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Defocus spot detection of astronomical optical system

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Received 2019 June 17; accepted 2019 October 17

Abstract Defocusing spot size detection is especially essential for aberration analysis and correction of optical systems. In the case of far defocusing, the celestial forms a pupil image on the detector, and the size of the image is linearly changed with the defocusing distance, and can be used to correct the optical system and analyze the image quality. Based on the focal plane attitude detection of Large Sky Area Multi-Object Fiber Spectroscopy Telescope (LAMOST), this paper uses a variety of methods to detect the size of the defocusing spot of LAMOST telescope. For the particularity of the spot, the average value spacing algorithm, the peak value spacing algorithm, the ellipse fitting algorithm, and the multi-peak Gaussian fitting algorithm are used to detect the spot size. This paper will introduce these four methods, in which the average value spacing algorithm is proposed by the author of this paper. The advantages and disadvantages of the four methods are compared. The experimental results show that the average value spacing algorithm can achieve better accuracy of spot size detection in the four algorithms.

Key words: methods: data analysis - techniques: image processing - telescopes

1 INTRODUCTION

The LAMOST telescope is the largest aperture telescope in China, and it is a Schmidt optical system with two large segmented mirrors. The telescope consists of a reflective active aspheric Schmidt correction plate Ma, a spherical primary mirror Mb and a focal plane. Ma is made up of 24 hexagonal sub-mirrors. Mb is made up of 37 hexagonal sub-mirrors. The radius of curvature of Mb is 40 meters, and the focal length of the telescope system is 20 meters (Cui et al. 2012; Wang 2000).

The size of the spherical focal plane of the LAMOST telescope is about 1.8 meters. The attitude of the focal plane is directly related to the imaging quality. The detection and correction of the focal plane attitude are especially important. The size of the spherical focal plane of the LAMOST telescope is defocused spot detection technology is a crucial telescope focal plane attitude detection technology (Sutherland et al. 2015; Schechter & Levinson 2011); the key to the technology is the size detection of a defocused spot.

In the case of far defocusing, the defocused spot on the focal plane has the same shape as the system pupil (Born et al. 2013). Theoretically, the size of the defocused spot changes linearly with the defocus distance. According to the size of the spot obtained before and after the focus, we can get the best focus position for correcting the focal plane attitude. Affected by atmospheric disturbances and system aberrations, the intensity distribution of defocused spot is uneven, and atmospheric disturbances can be smoothed by long exposures (Oboukhov 1962), but system aberrations still exist.

The LAMOST telescope is a Schmidt telescope with segmented mirrors. The shape of defocused spot is affected by both the Ma and the focal plane, forming a bright spot within the middle of the dark, which makes it difficult to detect the size of the spot.

Traditional defocus spot size detection of, such as laser spot size detection, small image spot, which contour close to the ellipse. Generally, using elliptical spot matching can achieve good dimensional accuracy (Fitzgibbon et al. 1999). For the telescope optical system, the defocus spot



Fig. 1 The light intensity distribution of the defocusing spot of LAMOST telescope, isolated peaks are produced by Ma submirror.

light intensity distribution is complex, and the shape is irregular. The light intensity distribution of LAMOST defocus spot is shown in Figure 1. In this case, we use four defocus spot size detection methods, including Ellipse fitting algorithm (EFA) (Fitzgibbon et al. 1999), Gaussian fitting algorithm (GFA) (Hossain et al. 2014; Fast et al. 1999), average spacing algorithm (ASA), peak spacing algorithm (PSA), the latter two ways are raised for telescope defocus spot by author.

The first part of the paper gives the principles of several algorithms, the emphasis is on the average spacing algorithm. The second part uses the simulation image obtained by ZEMAX software, by comparing the advantages and disadvantages of the four algorithms. The third part uses the LAMOST telescope to get the defocus spots, and to compare the advantages and disadvantages of the four algorithms.

2 PRINCIPLE OF DEFOCUS SPOT SIZE DETECTION METHOD

This paper mainly introduces the average spacing algorithm proposed for the specialty of telescope defocus spot size processing, and then briefly introduces the ellipse fitting algorithm, multi-peak Gaussian fitting, and peak spacing algorithm.

2.1 Average Spacing Algorithm

First, we need to preprocess the image obtained by the CCD, including image binarization, extracting the defocused spot, and determining the center position of the spot. For our defocused spot, the Niblack algorithm has a better binarization effect after comparing Niblack (Niblack 1986) and OTSU (Otsu 1979) image binarization algorithms. We use the Niblack algorithm to perform image binarization. We use the binarized defocus spot region to calculate the center of the spot, and obtain the light intensity distribution in the x-direction and y-direction through the center. The light intensity distribution in the x-direction is shown in Figure 2.

The most important thing for binarization is to determine the threshold value, and the binarization threshold value is calculated by the mean m and the standard deviation s of the gray values of the pixel points in an individual pixel and its neighborhood. For any point (x, y) on the image, the calculation method of the mean, and the calculation method of the variance are as follows, where f(i, j)is light intensity of position (i, j).

$$m(x,y) = \frac{1}{n^2} \sum_{i=x-\frac{n}{2}}^{x+\frac{n}{2}} \sum_{j=x-\frac{n}{2}}^{x+\frac{n}{2}} f(i,j).$$
(1)

$$s(x,y) = \sqrt{\frac{1}{n^2} \sum_{i=x-\frac{n}{2}}^{x+\frac{n}{2}} \sum_{j=x-\frac{n}{2}}^{x+\frac{n}{2}} (f(i,j) - m(x,y))^2}.$$
 (2)

Then the binarized threshold value T(x, y) of the point is obtained according to the mean and the standard deviation. The calculation formula is

$$T(x,y) = k \times s(x,y) + m(x,y), \tag{3}$$

$$x_0 = \frac{1}{n^2} \sum_{i=x-\frac{n}{2}}^{x+\frac{n}{2}} \sum_{j=x-\frac{n}{2}}^{x+\frac{n}{2}} x_{i,j} f(i,j), \qquad (4)$$



Fig. 2 Light intensity distribution in x direction and through center point.

$$y_0 = \frac{1}{n^2} \sum_{i=x-\frac{n}{2}}^{x+\frac{n}{2}} \sum_{j=x-\frac{n}{2}}^{x+\frac{n}{2}} y_{i,j} f(i,j).$$
(5)

After the defocusing spot is binarized, we can then obtain the intensity distribution in the x-the direction of the spot through the centre (x_0, y_0) .

According to the light intensity distribution curve, the non-bright spot area near the defocus spot is selected for calculating the background noise, including the central dark spot and the area near the outer contour of the defocus spot, obtaining the average background noise $I_{\rm noi}$. Then the light intensity at all positions is subtracted from the background noise $I_{\rm noi}$ to get a new light intensity distribution $I_{\rm new}$ and calculate the average value $I_{\rm ave}$ of the new light intensity distribution.

Then, the Lagrangian interpolation method (Li 2001) is used to change the light intensity distribution composed of n discrete points (x_1, y_1) to (x_n, y_n) into a continuous light intensity distribution function f(x) to improve the detection accuracy:

$$f(x) = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix} \begin{bmatrix} l_1(x) \\ l_2(x) \\ \vdots \\ l_n(x) \end{bmatrix}, \quad (6)$$

$$l_{i} = \prod_{j=1\& j \neq i}^{n} \frac{x - x_{j}}{x_{i} - x_{j}} .$$
 (7)

According to the obtained light intensity distribution f(x) function, the solution x_1, x_2 corresponding to $f(x) = I_{ave}$ is found, and the minimum and maximum values are taken when more than two solutions are exceeded. Then, we use $|x_1 - x_2|$ as the spot size.

6.0x10³ -5.0x10³ -2.0x10³ -1400 1600 1800 X Direction/pixel

Fig. 3 Multi-peak Gaussian fitting of light intensity distribution.

2.2 Ellipse Fitting Algorithm

After obtaining the contour of the spot, we perform a twodimensional ellipse fitting on the spot contour. The twodimensional elliptic function equation is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$
 (8)

To find the best-fitted ellipse, we choose the central second moment of the standard image spot as the optimal estimate. s_{xx} stand for the x-direction second-order central moment, s_{yy} represent the second-order central moment of the y-direction, s_{xy} stand for second-order central moment of xy direction, (x_0, y_0) is central point:

$$s_{xx} = \sum (x - x_0)^2 / N$$
, (9)

$$s_{yy} = \sum (y - y_0)^2 / N$$
, (10)

$$s_{xy} = \sum (x - x_0)(y - y_0)/N.$$
 (11)

Then the major axes a and minor axes b of the ellipse having the second-order central moment are

$$\begin{bmatrix} a^2 & 0\\ 0 & b^2 \end{bmatrix} = \begin{bmatrix} s_{yy} & s_{xy}\\ s_{xy} & s_{xx} \end{bmatrix}.$$
 (12)

We use the major axes as the size of spot.

2.3 Gaussian Fitting Algorithm

For the light intensity multi-peak case, we can use the Gaussian function to fit the spotlight intensity curve. The multi-peak Gaussian fitting can be divided into multiple Gaussian functions to fit the whole curve. After Gaussian fitting, the spacing of the centres of the two Gaussian curves, which have furthest spacing, is selected as the spot size.

For multi-peak Gaussian fitting, we use functions

$$f(x) = f(x_0) + \sum_{i=1}^{N} A_i \frac{1}{\sqrt{2\pi\sigma_i}} e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}$$
(13)



Fig. 4 3D simulation of the LAMOST optical system.



Fig. 5 The left panel shows the simulation results of the spot intensity distribution in 20 mm out of focus position; the right panel shows the light intensity distribution through the center.

to perform the fitting, first determine the value of the position μ_i of the peak and $f(x_0)$, and then select the *n* points (x_i, y_i) on the light intensity distribution curve. The residual value of the fitted result $f(x_i)$ and the actual light intensity y_i is The least squares method is used for optimal estimation, and the iterative process uses the Gauss-Newton method. Multi-peak Gaussian fitting of light intensity distribution is shown in Figure 3.

2.4 Peak Spacing Algorithm

$$Q = \sum_{i=1}^{N} (f(x_i) - y_i).$$
(14)

According to the light intensity distribution through the centre point (x_0, y_0) , the coordinates (x_i, y_i) of the max-

Table 1 Simulated Spot Size Measured by ZEMAX Software

| Defocus distance (mm) | Size of spot (µm) |
|--------------------------|----------------------|
| 20 | 4497.9 |
| 21 | 4727.7 |
| 22 | 4957.5 |
| 23 | 5187.4 |
| 24 | 5417.2 |
| 25 | 5647.0 |

 Table 2
 The Size of the Simulated Image Spot Obtained by the Four Algorithms

| Defocus distance (mm) | EFA (pixel) | GFA (pixel) | PSA (pixel) | ASA (pixel) |
|--------------------------|----------------|----------------|----------------|----------------|
| 20 | 271.126 | 165.065 | 172.000 | 205.507 |
| 21 | 284.801 | 174.518 | 180.000 | 215.591 |
| 22 | 296.478 | 182.085 | 189.000 | 225.814 |
| 23 | 311.794 | 190.655 | 197.000 | 235.228 |
| 24 | 324.958 | 198.661 | 207.000 | 244.121 |
| 25 | 338.438 | 205.474 | 214.000 | 254.573 |

imum values of the peak are selected, and we calculated the size of the image spots by using two particular peaks $|x_i - x_j|$.

The intensity distribution of the defocus spot reflects the aberrations of the system, including the effects of optical system aberrations and atmospheric disturbances, and eliminates high-frequency atmospheric disturbances through the prolonged exposure time. In the case where only systematic aberrations exist, the light intensity distribution of the defocused spot and the relative position of the peak remain unchanged.

3 SIMULATION AND CALCULATION

To analyze the performance of the above four algorithms for spot size detection, we used ZEMAX software to simulate the LAMOST optical path, which is shown in Figure 4. The ray tracing is used to obtain the theoretical spot size, and then the point spread function is used to obtain a defocused spot image shown in Figure 5.

Using ZEMAX's geometric dimension detection, including the 100% intensity range of the defocused spot as the spot size, we separately measure the size of the spot with a positive defocuses distance of 20 mm, 21 mm, 22 mm, 23 mm, 24 mm, 25 mm. The result is shown in Table 1.

The size measurement of the spot shows that the size of the spot is in an excellent linear relationship with the defocus distance. Because the size of the defocusing spot of the four algorithms is different, we use the linearity of the spot size of different distances to judge the pros and cons of the four spot size detection algorithms. The simulation image is subjected to size detection. The detection results of the four algorithms are given in Table 2. Table 3 shows the linearity of the detection results of the four algorithms. One algorithm has higher the linearity and the more stable the detection result than others, which detection accuracy is higher. Linear equations and standard error of four algorithms are shown in Table 4.

The simulation results show that the linearity of the average spacing algorithm is the best, the standard error of the slope and intercept is the smallest, the slope detection error is 0.1018, and the detection error of the intercept with the x coordinate axis is 0.25 pixel, the intercept of the y coordinate axis detection error is 2.45 pixel. The average spacing algorithm has a significant advantage in measuring the size of the defocused spot.

4 EXPERIMENT

The device used in the experiment is the LAMOST telescope which is the largest optical telescope in China. A general view is shown in Figure 6.

To evaluate the performance of the four algorithms mentioned above in the actual defocusing spot size detection, the guiding CCD on the focal plane of the LAMOST telescope is used to obtain a different defocused spot, which is consistent with the simulation. The defocus distance is 20 mm to 25 mm. Imaging was captured every 1 mm, and then the algorithm was evaluated by comparing the linearity of the defocusing spot size and the defocusing distance of the four different algorithms.

The brightness of different stars is different, resulting in different contrasts of the corresponding defocus spots, which can be used to test the influence of contrast to the four algorithms detection accuracy.

The 30-degree sky area was imaged near culmination, and the focal plane was defocused toward the direction of Ma. The defocus distance was 20, 21, 22, 23, 24 and 25 mm, respectively, and the exposure time was 60 s.

The defocusing spot size obtained by the LAMOST's eight guides CCDs were detected using four algorithms, and the results obtained by the eight detectors were similar. The image spot of the No.0 CCD is shown in Figure 7 and spot size detection value of four algorithms are shown in Figure 8.

We can see from Tables 5 and 6 that all four methods have excellent linearity, in which the linearity of the average spacing is the best, the peak spacing algorithms is second, and the Gaussian fitting algorithms is the worst. The experimental results are consistent with the results obtained by the simulation.

Four methods were used to measure the defocus spot size of different celestial, and the effect of contrast on the

| Equation | Residual Sum of Squares | Pearson's r | Adj. R-Square | Method |
|---------------|-------------------------|-------------|---------------|------------------|
| | 2.37817 | 0.99963 | 0.99907 | EFA |
| | 2.12506 | 0.99907 | 0.99768 | GFA |
| y = a + b * x | 1.67619 | 0.99934 | 0.99836 | PSA |
| | 0.82522 | 0.99975 | 0.99938 | ASA |
| | 6.70E-06 | 1 | 1 | Theoretical size |

Table 3 The Linearity of the Simulated Image Size by Four Algorithms



Fig. 6 LAMOST : a general review (Su & Cui 2004).



Fig.7 The left panel shows the defocus spot by guiding CCD of LAMOST; the right panel shows the defocus spot after binarization.

detection accuracy was obtained. The contrast of bright stars was 2.8, and the contrast of dark stars was 1.8. We take the data obtained by the camera at a defocus of 20 mm to 25 mm as an example. It can be seen that the average spacing algorithm is minimally affected by contrast, and peak spacing algorithm is most easy be affected by contrast. Defocus spot size of different brightness celestial can be controlled within 1 pixel by using average spacing algorithm. Figure 9 shows the results of different algorithms affected by contrast.

5 CONCLUSIONS

To calibrate the focal plane posture of the LAMOST telescope, the size of the defocused spot is detected based on

| Method | Value a and b | Value | Standard Error |
|------------------|-------------------|----------|----------------|
| EFA | Intercept | 0.94752 | 4.15913 |
| | Slope | 13.49563 | 0.18432 |
| GFA | Intercept | 4.11948 | 3.93158 |
| | Slope | 8.08697 | 0.17424 |
| PSA | Intercept | 0.95238 | 3.49175 |
| | Slope | 8.54286 | 0.15474 |
| ASA | Intercept | 11.35286 | 2.44999 |
| | Slope | 9.72383 | 0.10858 |
| Theoretical size | Intercept | 5.8139 | 0.00698 |
| | Slope | 9.18709 | 3.09E-04 |

 Table 4
 Linear Equations and Standard Error of Four Algorithms

Table 5 Linearity of Test Results of Four Detection Algorithms

| Equation | Residual Sum of Squares | Pearson's r | Adj. R-Square | Method |
|---------------|-------------------------|-------------|---------------|--------|
| y = a + b * x | 8.05758 | 0.99842 | 0.99605 | EFA |
| | 3.29794 | 0.99795 | 0.99488 | GFA |
| | 1.67619 | 0.99934 | 0.99836 | PSA |
| | 0.74965 | 0.99962 | 0.99904 | ASA |



Fig. 8 Defocus spot size measured by four detection algorithms.



Fig.9 Using four algorithms, we compare the spot size detection value of dark and bright celestial. It is obvious that the average spacing algorithm has the smallest difference.

| Method | Value a and b | Value | Standard Error |
|--------|-------------------|-----------|----------------|
| EFA | Intercept | -23.05308 | 7.65567 |
| | Slope | 12.05896 | 0.33928 |
| GFA | Intercept | 5.74942 | 4.89781 |
| | Slope | 6.77128 | 0.21706. |
| PSA | Intercept | 10.19048 | 4.19472 |
| | Slope | 6.22857 | 0.1859 |
| ASA | Intercept | 21.52029 | 2.33513 |
| | Slope | 7.46794 | 0.10349 |

Table 6 Linear Equations and Standard Error of Four Algorithms

the guiding CCD on the focus plane of the telescope. Four defocusing spot size detection algorithms were evaluated using the linearity of the defocusing distance, the spot size, and the spot detection results for stars of different brightness. The experimental and simulation results show that the average spacing algorithm that is proposed in this paper has good defocusing spot size detection accuracy.

The algorithm will be used in the focal plane attitude detection of the LAMOST telescope. The algorithm is useful for the detection of the defocused spot of other telescopes and the focal plane attitude detection of telescopes similar to LAMOST.

Acknowledgements The Guo Shou Jing Telescope (the Large Sky Area Multi-object Fiber Spectroscopic Telescope, LAMOST) is a national major scientific project built by Chinese Academy of Sciences. Funding for the project has been provided by the National Development and Reform Commission. LAMOST is operated and managed by National Astronomical Observatories, Chinese Academy of Sciences. The LAMOST data release web site is http://data.lamost.org/. This work was supported by the National Key Basic Research Program of China (2015CB857100) and the National Natural Science Foundation of China (Grant Nos. U1331204 and 14473050).

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