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The force-free dipole magnetosphere in nonlinear electrodynamics

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Abstract Quantum electrodynamics (QED) effects may be included in physical processes of magnetar and pulsar magnetospheres with strong magnetic fields. Involving the quantum corrections, Maxwell electrodynamics is modified to nonlinear electrodynamics. In this work, we study the force-free magnetosphere in nonlinear electrodynamics in a general framework. The pulsar equation describing a steady and axisymmetric magnetosphere is derived, which now admits solutions with corrections. We derive the first-order nonlinear corrections to the near-zone dipole magnetosphere in some popular nonlinear effective theories. The field lines of the corrected dipole tend to converge on the rotational axis so that the fields in the polar region are stronger compared to the pure dipole case.

Key words: stars: neutron — stars: magnetars — stars: magnetic field

1 INTRODUCTION

It is inferred that pulsars and magnetars possess very strong magnetic fields. The field strength can even exceed the critical value $B_Q = m_e^2c^3/\hbar e \simeq 4.4 \times 10^{13}$ G, above which quantum electrodynamics (QED) effects should not be ignored.

QED effects will affect the polarization and spectra of thermal radiation from the surface of magnetars and pulsars with strong magnetic fields. In the magnetospheres, the photon polarizations can be decomposed into two modes according to the propagating direction and the local magnetic field direction. Due to resonance between the vacuum and plasma birefringence, one of the polarization modes of an X-ray photon can be converted into the other. This gives rise to an energy-dependent polarization signature for the observed quiescent non-thermal X-ray emission (Lai & Ho 2003; Denisov & Svertilov 2003; Harding & Lai 2006; van Adelsberg & Lai 2006; Fernandez & Davis 2011; Kaspi & Beloborodov 2017; Krawczynski et al. 2019). Magnetar magnetospheres are opaque to high energy photons due to attenuation by magnetic photon splitting (below the energy threshold $2m_ec^2$) and pair production (above the threshold). This will distort the blackbody spectra of the surface thermal radiation and can be tested with future precise observations of the spectra and the polarizations (Wadiasingh et al. 2019; Hu et al. 2019), which meanwhile provides information on the surface magnetic fields in the magnetar magnetospheres.

Thus, the strength and geometry of the magnetic fields are crucial in these QED processes. It is expected that these processes work more effectively at low altitudes where the magnetic fields are higher. In the near-zone regions, the geometry of the pulsar magnetospheres is usually taken to be that of a force-free dipole structure. When including the QED corrections, Maxwell electrodynamics should be modified with additional nonlinear terms and the dipole magnetosphere must be corrected with nonlinear contributions, as analyzed in previous works (Heyl & Hernquist 1997; Ptri 2016; Xiong et al. 2016).

In this work, we consider the force-free magnetosphere in general nonlinear electrodynamics. In contrast to previous treatments (Freytsis & Gralla 2016; Petri 2016), we shall follow the traditional approach to do so. We first derive the pulsar equation, describing the steady and axisymmetric magnetospheres, in nonlinear electrodynamics. We then obtain the corrected dipole magnetosphere to leading order from the pulsar equation in some popular nonlinear effective theories, like the Euler-Heisenberg (EH) theory, Born-Infeld (BI) theory and Logarithmic theory.

2 NONLINEAR ELECTRODYNAMICS

The action of general electrodynamics takes the form

$$S = \int \sqrt{-g}\left[\frac{1}{4\pi}\mathcal{L}_{\rm EM}(s,p) + A_\mu J^\mu\right] d^4x, \tag{1}$$

where $\mathcal{L}_{\rm EM}(s,p)$ is the general Lagrangian of the electromagnetic (EM) fields with

$$s \equiv \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}(\mathbf{B}^2 - \mathbf{E}^2), \tag{2}$$

$$p \equiv \frac{1}{4}\widetilde{F}^{\mu\nu}F_{\mu\nu} = \mathbf{E}\cdot\mathbf{B}. \tag{3}$$

The dual field strength $\widetilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$.

The equations of motion can be derived from the action. It is obvious that the Bianchi identity is automatically satisfied

$$\nabla_\mu \widetilde{F}^{\mu\nu} = 0. \tag{4}$$

In Minkowski spacetime, the equation can be decomposed into

$$\nabla\times\mathbf{E} = -\dot{\mathbf{B}}, \tag{5}$$

$$\nabla\cdot\mathbf{B} = 0. \tag{6}$$

The relation between the current and fields is given by

$$\nabla_\mu G^{\mu\nu} = 4\pi J^\nu, \tag{7}$$

where J^ν is the conserved current and

$$G^{\mu\nu} = SF^{\mu\nu} + P\widetilde{F}^{\mu\nu}, \tag{8}$$

with

$$S \equiv \partial_s\mathcal{L}_{\rm EM}, \quad P \equiv \partial_p\mathcal{L}_{\rm EM}. \tag{9}$$

When $S = -1$ and $P = 0$, the equation reduces to the Maxwell theory case. In Minkowski spacetime, the equation can be re-expressed as

$$\nabla\cdot\mathbf{D} = 4\pi\rho, \tag{10}$$

$$\nabla\times\mathbf{H} = 4\pi\mathbf{j} + \dot{\mathbf{D}}, \tag{11}$$

where

$$\mathbf{D} = -S\mathbf{E} + P\mathbf{B}, \tag{12}$$

$$\mathbf{H} = -S\mathbf{B} - P\mathbf{E}. \tag{13}$$

3 THE FORCE-FREE CONDITION

The derivative of the Lagrangian with respect to the metric yields the energy-momentum tensor of EM fields

$$T^{\mu\nu}_{\rm EM} = -\frac{1}{4\pi}[SF^\mu{}_\alpha F^{\nu\alpha} + P\widetilde{F}^\mu{}_\alpha F^{\nu\alpha} - g^{\mu\nu}\mathcal{L}_{\rm EM}]. \tag{14}$$

The tensor satisfies

$$\nabla_\mu T^{\mu\nu}_{\rm EM} = J_\mu F^{\mu\nu}. \tag{15}$$

Equation (15), relating the divergence of the EM energy-momentum to the Lorentz force, takes the same form as in the Maxwell theory. It determines the change of the momenta of the charged particles in the system.

It is usually assumed that, in a steady magnetosphere filled with plasma, the charged particles in the magnetospheres with strong EM fields should feel no net force (at least in most regions). This means that the Lorentz force in Equation (15) should vanish

$$J_\mu F^{\mu\nu} = 0. \tag{16}$$

This is the force-free condition in general nonlinear electrodynamics, which also has the same form as in the Maxwell theory. This condition also says that the dynamics of the energy density in the system is dominated by the EM fields and that the inertia of the plasma in the system can be ignored. That is, $T^{\mu\nu}_{\rm EM}$ can be approximately taken as the energy-momentum density of the whole system so that it is conserved.

In components, the force-free equation is decomposed into

$$\mathbf{j}\cdot\mathbf{E} = 0, \quad \rho\mathbf{E} + \mathbf{j}\times\mathbf{B} = 0, \tag{17}$$

which implies

$$p = \mathbf{E}\cdot\mathbf{B} = 0, \tag{18}$$

so we simply have $\mathcal{L}_{\rm EM}(s,p) = \mathcal{L}_{\rm EM}(s)$ and $G^{\mu\nu} = SF^{\mu\nu}$ under the force-free condition.

4 THE PULSAR EQUATION

Under the force-free condition, the equations describing a force-free magnetosphere can be derived. As usual, we consider the simplest case: axisymmetric and steady magnetospheres in Minkowski spacetime. In spherical coordinates, the force-free condition in Equation (16) reads

$$\partial_r A_0 J^r + \partial_\theta A_0 J^\theta = 0, \tag{19}$$

$$\partial_r A_0 J^0 + F_{r\theta}J^\theta + \partial_r A_\phi J^\phi = 0, \tag{20}$$

$$\partial_\theta A_0 J^0 - F_{r\theta}J^r + \partial_\theta A_\phi J^\phi = 0, \tag{21}$$

$$\partial_r A_\phi J^r + \partial_\theta A_\phi J^\theta = 0. \tag{22}$$

For convenience, let us define the Poisson bracket as in Petrova & Flanchik (2018) (and also Compre et al. 2016)

$$[A, B] \equiv \mathcal{L}_T B = \partial_r A \partial_\theta B - \partial_\theta A \partial_r B, \tag{23}$$

where the tangent vector $T = \partial_r A \partial_\theta - \partial_\theta A \partial_r$. The necessary condition that A is a function of B (or vice versa) is that $[A, B] = 0$.

From Equations (19) and (22), we can find that

$$[A_0, A_\phi] = 0, \tag{24}$$

so A_0 should be a function of A_ϕ. We can define

$$dA_0 = -\Omega(A_\phi) dA_\phi. \tag{25}$$

As is known, Ω is the angular velocity of a magnetic field line. It is constant along the magnetic field line.

Terms in the Maxwell equation, Equation (7), with nonlinear corrections are expressed as

$$J^0 = -\frac{1}{4\pi} \nabla \cdot (S \nabla A_0) \,, \tag{26}$$

$$J^r = -\frac{1}{4\pi r^2 \sin\theta} \partial_\theta (\sin\theta S F_{r\theta}), \tag{27}$$

$$J^\theta = \frac{1}{4\pi r^2 \sin\theta} \partial_r (\sin\theta S F_{r\theta}), \tag{28}$$

$$J^\phi = \frac{1}{4\pi} \nabla \cdot \left(\frac{S \nabla A_\phi}{r^2 \sin^2\theta} \right). \tag{29}$$

From Equations (19), (22), (27) and (28), we find that

$$[A_0, \sin\theta S F_{r\theta}] = [A_\phi, \sin\theta S F_{r\theta}] = 0. \tag{30}$$

So, $\sin\theta S F_{r\theta}$ is also a function of A_ϕ. Let us denote

$$\psi \equiv 2\pi A_\phi, \quad I(\psi) \equiv -2\pi \sin\theta S F_{r\theta}. \tag{31}$$

Then the charge density is expressed as

$$J^0 = \frac{1}{8\pi^2} \nabla \cdot (S\Omega \nabla \psi) \,. \tag{32}$$

From Equation (20) or (21), we have

$$J^\phi = \Omega J^0 - \frac{I I'}{8\pi^2 r^2 \sin^2\theta S}, \tag{33}$$

where the prime denotes the derivative with respect to ψ.

By comparing Equations (29) and (33), we can derive the general pulsar equation

$$S \nabla \cdot \left(\frac{S \nabla \psi}{r^2 \sin^2\theta} \right) - S\Omega \nabla \cdot (S\Omega \nabla \psi) = -\frac{I I'}{r^2 \sin^2\theta}. \tag{34}$$

Specifically, the equation in spherical coordinates can be written as

$$\begin{aligned} &\frac{1}{r^2}(1 - r^2 \sin^2\theta \Omega^2) \left[r^2 S \partial_r (S \partial_r \psi) + S \partial_\theta (S \partial_\theta \psi) \right] \\ &- \sin^2\theta \Omega \Omega' S^2 [r^2 (\partial_r \psi)^2 + (\partial_\theta \psi)^2] \\ &- 2r \sin^2\theta \Omega^2 S^2 \partial_r \psi \\ &- \frac{1}{r^2}(1 + r^2 \sin^2\theta \Omega^2) \cot\theta S^2 \partial_\theta \psi = -I I' \,. \end{aligned} \tag{35}$$

When $S = -1$, this reduces to the pulsar equation in Maxwell's theory.

The EM fields in the unit basis of spherical coordinates are expressed as

$$\mathbf{D} = -S \mathbf{E} = \frac{S\Omega}{2\pi r} (r \partial_r \psi, \partial_\theta \psi, 0) \,, \tag{36}$$

$$\mathbf{H} = -S \mathbf{B} = \frac{1}{2\pi r^2 \sin\theta} (-S \partial_\theta \psi, r S \partial_r \psi, r I) \,. \tag{37}$$

With them, we have

$$\begin{aligned} s = &\frac{1}{8\pi^2 r^4 \sin^2\theta} \\ &\times \left\{ \frac{r^2 I^2}{S^2} + (1 - r^2 \sin^2\theta \Omega^2) [r^2 (\partial_r \psi)^2 + (\partial_\theta \psi)^2] \right\} \\ = &\frac{1}{8\pi^2 x^2} \left\{ \frac{I^2}{S^2} + (1 - x^2 \Omega^2) [(\partial_x \psi)^2 + (\partial_z \psi)^2] \right\}, \end{aligned} \tag{38}$$

which must be non-negative. The expression in cylindrical coordinates ($x = r \sin\theta$ and $z = r\cos\theta$) written in the second line indicates that the translational symmetry along the rotation axis remains in nonlinear electrodynamics, i.e., the action and the equations are invariant with respect to the transformation: $z \to z' = z + \epsilon$.

The spin-down rate is obtained

$$L = \int \mathbf{S} \cdot d\mathbf{s} = -\frac{1}{8\pi^2} \int I(\psi) \Omega(\psi) d\psi, \tag{39}$$

where the Poynting flux is

$$\mathbf{S} = \frac{1}{4\pi} \mathbf{E} \times \mathbf{H}. \tag{40}$$

So, the torque takes the same form as in the Maxwell theory.

5 THE NEAR-ZONE DIPOLE MAGNETOSPHERES

The pulsar equation is hard to solve even in the Maxwell theory, so it is expected that numerical methods are needed to solve Equation (34) in nonlinear electrodynamics. However, here we do not need to search for the global solutions. We only need to focus on the magnetosphere at low

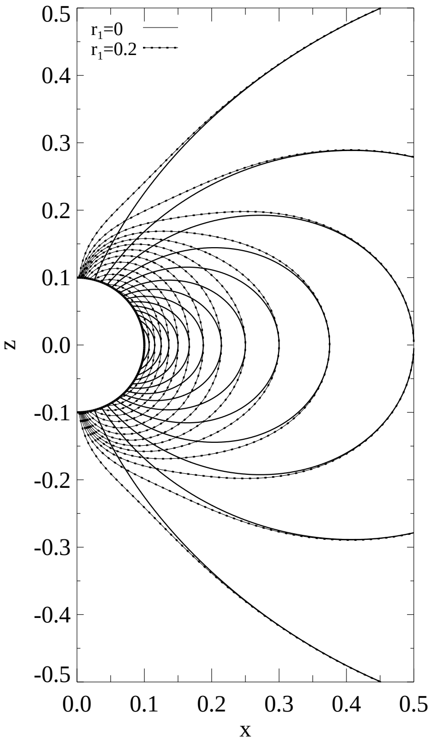

Fig. 1 Magnetic field lines based on Eq. (48) with different r_1. The radius of the star is $r_* = 0.1$.

altitudes where the EM fields are strong and the nonlinear corrections may be important.

As is done in the Maxwell theory, the near-zone magnetospheres on pulsars are usually regarded as a dipole structure, which serves as the inner boundary condition in numerical simulations of pulsar magnetospheres (Michel 1973; Contopoulos et al. 1999; Gruzinov 2005). This structure can be obtained from the pulsar equation at $r \to 0$, where the rotational velocities of the magnetic field lines are much less than the speed of light and the electric current is negligible by setting $I = 0$. In this limit, Equation (35) with $S = -1$ reduces to

$$\partial_\theta^2 \psi - \cot\theta \partial_\theta \psi + r^2 \partial_r^2 \psi = 0. \tag{41}$$

The equation is solved by the general form

$$\psi = \psi_{-n}(\theta) r^{-n}, \tag{42}$$

where $\psi_{-n}(\theta)$ is related to the associated Legendre polynomials for different n. For $n = 0$, it is a monopole, and, for $n = 1$, it is a pure dipole $\psi = \sin^2\theta / r$. The rotational effects in outer regions just deform this basic dipole geometry.

In what follows, we determine the force-free magnetospheres in the near regions in different nonlinear theories.

5.1 The BI Theory

The BI effective theory is a well-regularized nonlinear theory, leading to finite self-energy of point-like charge and absence of birefringence. It also arises from the world-volume action of D-branes in string theory. Some aspects of pulsar magnetospheres in BI effective theory were discussed previously in Denisov & Svertilov (2003) and Pereira et al. (2018). Here, we consider the corrected dipole geometry in the theory.

The Lagrangian of EM fields in the BI effective theory takes the form

$$\mathcal{L}_{\rm EM}(s,p) = b^2 \left(1 - \sqrt{1 + \frac{2s}{b^2} - \frac{p^2}{b^4}}\right), \tag{43}$$

where the only parameter is of dimension of mass squared: $b = M^2$. The lower bound of M is constrained to be 4×10^{-4} GeV by PVLAS (Della Valle et al. 2014) and 100 GeV by ATLAS in the Large Hadron Collider (LHC) (Ellis et al. 2017; Pereira et al. 2018).

From the Lagrangian, we can obtain the expression of S

$$S^2 = \frac{r^2(4\pi^2 b^2 r^2 \sin^2\theta - I^2)}{4\pi^2 b^2 r^4 \sin^2\theta + (1 - r^2 \sin^2\theta\Omega^2)[r^2(\partial_r\psi)^2 + (\partial_\theta\psi)^2]}. \tag{44}$$

At large distance r, $S^2 \to 1$ and so the pulsar equation recovers the one in Maxwell's theory. Inserting it into the nonlinear pulsar described by Equation (35), we can basically derive solutions, but it is difficult to do so. There do not even exist solutions that are only dependent on θ (like Michel's monopole solution in the Maxwell theory).

Let us take the near-zone limit with approximately vanishing Ω and I, for which Equation (35) with Equation (44) is simplified to

$$\begin{aligned} &\partial_\theta^2\psi - \cot\theta\partial_\theta\psi + r^2\partial_r^2\psi + \frac{1}{4\pi^2 b^2 r^3 \sin^2\theta} \\ &\times [r(\partial_r\psi)^2\partial_\theta^2\psi + r(\partial_\theta\psi)^2\partial_r^2\psi + r^2(\partial_r\psi)^3 \\ &- 2r\partial_r\psi\partial_\theta\psi\partial_r\partial_\theta\psi + 2\partial_r\psi(\partial_\theta\psi)^2] = 0. \end{aligned} \tag{45}$$

There exist exact solutions that are independent of the parameter b: $\psi = \cos\theta$ ($n = 0$), $\psi = r\cos\theta$ ($n = 1$) and $\psi = r^2\sin^2\theta$ ($n = 2$), which are also solutions to the pulsar equation in the Maxwell theory. So, the nonlinear terms do not alter the non-rotating monopole solution.

For the dipole solution, the situation is different. From the above equation, we can determine that the solution can be expanded in powers of b^{-2}

$$\psi = \psi^{(0)} + \psi^{(1)} + \psi^{(2)} + \cdots. \tag{46}$$

The zero-th order part $\psi^{(0)} = m \sin^2\theta/r$ is the pure dipole solution. $\psi^{(1)}$ is the first order correction at the order $\mathcal{O}(b^{-2})$. Inserting the expression into Equation (45), we get the leading order equation

$$\begin{aligned}\partial_\theta^2\psi^{(1)} - \cot\theta\partial_\theta\psi^{(1)} + r^2\partial_r^2\psi^{(1)} \\ = \frac{3m^3\sin^2\theta(1+\cos^2\theta)}{4\pi^2 b^2 r^7}\,.\end{aligned} \tag{47}$$

Thus, the dependence of $\psi^{(1)}$ on r should be of the form $\sim r^{-7}$. Up to first order, the final solution is

$$\psi = \frac{m\sin^2\theta}{r}\left[1 + \frac{r_1^6}{r^6}\left(1 - \frac{9}{16}\sin^2\theta\right) + \cdots\right], \tag{48}$$

where the first-order characteristic distance

$$r_1 = \left(\frac{m}{\sqrt{33}\pi b}\right)^{\frac{1}{3}}. \tag{49}$$

So, the correction becomes unimportant sharply at $r \gg r_1$. The first-order corrected part $\psi^{(1)}$ becomes important for $r_1 \gtrsim r \gg r_2$, where r_2 is the characteristic distance of $\psi^{(2)}$ (not derived here since the surface fields on neutron stars are not so strong that the higher order corrections are important). The distribution of the magnetic field lines from the solution is displayed in Figure 1. Compared with the dipole magnetosphere, the field lines around $r \sim r_1$ tend to converge on the rotation axis. The curvature of the field lines becomes larger at a distance less than but near the characteristic distance r_1.

With the solution, it is easy to check that the strength B and the energy density $(b^2/4\pi)(\sqrt{1+B^2/b^2}-1)$ of the magnetic field become larger than in the Maxwell theory case.

5.2 The Logarithmic Theory

In the Logarithmic theory (e.g., see Gaete & Helayel-Neto 2014; Guo et al. 2018), the self-energy of a point-like charge is finite and the birefringent phenomenon appears. Its action takes a logarithmic form

$$\mathcal{L}_{\mathrm{EM}}(s,p) = -b^2\ln\left(1 + \frac{s}{b^2} - \frac{p^2}{2b^4}\right). \tag{50}$$

Equation (35) with $I = \Omega = 0$ reduces to

$$\begin{aligned}&\partial_\theta^2\psi - \cot\theta\partial_\theta\psi + r^2\partial_r^2\psi + \frac{1}{8\pi^2 b^2 r^4\sin^2\theta} \\ &\times\left[r^2(\partial_r\psi)^2(\partial_\theta^2\psi + \cot\theta\partial_\theta\psi + 2r\partial_r\psi - r^2\partial_r^2\psi)\right. \\ &-(\partial_\theta\psi)^2(\partial_\theta^2\psi - \cot\theta\partial_\theta\psi - 4r\partial_r\psi - r^2\partial_r^2\psi) \\ &\left.-4r^2\partial_r\psi\partial_\theta\psi\partial_r\partial_\theta\psi\right] = 0.\end{aligned} \tag{51}$$

It is interesting that the equation has the same three exact solutions as Equation (45). The first order solution $\psi^{(1)}$ also takes the same form as Equation (47). So, the geometries of the field lines are the same and the solutions cannot be discriminated in the two theories up to first order.

5.3 The EH Theory

The various QED effects on the physical processes in magnetar magnetospheres, including the vacuum birefringence, photon splitting and pair production, were mostly discussed based on the EH effective theory (Euler & Kockel 1935; Heisenberg & Euler 1936). In the weak field limit, the Lagrangian expanded to leading orders is

$$\mathcal{L}_{\mathrm{EM}}(s,p) = -s + \beta\left(4s^2 + 7p^2\right) + \cdots, \tag{52}$$

where $\beta = e^2/(45hcB_K^2)$ with $B_k = m_e^2c^3/(\hbar e)$. So, the leading order terms are the same as in the BI (Eq. (43)) and the Logarithmic (Eq. (50)) Lagrangians under the force-free condition, just with different parameters b and β. The first-order corrected dipole geometry should be also the same as displayed in Figure 1. So, within the characteristic distance, the dipole structure assumes a multipole-like structure, consistent with the corrected dipole structure to leading orders in the EH theory (Heyl & Hernquist 1997; Ptri 2016).

6 CONCLUSIONS

The pulsar equation in general nonlinear electrodynamics is derived. The corrected dipole solutions in some popular nonlinear effective theories are obtained and discussed. These solutions take the same form up to the first order, which indicates that the field lines tend to converge on the rotation axis. So, the fields are stronger in the polar region and have larger curvature within the characteristic distance than in the pure dipole magnetosphere. This discrepancy should be taken into account when considering the quantum effects in the radiative transfer process of the surface emission.

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