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The influence of the observational strategies of pulsar timing on the properties of pulsar clocks

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Abstract Pulsars are very stable spinning stars, which have the potential to application in the work of time-keeping and autonomous navigation in deep space. For time application, an individual pulsar can be regarded as a clock. The accuracy and stability of a pulsar clock are mainly determined by various timing noises and the measurement errors; however, they would be affected by the concrete observational strategy. Taking four millisecond pulsars from the first data released by International Pulsar Timing Array (IPTA) as an example, we investigated the influences of different observational strategies on the properties of pulsar clocks by removing some data in various ways. We find that the long-term stabilities of pulsar clocks are nearly not affected by increasing the observational cadence with a fixed time span. It is also found that the capabilities of prediction by pulsar clocks are also hardly affected by different observational strategies, which is reflected by both the stable weighted root-mean-square (wrms) and the stability of the resulting pre-fit timing residuals, unless the data span is too short or the data period is too far from the start of prediction.

Key words: methods: data analysis — time — pulsars: general

1 INTRODUCTION

Pulsars (PSRs) are a special class of stars because they not only form ideal natural laboratories in studying physics and astrophysics but also are very important for applications such as time metrology (Petit & Tavella 1996; Ilyasov et al. 1998; Hobbs et al. 2012) and pulsar navigation in deep space (Becker et al. 2018). The application in time metrology mainly relies on millisecond pulsars (MSPs), a class of pulsars who have shorter spin periods and more stable rotations. The stability of some MSPs can reach up to the order of magnitude of 10^{-15} or even better when the observational time span is as long as a decade. Hence the extraordinarily regular spin of MSPs make them a kind of clock, which have a completely different working mechanism from atomic clocks based on quantum transition. Similar to the atomic time scale accomplished by atomic clocks, pulsar time scale can also be accomplished by pulsar clocks. Moreover, the ensemble pulsar time scale (EPT), which has a higher stability compared to an individual pulsar clock, can be established by a group of MSPs with some reasonable algorithms (Rodin 2008). Due to the superiority of the high long-term stability, the EPT has the potential to be applied in the work of time-keeping. Pulsar clocks are based on the accuracy and precision of pulsar timing model parameters, which are obtained by measuring the times of arrival (ToAs) of pulses. However, in pulsar timing various noises prevent the enhancement of the measurement precision because the remote pulses have been inevitably disturbed during the transformation in the complicated paths before arriving the Earth. Even so, one can still obtain the ToAs with high precision with the help of a precise timing model, in which various transformations and corrections are included (Lorimer & Kramer 2005; Edwards et al. 2006; Hobbs et al. 2006). However, the low frequency red noises are hard to model, which is the most significant factor to affect the long-term stability of pulsar clocks. Under the effects of these noises, differences in ToA distribution density, time span, and so on would give some impacts on the properties of pulsar clocks. Hence, the influences from different observational strategies on pulsar clocks are needed to be investigated. In this paper, we focus on the influence of observational strategies on the properties of the individual pulsar clocks aiming at the construction of the EPT because the latter would be additionally depended on the concrete algorithms.

The International Pulsar Timing Array (IPTA) (Verbiest et al. 2016; Perera et al. 2019) now provides the longest public observational data of MSPs, and some MSPs have been observed by several radio telescopes. In this paper, we will choose four MSPs from the first released data by IPTA (IPTADR1)[1]. The most popular software for pulsar timing, tempo2[2], will be employed as a tool for data precessing and analysis.

2 PROPERTIES OF PULSAR CLOCKS

Accuracy and stability are the two most important indices describing the properties of a clock. For atomic clocks, the accuracy is evaluated by the conformance of the generated frequency to the frequency standard. However, there is no definite frequency standard for pulsar clocks, and the accuracy is reflected by the uncertainties of the timing model parameters in different ways (Tong et al. 2017). For stability, Allan variance (σ_y^2) or Hadamard variance (σ_H^2) is often used to evaluate manufactured frequency sources. However, they are unsuitable to evaluate the stability of pulsar clocks because the sampling of pulsar data is quite irregular. In 1997, Matsakis et al. (1997) proposed a new method, called the statistic $\sigma_z(\tau)$, to evaluate the stability

$$\sigma_z(\tau) = \frac{\tau^2}{2\sqrt{5}} \langle c_3^2 \rangle^{1/2}, \tag{1}$$

where c_3 is cubic coefficient of a cubic polynomial fitting to the post-fit timing residual data in each subsequence at a length τ, and "$\langle\rangle$" means the ensemble average over the subsequences weighted by the inverse squares of the formal errors in c_3. τ is set to be $T, T/2, T/4...$, T is the length of residuals (Matsakis et al. 1997). Taylor (1991) showed that this polynomial fitting has a same result with the third difference. As shown in Barnes et al. (1971); Matsakis et al. (1997), if the power spectral density of a time series can be written as

$$S_x(f) \propto f^{\alpha-2}, \tag{2}$$

then $\sigma_z^2(\tau)$ also follows a power-law form

$$\sigma_z^2(\tau) \propto \tau^{\mu}, \tag{3}$$

where

$$\mu = \begin{cases} -(\alpha+1) & \text{if } \alpha < 3, \\ -4 & \text{otherwise.} \end{cases} \tag{4}$$

[1] `http://www.ipta4gw.org`

[2] `http://www.atnf.csiro.au/research/pulsar/tempo2`

Here f is the frequency in Fourier space. It can be seen from Equations (3) and (4) that, $\sigma_z(\tau)$ will not be decreased with the increasing length τ once $\alpha \leq -1$. Thus, the red noises with the index $\alpha \leq -1$ of their power spectral densities in the timing data would give negative impacts on the long-term stability of pulsar clocks which is the most concern for a time scale. Meanwhile, the red noises could affect the pulsar timing processes, leading to the deviations of the timing model parameters from their true values (Coles et al. 2011), and in turn, the accuracy of the pulsar clocks will get worse. Since the establishment of the EPT will be dominated by some MSPs which have more precise timing properties and lower timing noises, to avoid complexities, below we only choose four MSPs as the targets to be studied. These four MSPs are PSRs J0437–4715, J1713+0747, J1744–1134 and J1909–3744, which are basically the most important candidates for the establishment of the EPT. The timing data of these four MSPs are taken from Combination 'B', a default "tempo2" form (Verbiest et al. 2016), and the basic information are listed in Table 1.

Apart from the effects of various timing noises on the properties of pulsar clocks, the concrete observational strategy could also give some impacts on pulsar clocks. For a simple case, the parameter uncertainties are just inversely proportional to the square root of the number of ToAs, if there is only white noise in the timing data. However, the red noises normally exist in the timing data for most MSPs, which makes the effect of observational strategies on the properties of pulsar clocks be more complicated.

3 THE EFFECTS OF OBSERVATIONAL STRATEGY ON THE ESTABLISHED PULSAR CLOCKS

To find the effects from observational strategy on the established pulsar clocks, we first do a data preprocessing by combining the ToA data from different Pulsar Timing Arrays (PTAs) included in IPTA together for each pulsar and ascending them according to the Modified Julian Date (MJD). One can easily obtain the timing model parameters and the corresponding post-fit timing residuals of the whole data set for each pulsar. After that, we evenly kick the ToAs from the combined data set with different percentages. This can be treated effectively as operating the observations with different cadences. Actually, we also tried the case of kicking the data unevenly, but one could get the similar result as the previous one. Note that the start and the end of the combined data should be reserved in order to hold the time span fixed. Finally, we fit all the kicked data sets and get the corresponding timing model parameters for each pulsar. During the fitting procedure of the kicked data sets, all the jumps should not be fitted

Table 1 Basic observational information of the four pulsars in this work. Values come from Verbiest et al. (2016).

Pulsar Name	Time Span (yr)	Residual rms (μs)	Number of ToAs	Average Cadence (d)
J0437–4715	14.9	0.3	5052	5.1
J1713+0747	21.2	0.3	19972	5.1
J1744–1134	17.0	1.1	2589	8.4
J1909–3744	10.8	0.2	4025	4.4

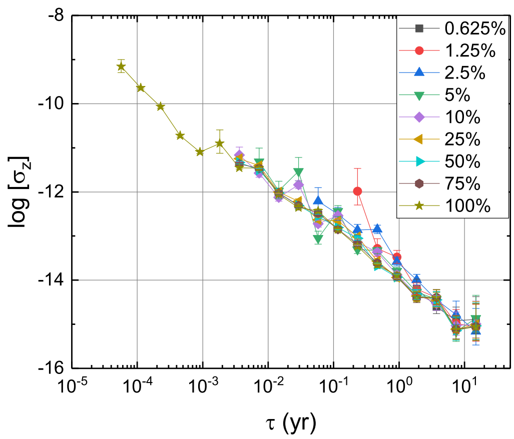

Fig. 1 Stabilities σ_z based on the post-fit timing residuals under different observational cadences for PSR J0437–4715. The legend gives the percentages of the left data compared to the whole data set.

because the jumps are only applicable for the whole data set.

Taking PSR J0437–4715 as an example, the stabilities based on different observational cadences are shown in Figure 1. It can be found that, even though the short-term stabilities cannot be estimated for sparse ToAs, the long-term stabilities that we concern do not show significant discrepancies for all the kicked data sets. A more detailed analysis shows that the values of σ_z do not change obviously while the data percentages $\gtrsim 10\%$. The situations are similar for the other three pulsars. Hence, pulsar clocks nearly do not lose stabilities if one decreases the observational cadence appropriately. Generally speaking, high cadence is preferred either in the construction of the EPT or the detection of gravitational waves. If the ToAs are very sparse, one can believe that it is difficult to detect the fluctuations of International Atomic Time (TAI) relative to Terrestrial Time (TT) published by the Bureau International des Poids et Mesures (BIPM) (Hobbs et al. 2012). We summarize the changes of the *wrms* timing residuals for the four pulsars in Table 2. It can be seen from Table 2, the changes of the *wrms* of the post-fit timing residuals for all the percentages larger than 10% are much smaller than those for all the different cadences that we employed.

4 THE EFFECTS OF OBSERVATIONAL STRATEGIES ON PREDICTIONS OF PULSAR CLOCKS

Pulsar time has the potential to apply in the area of autonomous navigation in deep space and the work of time-keeping. For example, a pulsar-aided clock by combining pulsar time and Galileo System Time (GST) has been proposed recently (Piriz et al. 2019). The aim of the aided clock is to achieve a high standard time scale in short-term and long-term. In the view of applications, the accuracy and stability of the predictions by pulsar clocks are the two important indices. Concretely, the influence factors contain the precision of the clock/timing model parameters and the level of timing noises. That is, a pulsar clock is described by both the clock model and the various noises. The properties of pulsar clocks can be learned by analyzing the pre-fit timing residuals, which actually stand for the clock differences between TT(BIPM) and pulsars.

To study the predictions of the pulsar clocks, we extract the last-year ToAs from the combined data set as the "future data" for examination and the left ToAs are used to construct different pulsar clocks with timing model parameters for different observational strategies. The pre-fit timing residuals can be used to correct the frequency of atomic clocks, it is necessary to calculate Allan deviation σ_y to evaluate the stability of predicted data by pulsar clocks. To do this, we use piecewise linear interpolation to make the pre-fit timing residuals equally spaced. For comparison, we also calculate σ_z of the pre-fit residuals that is similar to σ_H for atomic clocks. We first display the stabilities statistics of the one-year pre-fit timing residuals by using the longest data span for each pulsar in Figures 2 and 3. Although PSR J1713+0747 has the longest data span and largest number of ToAs, for most intervals the predicted properties are not better than the other pulsars.

4.1 Different Time Spans

First, we want to investigate the prediction properties of clock models constructed by different observational time spans. The shortest time span is chosen as one year which is closest to the one-year "future data". By repeating this process with the time span extending one year in turn till covering the whole data set, one can get a series of pre-fit timing residuals of the future one-year. We plotted the

Table 2 Comparisons of the *wrms* of the post-fit timing residuals for the four pulsars. Δ_1 in the second column denotes the maximum deviation compared to the whole data set for all the percentages larger than 10%. R is the *wrms* of the post-fit timing residuals for the whole data set of each pulsar. Δ_2 in the fourth column stands for the maximum deviation relative to R among all the different cadences that we employed.

Pulsar	$\Delta_1(\mu s)$	Δ_1/R	$\Delta_2(\mu s)$	Δ_2/R
J0437–4715	0.007	0.034	0.135	0.65
J1713+0747	0.006	0.025	0.031	0.13
J1744–1134	0.089	0.10	0.253	0.29
J1909–3744	0.005	0.028	0.125	0.69

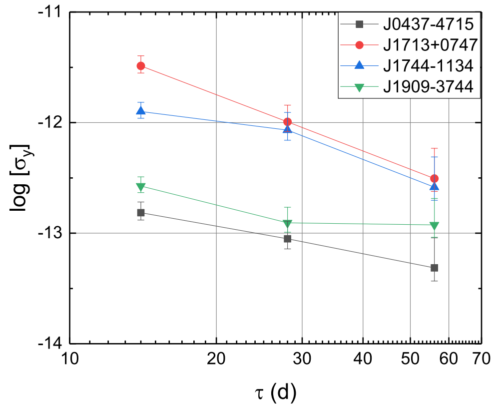

Fig. 2 Allan deviations of pre-fit residuals for the longest data span.

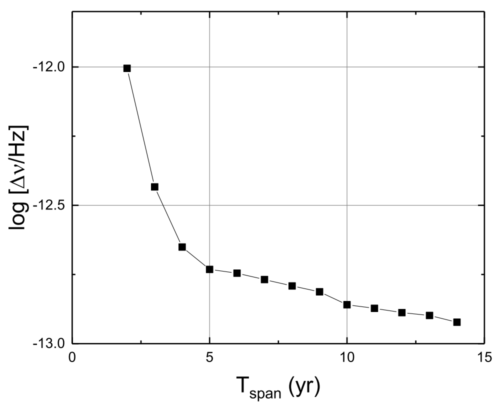

Fig. 4 The uncertainties of spin frequency for PSR J0437–4715 determined by different data spans.

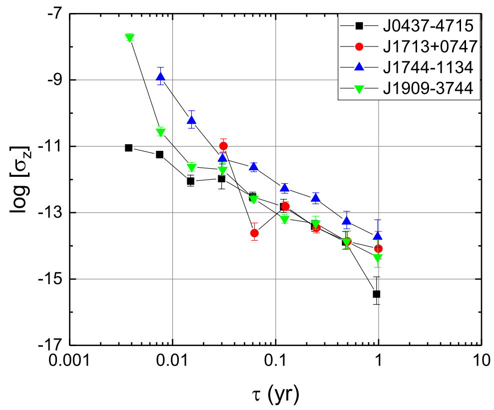

Fig. 3 σ_z of pre-fit residuals for the longest data span.

uncertainties of the spin frequency ($\Delta\nu$) of PSR J0437–4715 obtained with different time spans in Figure 4, where the first point corresponding to the time span equalling one year is not included for its big discrepancy. It can be seen that $\Delta\nu$ decays sharply as the more timing data available, and levels off when the available data exceed about 5 years. Moreover, we find that the uncertainties of other parameters have the similar properties for the other three pulsars.

Figure 5 displays the weighted root-mean-square (wrms) of the pre-fit timing residuals of the predicted one year data of the four pulsars with different available data spans. It can be seen that the wrms keeps relatively stable when the observational data span stretches over three years, except for PSR J1744–1134 exhibiting a little different. The stabilities σ_y and σ_z of the pre-fit residuals of PSR J0437–4715 are shown in Figure 6 and Figure 7, respectively. The common feature is that both σ_y and σ_z for various values of τ are nearly invariant when the data span over 2 years (Although the values of $\sigma_z(\tau = 1$ yr) show a decreasing trend when time span increases, this trend does not exist in the results of other three pulsars.).

We can then draw a conclusion from these analyses that, even though the uncertainties of model parameters can be decreased by prolonging the observational time span, the pre-fit timing residuals and stability statistics of the prediction cannot be improved significantly. It seems that, for the stability of the predictions of pulsar clocks, one does not need to accumulate too long timing data. It relaxes the requirement of pulsar observations for the construction of pulsar clocks. However, to guarantee the accuracy of pulsar clocks, longer timing data are anticipated.

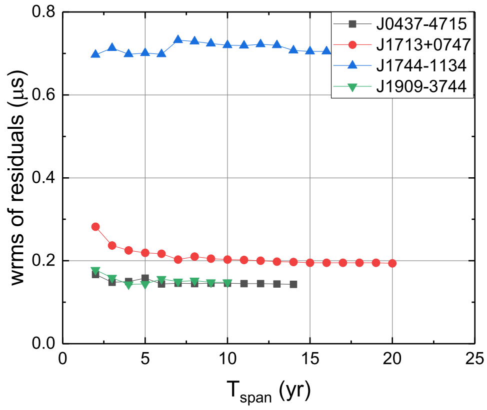

Fig. 5 The variable wrms of the pre-fit timing residuals for the four pulsars.

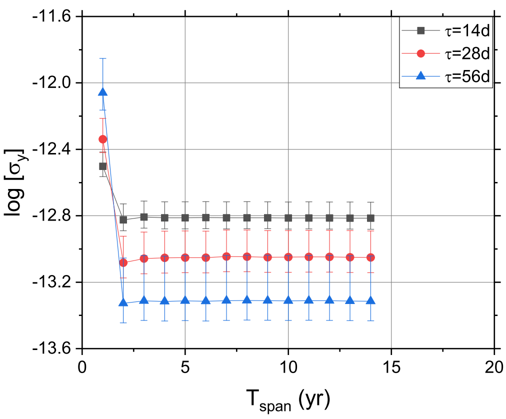

Fig. 6 The Allan deviation of pre-fit residuals for PSR J0437–4715. The legend marks the different values of τ (days).

4.2 Different Periods

In practical observations, there is a situation that the radio telescope often stops working some time accounting for device repair or upgrade. In this discontinuous case, we want to study the predicted abilities of the pulsar clocks, and estimate the accuracy lost due to the discontinuities. To investigate this, we first use the fractional data set closest to the "future data" with a fixed time span to construct the clock model, and employ it to make the one-year prediction. Then, we move the observational period away from the "future data" with a step of one year in turn, then many sets of clock models with the corresponding pre-fit timing residuals can be obtained. Note that, we assume that the DM models have been corrected with high precision for all the clock models, or else it would give rise to additional large-deviated predictions.

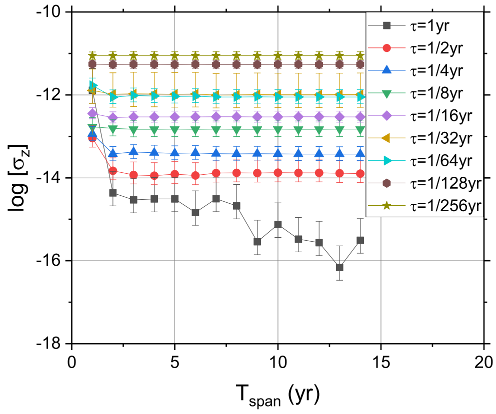

Fig. 7 σ_z of pre-fit residuals for PSR J0437–4715.

To give a comparison between different pulsars, we first set the fixed time span, $T_{\rm span}$, to be 5 yr, and the resulting wrms of the pre-fit timing residuals are displayed in Figure 8, where the symbol Δd of horizontal axis represents the time intervals from the end of the fixed five year to the start of the "future data". It is clearly that the wrms of the pre-fit timing residuals will become larger uniformly for all the four pulsars as Δd increases. To investigate the case for a longer fixed time span, we set it now to be 10 yr. Figure 9 shows the wrms of the pre-fit timing residuals of PSR J0437–4715 with the time span being 5 yr and 10 yr, respectively. It is interesting to find that the wrms of the pre-fit timing residuals exhibit quite stable as Δd increases for $T_{\rm span} = 10$ yr. The resulting σ_y and σ_z are shown in Figure 10 and Figure 11, respectively. In accord with Figure 9, σ_y and σ_z are nearly invariant as Δd increases for $T_{\rm span} = 10$ yr. It may be caused by the short data span of PSR J0437-4715. However, at least, it tells us that the clock model given by 10 yr data span is more stable than the case of 5 yr data span.

5 CONCLUSIONS

Since there are many objects to be observed by a radio telescope, the period left for pulsars would be very limited. Under this background, we investigated the influence of the observational strategies on pulsar timing aiming at the applications of pulsar clocks. We first estimated the stability of pulsar clocks with a fixed time span but different observation cadences, and found that it affects little the long-term stability which is often concerned. However, if the data are too sparse, then one can hardly estimate the short-term stability. It is worth pointing out that even though we reduced the data evenly, the result will be nearly the same if data are kicked unevenly. Therefore, in practical observations, one can decrease appropriately the observational density to establish the EPT without

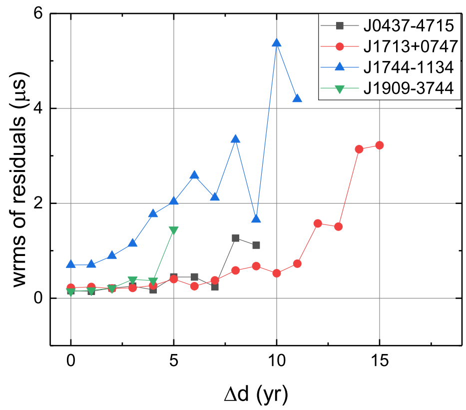

Fig. 8 Changes of the wrms of the pre-fit residuals with $T_{\rm span} = 5$ yr.

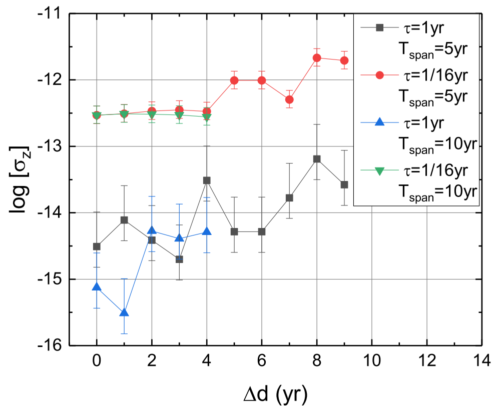

Fig. 11 Changes of σ_z of the pre-fit residuals of PSR J0437–4715 for $T_{\rm span} = 5$ yr and $T_{\rm span} = 10$ yr, respectively.

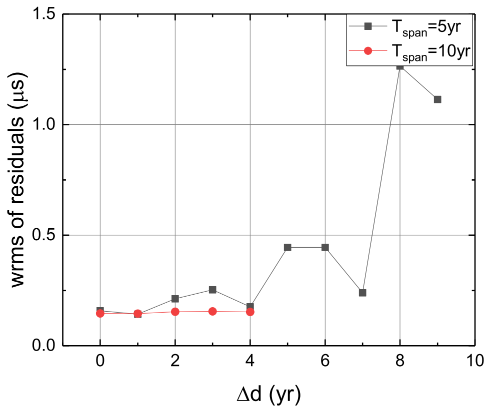

Fig. 9 Changes of the wrms of the pre-fit residuals of PSR J0437–4715 for $T_{\rm span} = 5$ yr and $T_{\rm span} = 10$ yr, respectively.

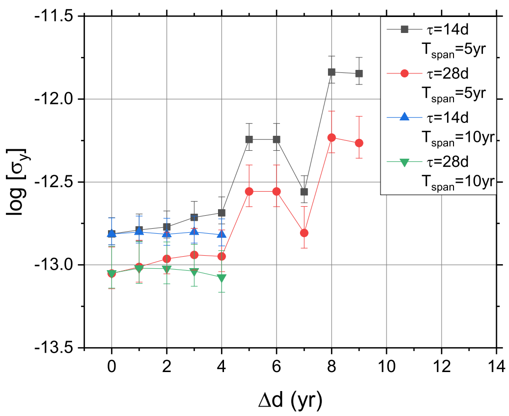

Fig. 10 Changes of Allan deviation of the pre-fit residuals of PSR J0437–4715 for $T_{\rm span} = 5$ yr and $T_{\rm span} = 10$ yr, respectively.

losing stability. However, if so, then the uncertainties of the timing model parameters will be larger.

From the point of view of the applications of pulsar clocks, we studied the predictions of the timing models obtained based on different observational strategies. It is found that, prolonging time span have limited influence on the stabilities of the one-year predictions by pulsar clocks if the timing data over two years are available to establish clock models. However, in the view of the accuracy of pulsar clocks, more timing data are preferred. The period where the timing data for fixed time span lies is another factor affecting the predictions by pulsar clocks. As is expected, the observational period closer to the start of the predictions gives a better result for whether the pre-fit timing residuals or the stabilities. However, if the time span is as long as 10 yr, then the differences of the predictions given by the data with different periods are very small.

Essentially, pulsar timing is affected by various timing noises, especially low-frequency red noises. If the timing noises can be suppressed to a negligible level, we believe the role of different observational strategies would be of less importance. However, the red noises exist generally in most millisecond pulsars, even for the best timed PSR J0437–4715 so far. Moreover. the red noises cannot be removed completely due to the complicated origins. Even though the uncertainties of timing model parameters will decrease with less dense observational data, the stabilities of the predicted clock are nearly not affected by reducing the observational cadence. This is different from the case of gravitational wave detection by PTAs, where more dense data are desirable.

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