, the sample size was not big, i.e., 50 SNe Ia from Riess et al. (1998), and 42 ones from Perlmutter et al. (1999). The population of SNe Ia discovered has been growing rapidly over the last two decades. The popular samples include the "gold" 2004 (157 data; Riess et al. 2004) and "gold" 2007 (182 data; Riess et al. 2007) samples, the Supernova Legacy Survey (SNLS) 1-year (115 data; Astier et al. 2006) and 3-year (252 data; Guy et al. 2010) samples, the "Equation of State: SupErNovae trace Cosmic Expansion" (ESSENCE) supernova survey sample (60 data; Miknaitis et al. 2007; Wood-Vasey et al. 2007; Davis et al. 2007), the Supernova Cosmology Project (SCP) Union (307 data; Kowalski et al. 2008), Union2 (557 data; Amanullah et al. 2010) and Union2.1 (580 data; Suzuki et al. 2012) compilations, the Constitution set (397 data; Hicken et al. 2009), the "Joint Light-curve Analysis" (JLA) compilation (740 data; Betoule et al. 2014) and the latest "Pantheon" sample (1048 data; Scolnic et al. 2018). Besides the dramatic increase in the population of SNe Ia, the techniques for measuring light curve parameters are also continually being improved to reduce systematic uncertainties (Riess et al. 1996; Perlmutter et al. 1997; Tonry et al. 2003; Wang et al. 2003, 2006; Guy et al. 2005, 2007; Conley et al. 2008). At present, the most popular techniques mainly include the SALT/SALT2 (Guy et al. 2005, 2007) and SiFTO (Conley et al. 2008) models which fit the light curves of supernovae by employing a spectral template.

The Cosmic Microwave Background (CMB) as one of the standard cosmological probes has revealed strong evidence (i.e. at more than 99% confidence level, CL) for a closed universe in the non-flat ΛCDM model, by using the near-term Planck CMB spectra (Planck Collaboration et al. 2018, 2019; Di Valentino et al. 2020). The observational constraints on cosmic curvature are widely studied with different probes (Gong et al. 2008; Liao et al. 2017; Wang et al. 2017; Denissenya et al. 2018; Cao et al. 2019; Liao 2019;

Qi et al. 2019; Wei & Melia 2020; Zhou & Li 2020). In this work, we intend to explore what type of cosmic curvature another standard cosmological probe, i.e., SNe Ia, may support. In our analysis, the SNe Ia dataset adopted is the Pantheon sample including 1048 data with $0.01 < z < 2.3$ (Scolnic et al. 2018). We also focus on investigating the relationship between cosmic curvature and the DE model. In practice, four cosmological models with different kinds of equation of state (EoS) for DE are taken into account. They are the ΛCDM model with the cosmological constant owning an EoS $\omega = -1$ (Peebles 1984), the ϕCDM model with the scalar field DE implementing a time-varying EoS $-1 < \omega < 0$ (Peebles & Ratra 1988), the ωCDM model with the phenomenological DE featuring an EoS $\omega =$ Constant (Ratra 1991), and the $\omega_0\omega_a$CDM model with the dynamical DE having a parameterized EoS $\omega(z) = \omega_0 + \omega_a \frac{z}{1+z}$ proposed in Chevallier & Polarski (2001) and Linder (2003).

The paper is organized as follows: in Section 2, we present the cosmological models under consideration, and demonstrate the methodology of using the SNe Ia data to put constraints on the model parameters. In Section 3, we carry out observational constraints on the effective energy density of the cosmic curvature Ω_{k0}, and other parameters in the considered cosmological models, and then mainly analyze the relationships between Ω_{k0} and the EoS of DE. The main conclusions and discussions are summarized in the last section.

2 METHODOLOGY AND DARK ENERGY MODELS

To put constraints on the cosmological parameters with the SNe Ia sample, one first needs to have the Friedmann equations for the cosmological models under consideration. According to the scope of this paper, cosmic curvature, parameterized through the effective energy density parameter Ω_{k0}, is taken to be a free parameter, rather than zero.

Among the various types of cosmological models, the most economical one may be the ΛCDM model (Peebles 1984), in which the accelerating expansion of the universe is powered by the DE component modeled as Einstein's cosmological constant, Λ, with an EoS parameter $\omega = p_\Lambda/\rho_\Lambda = -1$, where p_Λ and ρ_Λ are the fluid pressure and energy density respectively. The Friedmann equation of the ΛCDM model is

$$E^2(z;\mathbf{p}) = \Omega_{m0}(1+z)^3 + \Omega_\Lambda + \Omega_{k0}(1+z)^2, \quad (1)$$

where $E(z) = H(z)/H_0$ is the reduced Hubble parameter defined with the Hubble parameter $H(z)$ and the Hubble constant $H_0 = H(z=0)$. The model parameters are $\mathbf{p} = (\Omega_{m0}, \Omega_{k0})$, where Ω_{m0} is the matter density parameter, Ω_{k0} is the effective energy density parameter of the curvature and $\Omega_\Lambda = 1 - \Omega_{m0} - \Omega_{k0}$ is the energy density parameter of Λ. In this paper, we utilize the subscript 0 to denote the present-day value of a quantity.

In the ϕCDM model, DE is treated as the scalar field ϕ with a potential-energy density $V(\phi)$ decreasing gradually in ϕ, in which the DE density decreases slowly in time. For the scalar field DE, several kinds of $V(\phi)$ can satisfy the requirement of the late-time accelerating expansion of the universe (Samushia 2009). We consider the scalar field DE with a potential-energy density $V(\phi) = \frac{1}{2}\kappa m_p^2 \phi^{-\alpha}$, where $m_p = 1/\sqrt{G}$ is the Planck mass and G is the Newtonian constant of gravitation, and α and κ are constants which should be greater than or equal to zero (Ratra & Peebles 1988). The ϕCDM model under consideration has been extensively studied (Samushia et al. 2010; Chen & Ratra 2011, 2012; Mania & Ratra 2012; Chen & Xu 2016; Chen et al. 2015, 2016, 2017; Farooq et al. 2017; Ryan et al. 2019). It can reduce to the ΛCDM model in the case of taking $\alpha = 0$. The Friedmann equation of this model is

$$H^2(z;\mathbf{p}) = \frac{8\pi}{3m_p^2}(\rho_m + \rho_\phi) - \frac{k}{a^2}, \quad (2)$$

where the Hubble parameter is defined as $H(z) = \dot{a}/a$, $a(t)$ is the cosmic scale factor and $\dot{a} = da/dt$. The DE energy density is

$$\rho_\phi = \frac{m_p^2}{16\pi}(\frac{1}{2}\dot{\phi}^2 + V(\phi)). \quad (3)$$

The EoS is

$$\omega = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \quad (4)$$

One can figure out that this EoS satisfies $-1 < \omega < 1$. The motion equation for ϕ can be expressed as

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0. \quad (5)$$

The Hubble parameter $H(z)$ can be computed numerically with Equations (2) and (5), as well as the initial conditions described in Peebles & Ratra (1988). According to the usual convention, the effective energy density of the spatial curvature k is defined as $\Omega_k(a) \equiv -k/(a^2H(z)^2)$, so its present-day value is $\Omega_{k0} = \Omega_k(z=0) = -k/(a_0^2H_0^2)$. In the ϕCDM model, the model parameters are $\mathbf{p} = (\Omega_{m0}, \Omega_{k0}, \alpha)$.

In the ωCDM model, the EoS of DE is regarded as $\omega =$ Constant. It reduces to the ΛCDM model in the case of taking $\omega = -1$. One can obtain the Friedmann equation

$$\begin{aligned} E^2(z;\mathbf{p}) =& \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0} - \Omega_{k0}) \\ & (1+z)^{3(1+\omega)} + \Omega_{k0}(1+z)^2, \end{aligned} \quad (6)$$

where the model parameters are $\mathbf{p} = (\Omega_{m0}, \Omega_{k0}, \omega)$.

The $\omega_0\omega_a$CDM model can be deemed as an extension of the ΛCDM and ωCDM models, in which the DE is modeled as a dynamical component with the EoS parameterized as $\omega = \omega_0 + \omega_a z/(1+z)$. It reduces to the ΛCDM model in the case of taking $\omega_0 = -1$ and $\omega_a = 0$, and to the ωCDM model in the case of taking $\omega_0 =$ Constant and $\omega_a = 0$. Obviously, ω_a is a key parameter to denote the dynamic level of the DE. The Friedmann equation of the $\omega_0\omega_a$CDM model satisfies

$$E^2(z;\mathbf{p}) = \Omega_{m0}(1+z)^3 + \Omega_{de0}(1+z)^{3(1+w_0+w_a)} e^{-\frac{3w_a z}{1+z}} + \Omega_{k0}(1+z)^2, \quad (7)$$

where the present-day value of the DE density is $\Omega_{de0} = 1 - \Omega_{m0} - \Omega_{k0}$.

To constrain the cosmological parameters with the SNe Ia data, one should first figure out the corresponding observable and its theoretical (predicted) value. The observable given in the "Pantheon" data set is the corrected magnitude $m_{\rm cor}$ (see Table A17 of Scolnic et al. 2018), i.e.,

$$m_{\rm cor} \equiv m_B + K = \mu + M, \quad (8)$$

where μ is the distance modulus, m_B is the apparent B-band magnitude and M is the absolute B-band magnitude of a fiducial SN Ia. According to equation (3) in Scolnic et al. (2018), we can get the correction term $K = \alpha x_1 - \beta c + \Delta_M + \Delta_B$ which includes the corrections related to four different sources (for more details, see Scolnic et al. 2018). According to the definition of the distance modulus, one has

$$\mu = 5\log(d_L) + 25, \quad (9)$$

where d_L is the luminosity distance in Mpc. The observable $Y^{\rm obs} = \mu + M$ displayed in Equation (8) should correspond to the theoretical (predicted) value

$$\begin{aligned} Y^{th} &= 5\log(d_L) + 25 + M \\ &= 5\log[(1+z)D(z)] + Y_0, \end{aligned} \quad (10)$$

where the constant term Y_0 is written as $Y_0 = M + 5\log(\frac{cH_0^{-1}}{Mpc}) + 25$, and the normalized comoving distance $D(z)$ is defined by,

$$d_L(z) = \frac{c(1+z)}{H_0} D(z), \quad (11)$$

where c is the speed of light. The normalized comoving distance $D(z)$ can be expressed as

$$D(z) = \begin{cases} \frac{1}{\sqrt{-\Omega_{k0}}} \sin\left(\sqrt{-\Omega_{k0}} \int_0^z \frac{d\tilde{z}}{E(\tilde{z})}\right) & \text{if } \Omega_{k0} < 0, \\ \int_0^z \frac{d\tilde{z}}{E(\tilde{z})} & \text{if } \Omega_{k0} = 0, \\ \frac{1}{\sqrt{\Omega_{k0}}} \sinh\left(\sqrt{\Omega_{k0}} \int_0^z \frac{d\tilde{z}}{E(\tilde{z})}\right) & \text{if } \Omega_{k0} > 0. \end{cases} \quad (12)$$

The likelihood of the Pantheon sample is given by

$$\mathcal{L} \propto e^{-\chi^2/2}. \quad (13)$$

χ^2 is constructed as

$$\chi^2 = \Delta\overrightarrow{Y}^T \mathbf{C}^{-1} \Delta\overrightarrow{Y}, \quad (14)$$

where the residual vector for the SNe Ia data in the Pantheon sample is $\Delta\overrightarrow{Y}_i = [Y_i^{\rm obs} - Y^{th}(z_i; Y_0, \mathbf{p})]$. The covariance matrix $\mathbf{C}$ of the sample includes the contributions from both the statistical and systematic errors. The nuisance parameter, i.e., the constant term Y_0, is marginalized over with the analytical methodology presented in Giostri et al. (2012). The posterior probability distributions of model parameters are obtained with an affine–invariant Markov chain Monte Carlo (MCMC) ensemble sampler (emcee; Foreman-Mackey et al. 2013), where the likelihood can be worked out with Equations (13) and (14). We assume a flat prior for each parameter over a range of interest. In the framework of each cosmological model, the number of walkers is set as the number of model parameters times 40, and the number of steps is 3000.

3 ANALYSIS AND RESULTS

In the frameworks of the cosmological models under consideration, the observational constraints from the Pantheon sample are presented in Table 1, including the mean values and 68% confidence limits on the parameters. In the ΛCDM model, a closed universe is preferred with a mean value $\Omega_{k0} = -0.062$, but at a non-high CL ($\sim$ 25% CL) because of a high uncertainty. The result is consistent with that from Wang (2018), in which the non-flat ΛCDM model is constrained with the Pantheon sample via the MCMC code CosmoMC (Lewis 2013). In the ϕCDM model, it prefers a flat universe with $\Omega_{k0} = -0.004^{+0.228}_{-0.134}$ at 68% CL. An open universe is preferred in both the ωCDM and $\omega_0\omega_a$CDM models, according to $\Omega_{k0} = 0.127^{+0.280}_{-0.276}$ and $0.422^{+0.213}_{-0.338}$ at 68% CL, respectively. It turns out that the bound on Ω_{k0} is significantly dependent on the adopted DE model. Further, we employ Bayesian Information Criterion (BIC) to do the model comparison. BIC (Schwarz 1978) is defined as

$$\mathrm{BIC} = -2\ln\mathcal{L}_{\max} + k\ln N, \quad (15)$$

where $\mathcal{L}_{\max}$ is the maximum likelihood (i.e., $-2\ln\mathcal{L}_{\max} = \chi^2_{\min}$ under the Gaussian assumption), k is the number of model parameters and N is the size of the sample used in the analysis. BIC is widely utilized in a cosmological context (see e.g. Liddle 2004; Biesiada 2007; Li et al. 2013; Birrer et al. 2019; Chen et al. 2019). The favored model should be the one with a minimum BIC

Table 1 Observational constraints on the parameters of interest from the Pantheon SNe sample. The mean values with 68% confidence limits are displayed.

Model	Parameters				$\chi^2_{\rm min}$/d.o.f	BIC
ΛCDM	$\Omega_{k0}=-0.062^{+0.189}_{-0.169}$	$\Omega_{m0}=0.323^{+0.069}_{-0.075}$	...	...	1026.7/1048	1040.6
ϕCDM	$\Omega_{k0}=-0.004^{+0.228}_{-0.134}$	$\Omega_{m0}=0.215^{+0.060}_{-0.082}$	$\alpha=0.679^{+1.168}_{-0.505}$	...	1026.5/1048	1047.4
ωCDM	$\Omega_{k0}=0.127^{+0.280}_{-0.276}$	$\Omega_{m0}=0.288^{+0.076}_{-0.078}$	$\omega=-1.236^{+0.346}_{-0.722}$	...	1026.4/1048	1047.3
$\omega_0\omega_a$CDM	$\Omega_{k0}=0.422^{+0.213}_{-0.338}$	$\Omega_{m0}=0.219^{+0.134}_{-0.129}$	$\omega_0=-1.784^{+0.770}_{-1.291}$	$\omega_a=-0.110^{+6.116}_{-5.289}$	1025.6/1048	1054.4

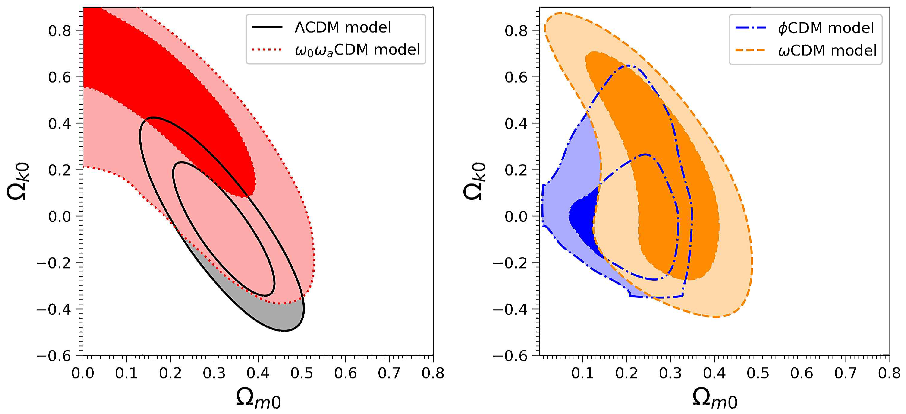

Fig. 1 Contours in the $(\Omega_{m0},\Omega_{k0})$ plane refer to the 2D marginalized distributions at 68% and 95% CLs, constrained with the Pantheon sample in the scenarios of ΛCDM, ϕCDM, ωCDM and $\omega_0\omega_a$CDM models.

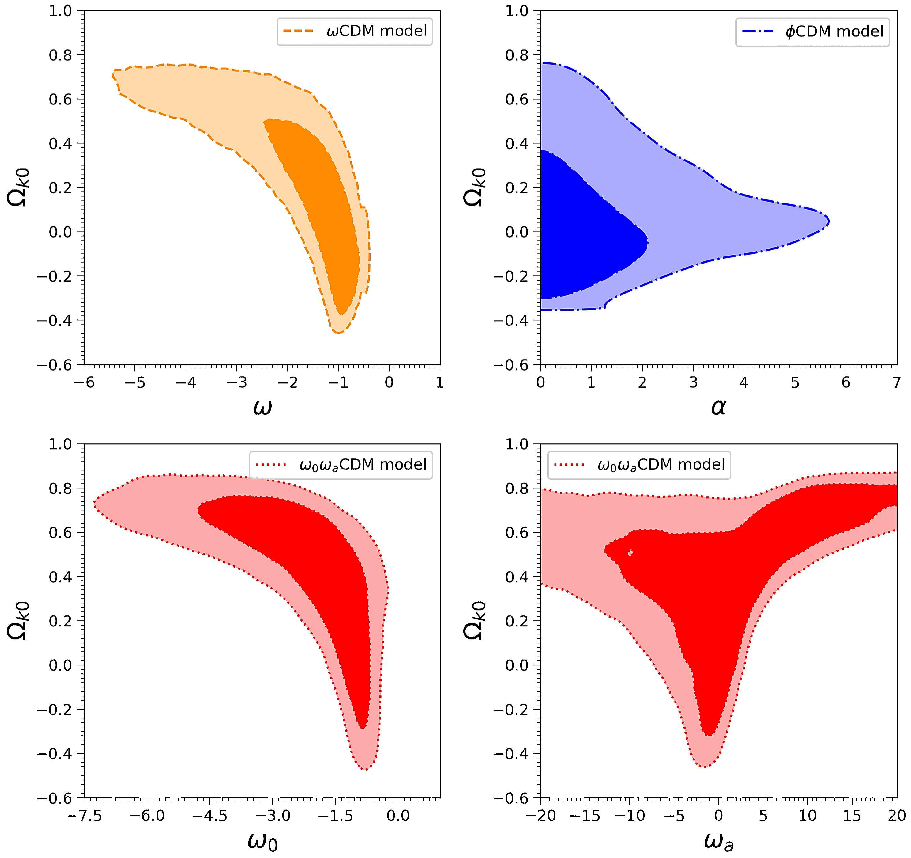

Fig. 2 The contours correspond to the 2D probability distributions at 68% and 95% CL for parameters of interest.

value. The BIC values for the ΛCDM, ϕCDM, ωCDM and $\omega_0\omega_a$CDM models are 1040.6, 1047.4, 1047.3 and 1054.4, respectively. So, the ΛCDM model is the one which fits the Pantheon SNe sample best.

To study the correlation between Ω_{k0} and Ω_{m0}, we display the two-dimensional (2D) probability distributions in the $(\Omega_{m0}, \Omega_{k0})$ plane for all the cosmological models under consideration in Figure 1. One can find a negative correlation between Ω_{k0} and Ω_{m0} in the ΛCDM, ωCDM and $\omega_0\omega_a$CDM scenarios. However, there is not an apparent correlation between them in the ϕCDM scenario. Then, we turn to study the relations between Ω_{k0} and other parameters besides Ω_{m0} in Figure 2. We find a negative correlation between Ω_{k0} and the DE EoS in the ωCDM model from the upper-left panel of Figure 2. The upper-right panel of Figure 2 demonstrates that there is not an obvious correlation between Ω_{k0} and α in the ϕCDM scenario. From the lower panels of Figure 2, we find a negative correlation between Ω_{k0} and ω_0, but no obvious correlation between Ω_{k0} and ω_a is discovered in the $\omega_0\omega_a$CDM model.

In the ΛCDM scenario, the mean value $\Omega_{k0} = -0.062$ constrained from the Pantheon SNe sample is close to but a bit smaller than the one with $\Omega_{k0} = -0.044$ from the Planck CMB spectra (Planck Collaboration et al. 2018). Nevertheless, the uncertainty of Ω_{k0} from the Pantheon sample is about 8 times larger than that from the Planck data, hence the former supports a closed universe at a much lower CL (at $\sim$ 25% CL) than that from the latter (at $\sim$ 99% CL). Moreover, as discussed in Di Valentino (2020), when jointing the Planck CMB along with the baryon acoustic oscillations (BAO) data, Ω_{k0} increases to $\Omega_{k0} = 0.0008^{+0.0038}_{-0.0037}$ at 95% CL (Planck Collaboration et al. 2018; Di Valentino et al. 2020). It turns out that the limit on Ω_{k0} changes significantly with the data sets adopted.

4 CONCLUSIONS

By considering four different kinds of DE models, we have studied the relation between the energy density of spatial curvature Ω_{k0} and the DE model with the recent SNe Ia data, i.e., the Pantheon sample. It turns out that the bound on Ω_{k0} is dependent notably on the adopted DE model, and a negative correlation exists between Ω_{k0} and the DE EoS. Briefly speaking, a closed universe is preferred in the ΛCDM model; a flat universe is heavily supported in the ϕCDM model; an open universe is favored in the ωCDM and $\omega_0\omega_a$CDM models.

In the scenario of the ΛCDM model, the limits on Ω_{k0} at 68% are $\Omega_{k0} = -0.062^{+0.189}_{-0.169}$ from the Pantheon sample, and $\Omega_{k0} = -0.044^{+0.018}_{-0.015}$ from the Planck CMB spectra (Planck Collaboration et al. 2019). Both the Pantheon SNe sample and the Planck CMB spectral data support a closed universe. Nevertheless, the uncertainty in Ω_{k0} from the former one is much larger than that from the latter one, thus the former one supports a closed universe at a much lower CL (at $\sim$ 25% CL) than that from the latter one (at $\sim$ 99% CL). In addition, when combining the Planck CMB with the BAO data, the value of Ω_{k0} changes to $\Omega_{k0} = 0.0008^{+0.0038}_{-0.0037}$ at 95% CL, which is in good agreement with a flat universe. It reflects the sample dependence of the limit on Ω_{k0}. Consequently, in view of the noticeable model-dependence and sample-dependence of the limit on Ω_{k0}, one should modestly apply the assumption of a flat universe.

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References

Amanullah, R., Lidman, C., Rubin, D., et al. 2010, ApJ, 716, 712
Astier, P., Guy, J., Regnault, N., et al. 2006, A&A, 447, 31
Betoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 568, A22
Biesiada, M. 2007, J. Cosmol. Astropart. Phys., 2007, 003
Birrer, S., Treu, T., Rusu, C. E., et al. 2019, MNRAS, 484, 4726
Cao, S., Qi, J., Biesiada, M., et al. 2019, Physics of the Dark Universe, 24, 100274
Chen, Y., & Ratra, B. 2011, Physics Letters B, 703, 406
Chen, Y., & Ratra, B. 2012, A&A, 543, A104
Chen, Y., Geng, C.-Q., Cao, S., Huang, Y.-M., & Zhu, Z.-H. 2015, J. Cosmol. Astropart. Phys., 2015, 010
Chen, Y., Ratra, B., Biesiada, M., Li, S., & Zhu, Z.-H. 2016, ApJ, 829, 61
Chen, Y., & Xu, L. 2016, Physics Letters B, 752, 66
Chen, Y., Kumar, S., & Ratra, B. 2017, ApJ, 835, 86
Chen, Y., Li, R., Shu, Y., & Cao, X. 2019, MNRAS, 488, 3745
Chevallier, M., & Polarski, D. 2001, International Journal of Modern Physics D, 10, 213
Conley, A., Sullivan, M., Hsiao, E. Y., et al. 2008, ApJ, 681, 482
Davis, T. M., Mörtsell, E., Sollerman, J., et al. 2007, ApJ, 666, 716
Denissenya, M., Linder, E. V., & Shafieloo, A. 2018, J. Cosmol. Astropart. Phys., 2018, 041
Di Valentino, E., Melchiorri, A., & Silk, J. 2020, Nature Astronomy, 4, 196
Farooq, O., Ranjeet Madiyar, F., Crandall, S., & Ratra, B. 2017, ApJ, 835, 26
Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306
Giostri, R., Vargas dos Santos, M., Waga, I., et al. 2012, J. Cosmol. Astropart. Phys., 2012, 027

Gong, Y., Wu, Q., & Wang, A. 2008, ApJ, 681, 27
Guy, J., Astier, P., Nobili, S., Regnault, N., & Pain, R. 2005, A&A, 443, 781
Guy, J., Astier, P., Baumont, S., et al. 2007, A&A, 466, 11
Guy, J., Sullivan, M., Conley, A., et al. 2010, A&A, 523, A7
Hicken, M., Wood-Vasey, W. M., Blondin, S., et al. 2009, ApJ, 700, 1097
Kowalski, M., Rubin, D., Aldering, G., et al. 2008, ApJ, 686, 749
Lewis, A. 2013, Phys. Rev. D, 87, 103529
Li, M., Li, X.-D., Wang, S., & Wang, Y. 2013, Frontiers of Physics, 8, 828
Liao, K. 2019, Phys. Rev. D, 99, 083514
Liao, K., Li, Z., Wang, G.-J., & Fan, X.-L. 2017, ApJ, 839, 70
Liddle, A. R. 2004, MNRAS, 351, L49
Linder, E. V. 2003, Phys. Rev. Lett., 90, 091301
Mania, D., & Ratra, B. 2012, Physics Letters B, 715, 9
Miknaitis, G., Pignata, G., Rest, A., et al. 2007, ApJ, 666, 674
Peebles, P. J. E. 1984, ApJ, 284, 439
Peebles, P. J. E., & Ratra, B. 1988, ApJL, 325, L17
Perlmutter, S., Gabi, S., Goldhaber, G., et al. 1997, ApJ, 483, 565
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Planck Collaboration, Aghanim, N., Akrami, Y., et al. 2018, arXiv e-prints, arXiv:1807.06209
Planck Collaboration, Aghanim, N., Akrami, Y., et al. 2019, arXiv e-prints, arXiv:1907.12875
Qi, J.-Z., Cao, S., Zhang, S., et al. 2019, MNRAS, 483, 1104
Ratra, B., & Peebles, P. J. E. 1988, Phys. Rev. D, 37, 3406
Ratra, B. 1991, Phys. Rev. D, 43, 3802
Riess, A. G., Press, W. H., & Kirshner, R. P. 1996, ApJ, 473, 88
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Riess, A. G., Strolger, L.-G., Tonry, J., et al. 2004, ApJ, 607, 665
Riess, A. G., Strolger, L.-G., Casertano, S., et al. 2007, ApJ, 659, 98
Ryan, J., Chen, Y., & Ratra, B. 2019, MNRAS, 488, 3844
Samushia, L. 2009, Constraining scalar field dark energy with cosmological observations, PhD thesis (Kansas State University)
Samushia, L., Dev, A., Jain, D., & Ratra, B. 2010, Physics Letters B, 693, 509
Schwarz, G. 1978, Annals of Statistics, 6, 461
Scolnic, D. M., Jones, D. O., Rest, A., et al. 2018, ApJ, 859, 101
Suzuki, N., Rubin, D., Lidman, C., et al. 2012, ApJ, 746, 85
Tonry, J. L., Schmidt, B. P., Barris, B., et al. 2003, ApJ, 594, 1
Wang, D. 2018, Phys. Rev. D, 97, 123507
Wang, G.-J., Wei, J.-J., Li, Z.-X., Xia, J.-Q., & Zhu, Z.-H. 2017, ApJ, 847, 45
Wang, L., Goldhaber, G., Aldering, G., & Perlmutter, S. 2003, ApJ, 590, 944
Wang, L., Strovink, M., Conley, A., et al. 2006, ApJ, 641, 50
Wei, J.-J., & Melia, F. 2020, ApJ, 888, 99
Wood-Vasey, W. M., Miknaitis, G., Stubbs, C. W., et al. 2007, ApJ, 666, 694
Zhou, H., & Li, Z. 2020, ApJ, 889, 186