RAA 2019 Vol. **19** No. **4**, 53(12pp) doi: 10.1088/1674–4527/19/4/53

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Research in Astronomy and Astrophysics

Field theory of the correlation function of mass density fluctuations for self-gravitating systems

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Received 2018 July 6; accepted 2018 October 16

Abstract The mass density distribution of Newtonian self-gravitating systems is studied analytically in the field theoretical method. Modeling the system as a fluid in hydrostatic equilibrium, we apply Schwinger's functional derivative on the average of the field equation of mass density, and obtain the field equation of 2-point correlation function $\xi(r)$ of the mass density fluctuation, which includes the next order of nonlinearity beyond the Gaussian approximation. The 3-point correlation occurs hierarchically in the equation, and is cut off by the Groth-Peebles ansatz, making it closed. We perform renormalization and write the equation with three nonlinear coefficients. The equation tells us that ξ depends on the point mass m and the Jeans wavelength scale λ_0, which are different for galaxies and clusters. Applying this to large scale structures, it predicts that the profile of ξ_{cc} for clusters is similar to ξ_{gg} for galaxies but with a higher amplitude, and that the correlation length increases with the mean separation between clusters, i.e., a scaling behavior $r_0 \simeq 0.4d$. The solution yields the galaxy correlation $\xi_{gg}(r) \simeq (r_0/r)^{1.7}$ valid only in a range $1 < r < 10\,h^{-1}$ Mpc. At larger scales the solution ξ_{gg} deviates below the power law and goes to zero around $\sim 50\,h^{-1}$ Mpc, just as the observations show. We also derive the field equation of the 3-point correlation function in the Gaussian approximation and its analytical solution, for which the Groth-Peebles ansatz with $Q = 1$ holds.

Key words: cosmology: theory — cosmology: large-scale structure of universe

1 INTRODUCTION

Understanding the matter distribution in the universe on large scales is one of the major goals of modern cosmology. The large scale structure is determined by self-gravity of matter. Since the number of galaxies as typical objects is enormous, one needs statistics to study the distribution. In this regard, the 2-point correlation function $\xi_{gg}(r)$ of galaxies and $\xi_{cc}(r)$ of clusters serves as a powerful tool (Bok 1934; Totsuji & Kihara 1969; Peebles 1980). It not only provides the statistical information, but also contains the underlying dynamics due to gravity. Observational surveys have been carried out for galaxies and for clusters, such as the Automatic Plate Measuring (APM) galaxy survey (Loveday et al. 1996), the Two-degree-Field Galaxy Redshift Survey (2dFGRS) (Peacock et al. 2001), Sloan Digital Sky Survey (SDSS) (Abazajian et al. 2009), etc. All these surveys suggest that the correlation of galaxies has a power law form $\xi_{gg}(r) \propto (r_0/r)^{\gamma}$ with $r_0 \sim 5.4\,h^{-1}$ Mpc and $\gamma \sim 1.7$ in a range $(0.1 \sim 10)\,h^{-1}$ Mpc (Totsuji & Kihara 1969; Groth & Peebles 1977; Peebles 1980; Groth & Peebles 1986; Soneira & Peebles 1978). The correlation of clusters is found to have a similar form: $\xi_{cc}(r) \sim 20\xi_{gg}(r)$ in a range $(5 \sim 60)\,h^{-1}$ Mpc, with an amplified magnitude (Bahcall & Soneira 1983; Klypin & Kopylov 1983). For quasars $\xi_{qq}(r) \sim 5\xi_{gg}(r)$ (Shaver 1988).

On the theoretical side, numerical computation is the most used method and significant progress has been made in study of the large scale structure. On the other hand, analytical studies are also important in understanding the physical mechanism underlying the clustering. Davis & Peebles (1977) used the BBKGY method and got five equations for five unknown functions, among which the equation of the 2-point correlation was not in a closed form. Bernardeau et al. (2002); Crocce & Scoccimarro (2006b,a) developed a perturbation theory for density field and analyzed the nonlinear propagator. Matarrese et al. (1998); Wang & Zhang (2017); Zhang et al. (2017) gave the analytical solutions of 2nd-order density contrast and metric perturbations during the matter-dominant stage, but the equation of correlation was not given. Saslaw (1968, 1969, 1985, 2000) used macroscopic thermodynamic variables, such as internal energy, entropy, pressure, etc, for adequate descriptions, whereby the power-law form of $\xi_{gg}(r)$ was used to calculate modifications to energy and pressure. Similarly, de Vega et al. (1996b, 1998) used the grand partition function of a self-gravitating gas to study a possible fractal structure of the correlation function of galaxies. However, the field equation of ξ has not so far been given in all these studies.

In Zhang (2007), we studied the mass density distribution of self-gravitating systems in hydrostatic equilibrium by a field-theoretical method. The starting point is the field equation of the mass density field ψ. We expand the density field as $\psi = \psi_0 + \delta\psi$, where $\psi_0 = \langle\psi\rangle$ is the mean density and $\delta\psi$ is the fluctuation field. We employ the technique of the generating functional $Z[J]$ as a path integral over the field ψ, where J is the external source. The connected 2-point correlation function is $G^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \langle\delta\psi(\boldsymbol{r}_1)\delta\psi(\boldsymbol{r}_2)\rangle = \delta^2 \ln Z[J]/\delta J(\boldsymbol{r}_1)\delta J(\boldsymbol{r}_2)$, also denoted by $\xi(r)$ with $r = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$ (Zhang 2007; Zhang & Miao 2009). By taking the functional derivative $\delta/\delta J$ of the equation of ψ_0, the field equation of $G^{(2)}$ was derived in Gaussian approximation with nonlinear terms of $\delta\psi$ being neglected. The analytic solution of the correlation function contains the Jeans wavelength as the unique scale of self-gravitating systems, and, as a prominent property, the amplitude of the correlation is proportional to the mass of the particle. This feature naturally explains the observational fact that clusters have a correlation amplitude higher than galaxies, and, similarly, richer clusters have a correlation amplitude higher than poor ones. When applying this to large scale structure, the solution agreed qualitatively with the observed correlation of galaxies and of clusters, however, at small scales of $r < 3\,h^{-1}$ Mpc, the correlation is too low to account for what is observed.

To improve the Gaussian approximation, Zhang & Miao (2009) considered nonlinear terms of density fluctuation to the order of $(\delta\psi)^2$ and gave the nonlinear equation of $G^{(2)}$. Due to hierarchy, the equation contains the 3-point correlation $G^{(3)}$, which can be expressed as the products of $G^{(2)}$ by the Kirkwood-Groth-Peebles ansatz (Kirkwood 1935; Groth & Peebles 1977) leading to the closed equation of $G^{(2)}$. After necessary renormalization to absorb the quantities like $G^{(2)}(0)$, a nonlinear equation is obtained. The correlation is enhanced at small scales $r = (0.3 \sim 3)\,h^{-1}$ Mpc, substantially improving the Gaussian result. However the treatment is not complete, as a nonlinear term is not properly included.

This paper extends the previous preliminary work (Zhang 2007; Zhang & Miao 2009) with a complete treatment of all terms $(\delta\psi)^2$, and presents the detailed derivation of the field equation of $G^{(2)}$ and the renormalization procedure. In addition, this paper also presents the field equation of 3-point correlation $G^{(3)}$ in the Gaussian approximation. These will complete the work of Zhang & Chen (2015), which listed only the brief results on $G^{(2)}$ without details. With one set of fixed values of nonlinear coefficients, the solution $G^{(2)}$ of the resulting field equation will confront the observational data of both galaxies and clusters.

In Section 2, we derive the field equation of the density field ψ by hydrostatics, and write down the generating functional $Z[J]$. Section 3 outlines the derivation of the nonlinear field equation of $\xi(r)$, using the functional derivative technique. Section 4 gives the main predictions by the equation on the properties of clustering.

In Section 5, we present the solution $\xi(r)$ to confront with observations of galaxies, and also compare this with numerical simulations. In addition, we give the projected correlation function. In Section 6, we apply the same solution to the system of clusters with greater mass m. Section 7 gives the 3-point correlation function $G^{(3)}$ in the Gaussian approximation. Section 8 contains conclusions and discussions. Appendix A gives the functional $Z[J]$ of the many-body self-gravitating system in terms of path integral over the gravitational field. Appendix B presents the details of the derivation of the field equation of $\xi(r)$, including the use of Kirkwood-Groth-Peebles ansatz and the renormalization procedure. Appendix C gives the derivation of the field equation of $G^{(3)}$ with the Gaussian approximation (see Appendix A, B and C at `http://www.raa-journal.org/docs/Supp/ms4263Appendix.pdf`).

We use a unit in which the speed of light $c = 1$ and the Boltzmann constant $k_B = 1$.

2 FIELD EQUATION OF MASS DENSITY FOR A SELF-GRAVITATING SYSTEM

Galaxies, or clusters, distributed in the universe can be approximately described as a fluid at rest in the gravitational field, i.e., as self-gravitating hydrostatics. This modeling is an approximation since the cosmic expansion is not considered. The system of galaxies in the expanding universe is in an asymptotically relaxed state, i.e., a quasi thermal equilibrium (Saslaw 2000). In this paper, under the approximation of hydrostatic equilibrium, we study the system of galaxies within a small redshift range. Let us examine how far this approximation is from the actual situation. The time scale of the cosmic expansion is $t_e \equiv 1/H_0 = \sqrt{3/8\pi G\rho_0}$, and the dynamical time for galaxies moving in the background is $t_d \sim \sqrt{3/16\pi G\rho_0}$ (Binney & Tremaine 1987), and the two time scales are roughly of the same order of magnitude, so the hydrostatic equilibrium is not a bad approximation, as will be demonstrated further in Section 5.

In general, a fluid is described by the continuity equation, the Euler equation and the Poisson equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla p + \nabla \Phi, \tag{2}$$

$$\nabla^2 \Phi = -4\pi G \rho. \tag{3}$$

For the hydrostatic case, $\dot{\rho} = 0$ and $\boldsymbol{v} = 0$, the Euler equation takes the form (Landau & Lifshitz 1987)

$$\frac{1}{\rho}\nabla \rho = \frac{1}{c_s^2}\nabla \Phi, \tag{4}$$

with $c_s^2 \equiv \partial p/\partial \rho$ being a constant sound speed, which describes the mechanical equilibrium of the fluid. Taking the gradient on both sides of this equation and using Equations (3) and (4) lead to

$$\nabla^2 \rho - \frac{1}{\rho}(\nabla \rho)^2 + \frac{4\pi G}{c_s^2}\rho^2 = 0. \tag{5}$$

We call Equation (5) the field equation of mass density for the self-gravitating fluid system. For convenience, we introduce a dimensionless density field $\psi(\boldsymbol{r}) \equiv \rho(\boldsymbol{r})/\rho_0$, where $\rho_0 = m n_0$ is the mean mass density of the system. Then Equation (5) takes the form

$$\nabla^2 \psi - \frac{1}{\psi}(\nabla \psi)^2 + k_J^2 \psi^2 = 0, \tag{6}$$

with $k_J \equiv \sqrt{4\pi G\rho_0}/c_s$ being the Jeans wavenumber. This is highly nonlinear in ψ as it contains $1/\psi$. Equation (6) also follows from $\delta \mathcal{H}(\psi)/\delta\psi = 0$ with the effective Hamiltonian density

$$\mathcal{H}(\psi) = \frac{1}{2}\left(\frac{\nabla \psi}{\psi}\right)^2 - k_J^2 \psi. \tag{7}$$

To employ Schwinger's technique of functional derivatives (Schwinger 1951a,b), an external source $J(\boldsymbol{r})$ is introduced to couple with the field ψ

$$\mathcal{H}(\psi, J) = \frac{1}{2}\left(\frac{\nabla \psi}{\psi}\right)^2 - k_J^2 \psi - J\psi, \tag{8}$$

and the mass density field equation in the presence of J is

$$\frac{1}{\psi^2}\nabla^2 \psi - \frac{1}{\psi^3}(\nabla \psi)^2 + k_J^2 + J = 0.$$

When $\psi \neq 0$ and $\psi \neq \infty$, one has

$$\nabla^2 \psi - \frac{1}{\psi}(\nabla \psi)^2 + k_J^2 \psi^2 + J\psi^2 = 0. \tag{9}$$

This is the starting equation which we shall use to derive the field equation of 2-point correlation function $G^{(2)}(r)$. When $\psi \neq 0$ and $\psi \neq \infty$, the generating functional for the correlation functions of ψ is defined as

$$Z[J] = \int D\psi e^{-\alpha \int d^3\boldsymbol{r} \mathcal{H}(\psi, J)}, \tag{10}$$

where $\alpha \equiv c_s^2/4\pi G m$ with c_s being the sound speed and m being the mass of a single particle. It is known that, by coarse-graining procedures (Baumann et al. 2012), integrating out short-wavelength nonlinear cosmological perturbations introduces additional effects into the dynamics on large scales, and yields an effective fluid characterized by a few parameters, such as pressure, viscosity and sound speed.

By setting $\psi(\boldsymbol{r}) \equiv e^{\phi(\boldsymbol{r})}$, where $\phi \equiv \Phi/c_s^2$ is the gravitational potential, Equations (6) and (7) can also be transformed into the well-known Lane-Emden equation (Emden 1907; Ebert 1955; Bonnor 1956; Antonov 1962; Lynden-Bell & Wood 1968)

$$\nabla^2 \phi + k_J^2 e^{\phi} = 0, \tag{11}$$

and the associated Hamiltonian

$$\mathcal{H}(\phi) = \frac{1}{2}(\nabla \phi)^2 - k_J^2 e^{\phi}. \tag{12}$$

In fact, Equation (12) derived from a hydrostatic model can also be derived using the following approach. A universe filled with galaxies and clusters can be modeled as a self gravitating gas assumed to be in thermal quasi-equilibrium (Saslaw 1985, 2000). Note that the universe is expanding with a time scale $\sim 1/H_0 = (3/8\pi G\rho_0)^{1/2}$, and the time scale of propagation of fluctuations $\sim \lambda_J/c_s \sim 1/(4\pi G\rho_0)^{1/2}$, with both being of the same order of magnitude. The thermal equilibrium is an approximation. For such a system of N particles of mass m, the Hamiltonian is

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} - \sum_{i<j}^{N} \frac{Gm^2}{r_{ij}} \tag{13}$$

with $r_{ij} = |\boldsymbol{r}_i - \boldsymbol{r}_j|$, and the grand partition function is

$$Z = \sum_{N=0}^{\infty} \frac{z^N}{N!} \int \prod_{i=1}^{N} \frac{d^3p_i \, d^3r_i}{(2\pi)^3} e^{-H/T}, \quad (14)$$

where z is the fugacity. As shown in Appendix A, this can be written as a path integral over the field ϕ

$$Z = \int D\phi \, e^{-\alpha \int d^3r \mathcal{H}(\phi)}, \quad (15)$$

where $\mathcal{H}(\phi)$ is given in Equation (12) with $c_s^2 = T/m$. Thus, the hydrodynamic model gives the same result as a self-gravitating gas in thermal equilibrium. In this paper, we shall use Equations (6) and (7), which better suits studying the mass distribution.

3 FIELD EQUATION OF THE 2-POINT CORRELATION FUNCTION OF DENSITY FLUCTUATIONS

In this section we outline the field equation of the 2-point correlation function of density fluctuations, and the comprehensive details are attached in Appendix B. Consider the fluctuation field $\delta\psi(\boldsymbol{r}) \equiv \psi(\boldsymbol{r}) - \langle\psi(\boldsymbol{r})\rangle$, with mean

$$\langle\psi(\boldsymbol{r})\rangle = \frac{1}{Z} \int D\psi \, \psi e^{-\alpha \int d^3\boldsymbol{r} \mathcal{H}(\psi)} = \frac{\delta \log Z[J]}{\alpha \delta J(\boldsymbol{r})} \, |_{J=0}, \quad (16)$$

where one sets $J = 0$ after taking the functional derivative. In a general system, the mean density $\langle\psi(\boldsymbol{r})\rangle$ can vary in space, but for the homogeneous and isotropic universe it is a constant $\langle\psi(\boldsymbol{r})\rangle = \psi_0$. The 2-point correlation function of $\delta\psi$, i.e., the *connected* 2-point Green function, is given by the functional derivative of $\log Z[J]$ with respect to J (Binney et al. 1992)

$$\begin{aligned} G^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2) &\equiv \langle\delta\psi(\boldsymbol{r}_1)\delta\psi(\boldsymbol{r}_2)\rangle \\ &= \alpha^{-2} \frac{\delta^2}{\delta J(\boldsymbol{r}_1)\delta J(\boldsymbol{r}_2)} \log Z[J]|_{J=0} \\ &= \alpha^{-1} \frac{\delta\langle\psi(\boldsymbol{r}_2)\rangle_J}{\delta J(\boldsymbol{r}_1)} \, |_{J=0} \, , \end{aligned} \quad (17)$$

where $\langle\psi(\boldsymbol{r})\rangle_J \equiv \frac{\delta}{\alpha\delta J(\boldsymbol{r})} \log Z[J]$ before setting $J = 0$. One can take $G^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2) = G^{(2)}(r_{12})$ for a homogeneous and isotropic universe. Analogously, the n-point correlation function of $\delta\psi$ is

$$\begin{aligned} G^{(n)}(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_n) &\equiv \langle\delta\psi(\boldsymbol{r}_1) \cdots \delta\psi(\boldsymbol{r}_n)\rangle \\ &= \alpha^{-n} \frac{\delta^n \log Z[J]}{\delta J(\boldsymbol{r}_1) \cdots \delta J(\boldsymbol{r}_n)} |_{J=0} \\ &= \alpha^{-(n-1)} \frac{\delta^{n-1} \langle\psi(\boldsymbol{r}_n)\rangle_J}{\delta J(\boldsymbol{r}_1) \cdots \delta J(\boldsymbol{r}_{n-1})} |_{J=0} \end{aligned} \quad (18)$$

for $n \geq 3$. To derive the field equation of $G^{(2)}(r)$, as is routine (Goldenfeld 1992), one takes the functional derivative of the ensemble average of Equation (9) with respect to $J(\boldsymbol{r}')$,

$$\begin{aligned} \frac{\delta}{\delta J(\boldsymbol{r}')} \Big(&\langle\nabla^2\psi(\boldsymbol{r})\rangle_J - \langle\frac{1}{\psi(\boldsymbol{r})}(\nabla\psi(\boldsymbol{r}))^2\rangle_J \\ &+ k_J^2 \langle\psi(\boldsymbol{r})^2\rangle_J + J(\boldsymbol{r})\langle\psi(\boldsymbol{r})^2\rangle_J \Big) = 0 \, , \end{aligned} \quad (19)$$

and then sets $J = 0$. To systematically deal with the nonlinearity of $1/\psi$, we expand ψ in terms of the fluctuation $\delta\psi$, up to second order $(\delta\psi)^2$. Then Equation (19) leads to the following equation for $G^{(2)}$

$$\begin{aligned} &\nabla^2 G^{(2)}(\boldsymbol{r}) + k_0^2 \psi_0 G^{(2)}(\boldsymbol{r}) + \frac{1}{2\psi_0^2} \nabla^2 G^{(2)}(0) G^{(2)}(\boldsymbol{r}) \\ &- \Big(\frac{1}{2\psi_0}\nabla^2 + k_J^2\Big) G^{(3)}(0, \boldsymbol{r}, \boldsymbol{r}) + \frac{2}{\psi_0^2} \nabla G^{(2)}(0) \cdot \nabla G^{(2)}(\boldsymbol{r}) \\ &= -\frac{1}{\alpha} \big(\psi_0^2 - G^{(2)}(0)\big) \delta^{(3)}(\boldsymbol{r}) \, , \end{aligned} \quad (20)$$

where the characteristic wavenumber $k_0 \equiv \sqrt{2} k_J$. (See Appendix B for detailed calculations.) This equation is of the same form as equation (4) in our previous paper (Zhang & Miao 2009), except that we now keep $-k_J^2 G^{(3)}$ on the right hand side and $\frac{1}{\alpha} G^{(2)}(0)\delta^{(3)}(\boldsymbol{r})$ in the source on the left hand side. These new terms come from an improved treatment which properly includes high order contributions. Note that $G^{(3)}$ occurs in Equation (20). There are various ways to cut off this hierarchy. One of them is to use the Kirkwood-Groth-Peebles ansatz (Kirkwood 1935; Groth & Peebles 1977)

$$\begin{aligned} G^{(3)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = Q \Big(&G^{(2)}(r_{12}) G^{(2)}(r_{23}) \\ &+ G^{(2)}(r_{23}) G^{(2)}(r_{31}) + G^{(2)}(r_{31}) G^{(2)}(r_{12}) \Big) \, , \end{aligned} \quad (21)$$

where Q is a dimensionless parameter. The observational data of galaxy surveys have indicated that $Q \simeq 1 \pm 0.2$, which is also supported by numerical simulations (Peebles 1993). Here we adopt this ansatz. Substituting Equation (21) into Equation (20), after a necessary renormalization to absorb the quantities like $G^{(2)}(0)$, $\nabla G^{(2)}(0)$ and $\nabla^2 G^{(2)}(0)$, we obtain the field equation of the 2-point correlation function

$$\begin{aligned} &(1 - b\xi)\nabla^2\xi + k_0^2(1 - c\xi)\xi + (\boldsymbol{a} - b\nabla\xi) \cdot \nabla\xi \\ &= -\frac{1}{\alpha} \delta^{(3)}(\boldsymbol{r}) \, , \end{aligned} \quad (22)$$

where $\xi(r) \equiv G^{(2)}(\boldsymbol{r})$, $\boldsymbol{a}$, b and c are three constant parameters. The special case of $\boldsymbol{a} = b = c = 0$ is the Gaussian approximation and Equation (22) reduces to the Helmholtz equation (B.9) Thus, the terms $\boldsymbol{a}$, b and c represent the nonlinear contributions beyond the Gaussian approximation. Equation (22) in the radial direction is

$$(1-b\xi)\xi'' + ((1-b\xi)\frac{2}{x} + a)\xi' + \xi - b\xi'^{\,2} - c\xi^2 = -\frac{1}{\alpha}\frac{\delta(x)k_0}{x^2}\,, \tag{23}$$

where $\xi' \equiv \frac{d}{dx}\xi$, $x \equiv k_0 r$ and $k_0 = (8\pi G m n)^{1/2}/c_s$. The effects of nonlinear terms b and c can enhance the amplitude of ξ at small scales and increase the correlation length. The term a plays the role of effective viscosity, and a greater a leads to a strong damping of the oscillations of ξ at large scales, as shown in Figure 1. The value of a should be large enough to ensure $1+\xi(r) \geq 0$ for the whole range $0 < r < \infty$.

Taking the normalization that the nonlinear solution ξ and the Gaussian solution $\xi_{\rm gauss}$ (Zhang 2007) are equal ($\xi/\xi_{\rm gauss} = 1$) at $r = 10\,h^{-1}\,$Mpc as in Figure 2, the nonlinear terms enhance as follows

$$\xi/\xi_{\rm gauss} \sim 1.6 \ \text{ at } \ r = 5\,h^{-1}\,\text{Mpc},$$
$$\xi/\xi_{\rm gauss} \sim 3 \quad \text{at } \ r \leq 1\,h^{-1}\,\text{Mpc}.$$

Since the terms a, b, c reflect the nonlinearity of theory and are universal to all specific systems, we use the same set of values for (a, b, c) to account for both galaxies and clusters. The solution is not very sensitive to the values of parameters in a range of $a \sim (1 \sim 3)$, $b \sim (0.001 \sim 0.008)$, $c \sim (0.1 \sim 0.7)$ in confronting the observational data of galaxies and clusters. The solution is sensitive to the boundary values of amplitude ξ and slope ξ'. At a fixed set (a, b, c), we take $k_0 = 0.055\,h\,\text{Mpc}^{-1}$ and tune the boundary amplitude and slope to fit various survey samples of galaxies, and then we take $k_0 = 0.03\,h$ Mpc^{-1} and fit samples of clusters.

The solution $\xi(r)$ will confront the observational data of galaxies and clusters in Sections 5 and 6.

4 GENERAL PREDICTIONS OF FIELD EQUATION

Inspection of Equation (22) already reveals its predictions of the important properties of correlation.

(1) Equation (22) can apply to the system of galaxies, as well as to the system of clusters, and the only difference is their respective m and k_0 contained in the equation. Thus, the solutions of Equation (22) for galaxies have a profile similar to that for clusters. This explains the observational fact that the correlation functions of galaxies and of clusters have the same power-law form, $\xi_{gg}, \xi_{cc} \propto r^{-1.8}$, but different amplitudes and ranges (Bahcall & Soneira 1983; Klypin & Kopylov 1983).

(2) The $\delta^{(3)}(\boldsymbol{r})$ source of Equation (22) is proportional to $1/\alpha = 4\pi G m/c_s^2$, which determines the overall amplitude of a solution ξ. In fitting with observational data, the parameter $1/\alpha$ corresponds to the amplitude of ξ at some r as the boundary value. As it turns out, the resulting c_s is roughly of order $10^2 \sim 10^3\,\text{km}\,\text{s}^{-1}$, which is comparable to the peculiar velocity of galaxies (Hawkins et al. 2003; Masters et al. 2006). Therefore, $1/\alpha$ is essentially determined by m, and a greater m will yield a higher amplitude

$$\xi(r) \propto m. \tag{24}$$

This general prediction naturally explains a whole chain of observed facts: luminous galaxies are more massive and have a higher correlation amplitude than ordinary galaxies (Zehavi et al. 2005), clusters are much more massive and have a much higher correlation than galaxies and rich clusters have a higher correlation than poor clusters (Bahcall & Soneira 1983; Kaiser 1984; Bahcall 1996; Einasto et al. 2002; Bahcall et al. 2003).

(3) The power spectrum, as the Fourier transform of $\xi(r)$, is proportional to the inverse of the spatial number density

$$P(k) \propto 1/n_0, \tag{25}$$

(see Eq. (B.13)). The observed $P(k)$ of clusters is much higher than that of galaxies, which is explained by Equation (25) as n_0 of clusters being much lower than that of galaxies (Bahcall 1996). Since a greater m implies a lower n_0 for a given mean mass density $\rho_0 = m n_0$, the properties (24) and (25) reflect the same physical law of clustering from different perspectives.

(4) The characteristic length

$$\lambda_0 = 2\pi/k_0 = \left(\frac{\pi}{2}\right)^{1/2}\frac{c_s}{\sqrt{G\rho_0}} \propto \frac{c_s}{\sqrt{\rho_0}}$$

appears in Equation (22) as the only scale and underlies the scale-related features of the solution $\xi(r)$. Observations reveal that clusters have a longer "correlation length" than galaxies. This can be explained by the following. At a fixed λ_0, $\xi_{cc}(r)$ has a higher amplitude and drops to its first zero at a larger distance, leading to an apparently longer "correlation length" than $\xi_{gg}(r)$. Another possibility may also contribute to this effect: if ρ_0 of the region covered by cluster surveys is

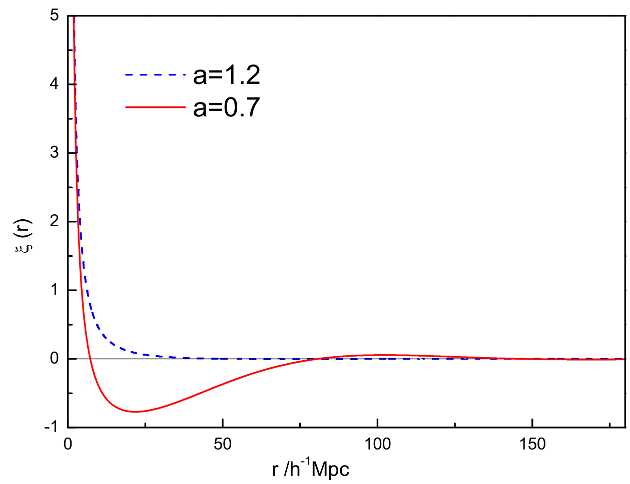

Fig. 1 A large viscosity coefficient a will cause strong damping on the oscillations of $\xi(r)$ at large distances. In this graph, b, c and k_0 are fixed for demonstration.

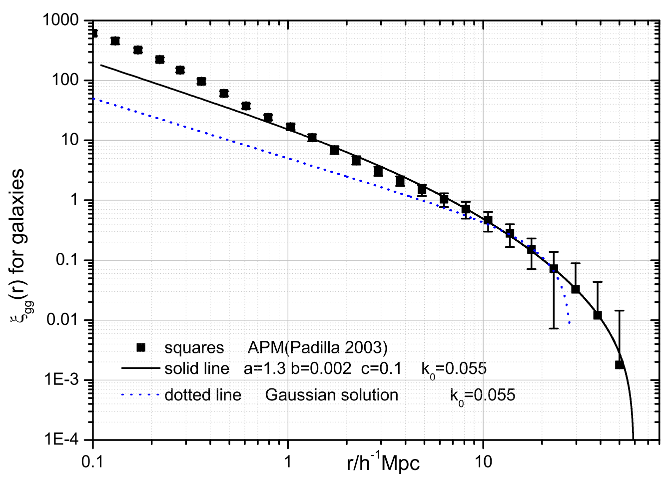

Fig. 2 Comparison of the nonlinear solution with the Gaussian as a demonstration.

lower than that of galaxy surveys, λ_0 for clusters will be longer accordingly. As will be seen in the following Sections 5 and 6, to use the solution to match the data of both galaxies and clusters, one has to take a longer λ_0 for clusters than for galaxies (Collins et al. 2000; Bahcall & Soneira 1983).

5 APPLYING TO GALAXIES

Now we give the solution $\xi_{gg}(r)$ of Equation (23) for a fixed set of parameters (a, b, c), and confront the observed correlation from major galaxy surveys. We choose $\xi_{gg}(r_0) > 0$ and $\xi'_{gg}(r_0) < 0$ as the boundary condition at a certain point $r_0 \sim 0.1\,h^{-1}$Mpc, corresponding to $\cos(k_0 r)/r$ in Equation (B.10) for the Gaussian case (Zhang 2007). This choice is similar to the choice of the adiabatic mode in the initial condition of cosmic microwave background (CMB) anisotropies (Hu & Sugiyama 1995). We shall also convert $\xi_{gg}(r)$ into its associated projected correlation function $w_p(r_p)$ simultaneously.

First, we consider the correlation function $\xi_{gg}(r)$. For demonstration, we take the parameters $(a, b, c) = (1.2, 0.003, 0.1)$, though other values of (a, b, c) can be also chosen. Figure 3 (left) shows the solution $\xi_{gg}(r)$ with $k_0 = 0.055\,h\,\text{Mpc}^{-1}$ and the observed $\xi_{gg}(r)$ by the galaxy surveys of 2dFGRS (Hawkins et al. 2003) and SDSS (Zehavi et al. 2005) with a median $z \sim 0.1$.

Figure 3 (right) also shows the data of SDSS Data Release 9 (DR9) with a median redshift $z \sim 0.53$, which we have converted from the data of the projected correlation function in Nuza et al. (2013). Since the samples in the right and left have different z corresponding to different evolutionary stages, we have accordingly chosen two different values k_0 and boundary conditions $\xi_{gg}(r_b)$ and $\xi'_{gg}(r_b)$ to compare with the data. It is seen that the theoretical $\xi_{gg}(r)$ matches the observational data. The power law $\xi_{gg} \propto r^{-1.7}$ is valid only in $r = (0.1 \sim 10)\, h^{-1}\,$Mpc, and deviates from both data and solution on large scales. Moreover, the solution predicts that $\xi_{gg}(r)$ decreases to zero and becomes negative around $\sim 70\, h^{-1}\,$Mpc, where data are not available currently. On small scales $r \leq 1\, h^{-1}\,$Mpc, the solution improves the Gaussian approximation (Zhang 2007), but is still lower than the data. This insufficient clustering at $r \leq 1\, h^{-1}\,$Mpc is possibly due to neglect of high-order nonlinear terms $(\delta\psi)^3$ in perturbations. Note that Equation (22) has been derived assuming $\delta\psi < 1$, and extrapolating the solution ξ_{gg} down to smaller scales is only an approximation.

Here we reexamine the assumption of hydrostatic equilibrium in the quasi-linear regime in the expanding universe. As is known, in the quasi-linear regime, the density fluctuation $\delta\psi \propto a(t)^{0.3}$ approximately, where $a(t)$ is the scale factor in the present stage of accelerating expansion. So, the time-evolving correlation function $\xi_{gg}(r,t) = \langle \delta\psi\delta\psi \rangle \propto a^{0.6}(t) = 1/(1+z)^{0.6}$. We have used the static calculated $\xi_{gg}(r)$ to compare with the observed correlation function $\xi_{gg}(r,t)$ in an expanding background. Let us estimate the errors in doing this. Take the static $\xi_{gg}(r)$ to correspond to the observed $\xi_{gg}(r, z = 0)$. Within the quasi-linear regime, the ratio $\xi_{gg}(r)/\xi_{gg}(r,t) \simeq (1+z)^{0.6} \simeq 1 + 0.6z$ for $z \ll 1$, and the error is of order $0.6z$. For the sample of $\sim 200\,000$ galaxies of SDSS (Zehavi et al. 2005), the redshift range is $z = (0.02 \sim 0.167)$ with a median $z \sim 0.1$. Take its maximum $z = 0.167$ and the ratio $\xi_{gg}(r)/\xi_{gg}(r,t) \simeq (1+0.167)^{0.6} \sim 1.097$, giving an error $0.6z \simeq 0.1$. Thus, using the static $\xi_{gg}(r)$ to describe the time-evolving $\xi_{gg}(r,t)$ of SDDS has a small error for $z \ll 1$. This analysis has also been supported by studies of numerical simulations. Hamana et al. (2001) have simulated the time-evolving correlation for ΛCDM model and demonstrated that $\xi_{gg}(r,z)$ has changed by a small amount during $z = 0.4 \sim 0$. The profiles of $\xi_{gg}(r, z = 0)$ and $\xi_{gg}(r, z = 0.4)$ are very similar on a whole range $r = (0.1 - 60)\, h^{-1}\,$Mpc (and are also similar to our theoretical profile $\xi_{gg}(r)$), and the ratio $\frac{\xi_{gg}(r,z=0)}{\xi_{gg}(r,z=0.4)} \sim 1.3$ for $r = (5 - 40)\, h^{-1}\,$Mpc. Similar results are also found in other numerical studies (Yoshikawa et al. 2001; Taruya et al. 2001). Hence the hydrostatic assumption, as an approximation, can be applied to a system of galaxies with $z \ll 1$, causing only a small error.

In fact, the observed data of $\xi_{gg}(r)$ are inevitably contaminated by redshift distortions to varying degrees. Since our analytical solution $\xi_{gg}(r)$ is given in real space, it is more realistic to compare our result directly with those of numerical simulations in real space that are free of distortions.

Figure 4 shows that on scales $r > 1\, h^{-1}\,$Mpc our solution $\xi_{gg}(r)$ agrees very well with the simulated one given by Hamana et al. (Hamana et al. 2001). Here the viscosity parameter $a = 2$ or 3 has been taken, greater than $a = 1.2$ used in Figure 3. Similarly, the insufficiency of amplitude on small scales $r < 1\, h^{-1}\,$Mpc should be improved by including higher order nonlinear terms.

Next, we consider the projected correlation function. For sky surveys of galaxies and clusters, the measurement of distances is through their cosmic redshift z. Galaxies or clusters have peculiar velocities, causing redshift distortion to the measured distance. To eliminate this distorting effect, one integrates over the distance parallel to the line of sight. This leads to the projected correlation function (Peebles 1980)

$$W_p(r_p) = 2\int_0^\infty \xi\left(\sqrt{r_p^2 + y^2}\right) dy = 2\int_{r_p}^\infty \xi(r)\frac{r dr}{\sqrt{r^2 - r_p^2}}\,, \tag{26}$$

where r_p is the separation of two points vertical to the line of sight, not distorted by the peculiar velocities.

In Figure 5 (left) the theoretical $W_p(r_p)$ is converted from the solution $\xi_{gg}(r)$ in Figure 3 (left), and compares the observational data of projected correlation function of 2dFGRS (Hawkins et al. 2003) and SDSS (Zehavi et al. 2005). In Figure 5 (right) the theoretical $W_p(r_p)$ with $k_0 = 0.03\, h\,\mathrm{Mpc}^{-1}$ compares the data of SDSS R9 (Nuza et al. 2013) with a median redshift $z \sim 0.53$. Overall, the theoretical $W_p(r_p)$ traces the observational data well in the range $r_p = (1 \sim 40)\, h^{-1}\,$Mpc, but is lower than the data on small scales $r_p \leq 1\, h^{-1}\,$Mpc, the same insufficiency mentioned before.

6 APPLYING TO CLUSTERS

Clusters are believed to trace the cosmic mass distribution on even larger scales, and the observational data cover spatial scales that exceed those of galaxies. Now we apply the solution with the same two sets of (a, b, c) as in Section 5 to the system of clusters. Clusters have a greater mass m than that of galaxies, leading to a higher overall amplitude of $\xi_{cc}(r)$. In addition, to match the observational data of clusters, a small value $k_0 = 0.03\, h\,\mathrm{Mpc}^{-1}$ is required, which is lower than that for galaxies.

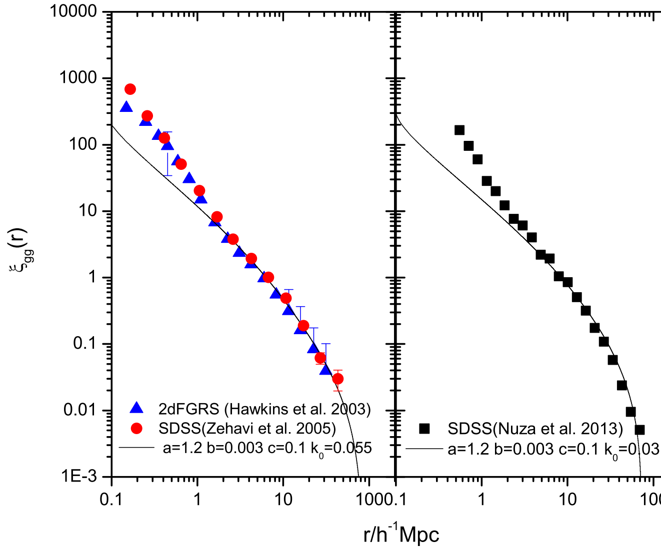

Fig. 3 *Left*: The solution $\xi_{gg}(r)$ with $k_0 = 0.055\,h\,\mathrm{Mpc}^{-1}$ is compared with the data of galaxies by 2dFGRS having a median redshift $z \sim 0.11$ (Hawkins et al. 2003) and SDSS having a median $z \sim 0.1$ (Zehavi et al. 2005); *Right*: The solution $\xi_{gg}(r)$ with $k_0 = 0.03\,h\,\mathrm{Mpc}^{-1}$ is compared with SDSS DR9 having a median $z \sim 0.53$ (Nuza et al. 2013).

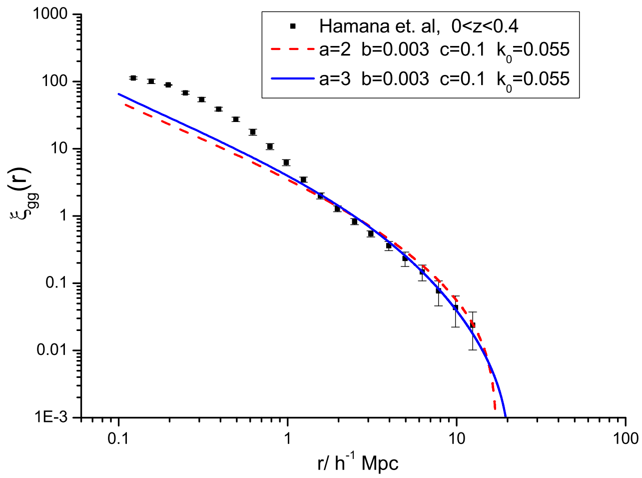

Fig. 4 The solution $\xi_{gg}(r)$ is compared with that of the simulations of Hanama et al. (Hamana et al. 2001). Here $a = 2, 3$ is taken respectively, greater than that used in Fig. 3 for the survey data.

In Figure 6, two solutions $\xi_{cc}(r)$ with different amplitudes are given, and are compared with two sets of data with richness $N > 10$ and $N > 16$ from the SDSS (Estrada et al. 2009). Interpreted by the field equation (22), the $N > 16$ clusters have a greater m than the $N > 10$ ones. The solutions match the data available on the whole range $r = (18 \sim 60)\,h^{-1}\,\mathrm{Mpc}$, and there is no small-scale insufficiency of correlation that occurred for the galaxy case. This means that the order of $(\delta\psi)^2$ in perturbations is accurate enough to account for the correlation of clusters. Since $k_0 = 0.03\,h\,\mathrm{Mpc}^{-1}$ for clusters and $k_0 = 0.055\,h\,\mathrm{Mpc}^{-1}$ for galaxies, it can be inferred that the mean density ρ_0 involved in this cluster survey is lower by $(0.03/0.055)^2 \sim 0.3$ than that in the galaxy case.

Observations show that the cluster correlation scale increases with the mean spatial separation between clusters (Szalay & Schramm 1985; Bahcall & West 1992; Bahcall 1996; Croft et al. 1997; Gonzalez et al. 2002). For a power-law $\xi_{cc} = (r_0/r)^{1.8}$ fitting, the data indicate a “correlation length”

$$r_0 \simeq 0.4 d_i, \tag{27}$$

where $d_i = n_i^{-1/3}$ and n_i is the mean number density of clusters of type i. For SDSS, the scaling can be also fitted by $r_0 \simeq 2.6 {d_i}^{1/2}$ (Bahcall et al. 2003), and for the 2dF galaxy groups, $r_0 \simeq 4.7 {d_i}^{0.32}$ (Zandivarez et al. 2003).

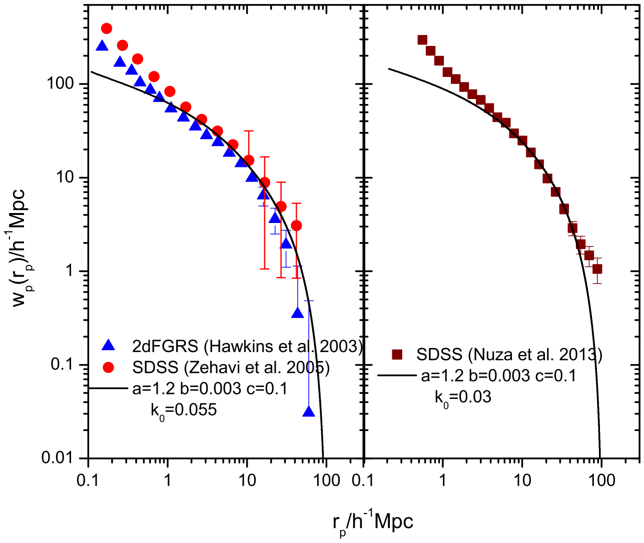

Fig. 5 The projected correlation function $W_p(r_p)$ converted from $\xi_{gg}(r)$ confronts the data. *Left*: 2dFGRS (Hawkins et al. 2003) and SDSS (Zehavi et al. 2005) with a median $z \sim 0.1$, *Right*: SDSS DR9 (Nuza et al. 2013) with a median $z \sim 0.5$.

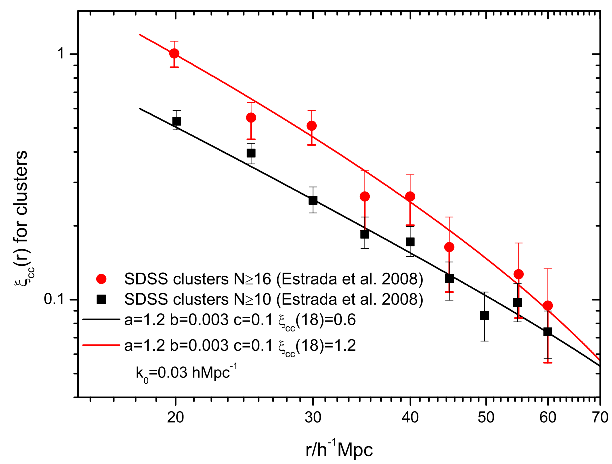

Fig. 6 The solution $\xi_{cc}(r)$ matches the cluster data of SDSS with two types of richness (Estrada et al. 2009). Notice that values of (a, b, c) are the same as those for galaxies, but $k_0 = 0.03\,h\,\mathrm{Mpc}^{-1}$, taken for clusters, is smaller than that for galaxies.

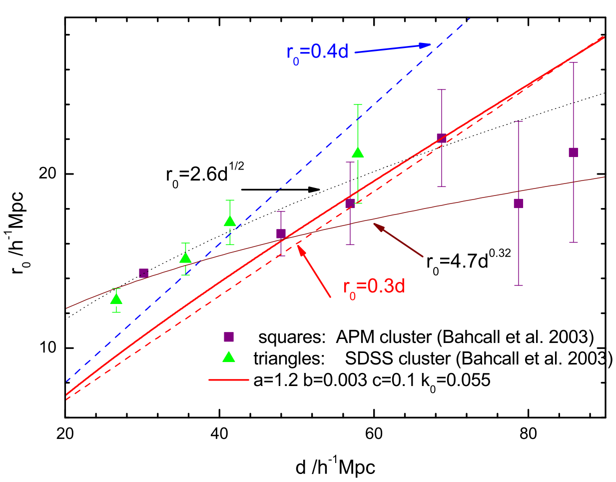

Fig. 7 The solution $\xi_{cc}(r)$ with $k_0 = 0.03\,h\,\mathrm{Mpc}^{-1}$ gives the scaling $r_0 \simeq 0.4d$. But with a greater $k_0 = 0.055\,h\,\mathrm{Mpc}^{-1}$, ξ_{cc} would give a flatter scaling $r_0 \simeq 0.3d$, which seems to fit the data of APM clusters better (Bahcall et al. 2003).

This kind of $r_0 - d_i$ empirical scaling has been a theoretical challenge (Bahcall 1996), and was thought to be either caused by a fractal phenomenon (Szalay & Schramm 1985), or by the statistics of rare peak events (Kaiser 1984). Interpreted by our theory, the scaling behavior is completely contained in the solution $\xi_{cc}(k_0 r)$, where the characteristic wavenumber $k_0 \propto \rho_0^{1/2} \propto d^{-3/2}$ appears together with r in the variable of the function ξ_{cc}. To comply with the empirical power-law, we take the theoretical "correlation length" as $r_0(d) \propto \xi_{cc}^{1/1.7}$, where ξ_{cc} is the solution depending on d.

Figure 7 shows that the solution ξ_{cc} with $k_0 = 0.03\,h\,\mathrm{Mpc}^{-1}$ gives the scaling $r_0(d) \simeq 0.4d$, agreeing well with the observation (Bahcall 1996). If a greater $k_0 = 0.055\,h\,\mathrm{Mpc}^{-1}$ is taken, the solution ξ_{cc} would yield a flatter scaling $r_0(d) \simeq 0.3d$, which fits the data of APM clusters better (Bahcall et al. 2003). Our solution $\xi_{cc}(k_0 r)$ tells us that a higher background density ρ_0 corresponds to a flatter slope of the scaling $r_0(d)$, and the scaling is naturally explained.

Extended to very large scales, the observed $\xi_{cc}(r)$ exhibits a pattern of periodic oscillations with a characteristic wavelength $\sim 120\,h^{-1}\,\mathrm{Mpc}$ (Einasto et al. 1997b,a). It was originally found in the galaxy distribution in narrow pencil beam surveys (Broadhurst et al. 1990), but also occurs in the correlation function of galaxies (Tucker et al. 1997) and of quasars (Yahata et al. 2005). There have been various interpretations. Our solution $\xi(r)$ with small values (a, b, c) exhibits periodic oscillations with a damped amplitude at increasing r (Zhang 2007). Although the data from samples of a cylindrical volume show large amplitude at $r \sim 400 - 600\,h^{-1}\,\mathrm{Mpc}$, these high amplitude cases would be damped in a full three-dimensional sample (Einasto et al. 2002).

7 3-POINT CORRELATION FUNCTION IN GAUSSIAN APPROXIMATION

It is also interesting to consider the 3-point correlation function $G^{(3)}(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{r}'')$ in the Gaussian approximation in our theory. As given in Appendix C, the field equation of $G^{(3)}$ in the Gaussian approximation is

$$\begin{aligned}
&\nabla_r^2 G^{(3)}(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{r}'') + 2k_J^2 \psi_0 G^{(3)}(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{r}'') \\
&- \frac{2}{\psi_0} \nabla G^{(2)}(\boldsymbol{r}, \boldsymbol{r}'') \cdot \nabla G^{(2)}(\boldsymbol{r}, \boldsymbol{r}') \\
&+ 2k_J^2 G^{(2)}(\boldsymbol{r}, \boldsymbol{r}') G^{(2)}(\boldsymbol{r}, \boldsymbol{r}'') \\
&+ \frac{2}{\alpha} \psi_0 \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}'') G^{(2)}(\boldsymbol{r}, \boldsymbol{r}') \\
&+ \frac{2}{\alpha} \psi_0 \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}') G^{(2)}(\boldsymbol{r}, \boldsymbol{r}'') = 0 \, .
\end{aligned} \tag{28}$$

To look for its solution, let $G^{(3)}$ be in the form of the Kirkwood-Groth-Peebles ansatz (21) with

$$Q = 1/\psi_0. \tag{29}$$

In fact, $Q = 1$ since $\psi_0 = 1$ by $\langle \rho \rangle = \rho_0$. Using Equation (B.9) for the 2-point function $G^{(2)}$ in the Gaussian approximation and the property of δ-function, the field equation (28) is satisfied automatically (see Appendix C). Thus, at the Gaussian approximation of our theory, the analytical solutions are

$$\begin{aligned}
&G^{(3)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = G^{(2)}(r_{12}) G^{(2)}(r_{23}) \\
&\quad + G^{(2)}(r_{23}) G^{(2)}(r_{31}) + G^{(2)}(r_{31}) G^{(2)}(r_{12}) \, ,
\end{aligned} \tag{30}$$

where the Gaussian 2-point correlation function (Zhang 2007)

$$G^{(2)}(\boldsymbol{r}) = A_1 \frac{Gm}{c_s^2} \frac{\cos(k_0 r)}{r} + A_2 \frac{Gm}{c_s^2} \frac{\sin(k_0\, r)}{r}, \tag{31}$$

with the coefficients satisfying $A_1 + A_2 = 1$. This result proves that the Kirkwood-Groth-Peebles ansatz (21) with $Q = 1$ holds exactly in the Gaussian approximation. As for the nonlinear field equation of $G^{(3)}$ beyond the Gaussian approximation, it will be much more involved, and will be studied in the future.

8 CONCLUSIONS AND DISCUSSION

We have presented a field theory of density fluctuations of a Newtonian self-gravitating system, derived the nonlinear field equation of the correlation function of second order of perturbations, and applied it to systems of galaxies and clusters in the universe.

As a starting point, we have obtained the field equation (6) of the mass density field ψ under the condition of hydrostatic equilibrium. It is better suited for studying the mass distribution than the Lane-Emden equation of gravitational potential. In dealing with the high nonlinearity, the mass density field is expanded as $\psi = \psi_0 + \delta\psi$, where the mean density ψ_0 is a constant for the background of the universe and in this paper, the fluctuation is kept to the order $(\delta\psi)^2$. This approach can also be applied to other finite self-gravitating systems, in which a given background density ψ_0 varies in space. As the main result, the field equation (22) of 2-point correlation function $G^{(2)}$ of density fluctuation has been derived, whereby the Kirkwood-Groth-Peebles ansatz is adopted to cut off the hierarchy and renormalization is performed. As for the 3-point correlation $G^{(3)}$, it is very revealing to find that its field equation in the Gaussian approximation is automatically satisfied when the Kirkwood-Groth-Peebles ansatz is used with $Q = 1$. Thus the ansatz holds as an exact relation between $G^{(2)}$ and $G^{(3)}$ at Gaussian level in our theory. The equation of $G^{(2)}$ is Helmholtz-like and nonlinear up to order $(G^{(2)})^2$, with three parameters (a, b, c) representing nonlinear effects beyond the Gaussian approximation. Notably, the Jeans wavelength λ_0 occurs as the only scale, and the mass m appears as the magnitude of the source. The result simultaneously explains several seemingly unrelated features of the large scale structure of the universe, such as the profile similarity of ξ_{cc} for clusters to ξ_{gg} for galaxies, the differences in amplitude and in correlation length of ξ_{cc} and ξ_{gg} respectively, the $r_0 - d$ scaling, and the pattern of periodic oscillations in ξ_{cc} with a wavelength $\lambda_0 \sim 120\, h^{-1}$ Mpc. With the same set of (a, b, c), the solution ξ_{gg} agrees with observations of galaxies over a range $(1 \sim 50)\, h^{-1}$ Mpc, and the solution ξ_{cc} of larger m and λ_0 matches observations of clusters over the whole range $(4 \sim 100)\, h^{-1}$ Mpc. Thus, our theory sheds light on understanding the large scale structure of universe.

There are several possible improvements that can be made to the present work. To improve the correlation at small scales $r \leq 1\, h^{-1}$ Mpc, higher order fluctuations are needed, and this can be carried out systematically by perturbation. To include the evolution effects, the field equation of correlation should be extended to the case of cosmic expansion. Finally, our formulation of perturbation can be systematically used to derive the nonlinear field equations of $G^{(3)}$ beyond Gaussian approximation, as well as other improvements. These would need more studies.

Acknowledgements Y. Zhang is supported by the National Natural Science Foundation of China (Grant Nos. 11421303, 11675165 and 11633001), SRFDP and CAS, the Strategic Priority Research Program "The Emergence of Cosmological Structures" of the Chinese Academy of Sciences (Grant No. XDB09000000).

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