

Reconstructing the temporal evolution of the speed of light in a flat FRW Universe

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Abstract We present a novel method to reconstruct the temporal evolution of the speed of light $c(z)$ in a flat Friedmann-Robertson-Walker (FRW) Universe using astronomical observations. After validating our pipeline with mock datasets, we apply our method to the latest baryon acoustic oscillations (BAO) and supernovae observations, and reconstruct $c(z)$ in the redshift range of $z \in [0, 1.5]$. We find no evidence of a varying speed of light, although we see some interesting features of $\Delta c(z)$, the fractional difference between $c(z)$ and c_0 (the speed of light in the International System of Units), e.g., $\Delta c(z) < 0$ and $\Delta c(z) > 0$ at $0.2 \lesssim z \lesssim 0.5$ and $0.8 \lesssim z \lesssim 1.3$, respectively, although the significance of these features is currently far below statistical importance.

Key words: cosmology — dark energy — large scale structure

1 INTRODUCTION

In this era of precision cosmology, we are fortunately equipped with high quality observational data of various kinds, which enable testing the fundamental laws of the Universe. The physical laws of the Universe are built with a few “constants,” including the gravitational constant G , electron charge e , speed of light c and so on. Actually, assuming the constancy of these “constants” at all cosmic times and scales is a significant extrapolation of our knowledge on rather limited temporal and spacial scales, which is subject to observational scrutiny.

Theorists including Dirac started thinking about building models for varying G or e , well before any observational tests became feasible (Dirac 1937; Bekenstein 1982). However, the constancy of c was much more sacred (Barrow 1999), as it is the pillar of special relativity, built by Einstein in 1905. Making c vary is much more destructive to the structure of formalisms in modern physics than varying other constants. Nevertheless, a varying speed of light (VSL) may solve the cosmological constant prob-

lem and build a new framework of cosmic structure formation, as an alternative to inflation. This is sufficiently attractive for theorists to develop viable VSL theories (see Magueijo 2003 for a review of recent developments in VSL). Meanwhile, techniques for testing VSL observationally have been developed in parallel. Recently, a new method to constrain VSL at a single redshift using baryon acoustic oscillations (BAO) was developed (Salzano et al. 2015) and has been applied to cosmological observations (Cao et al. 2017; Cai et al. 2016).

The method developed in (Salzano et al. 2015) is briefly summarised as follows. Given the relation between angular diameter distance $D_A(z)$ and Hubble function $H(z)$, the speed of light c at a specific redshift $z = z_M$ is just a product of $D_A(z_M)$ and $H(z_M)$, i.e.,

$$c(z_M) = D_A(z_M)H(z_M), \quad (1)$$

where z_M is the peak location of $D_A(z)$, i.e., $\partial D_A / \partial z|_{z=z_M} = 0$. Note that the exact value of z_M is model-dependent, and $z_M \sim 1.7$ for a flat Λ CDM

model, which is consistent with the Planck 2018 observations (Planck Collaboration et al. 2018).

This method has both pros and cons. On the positive side,

- It is model-independent, simply because it does not require any theoretical modeling at all in the first place. As shown in Equation (1), it only needs the peak redshift z_M , and the product of D_A and H measured at z_M , which can be derived from the BAO data alone;
- It is free from degeneracy with other cosmological parameters such the fractional matter density, the equation of state of dark energy, etc., because what we need is essentially a product of D_A and H , which is directly observable.

However, this method has also drawbacks, including,

- It is difficult to determine z_M accurately, as $D_A(z)$ is rather flat around its peak;
- For a wide range of cosmologies, z_M is as large as ~ 1.7 , making it inaccessible for most current BAO observations. Even for future deep BAO surveys, including that by the Dark Energy Spectroscopic Instrument (DESI) (DESI Collaboration et al. 2016), it is challenging to get decent D_A or H measurements at such high redshifts;
- This method only constrains c at one single redshift, which is far from sufficient for tests against various VSL models.

In this work, we generalise (Salzano et al. 2015) to propose a new method for testing VSL at multiple redshifts in the framework of a flat Friedmann-Robertson-Walker (FRW) Universe. After presenting the methodology in Section 2, we perform validation tests using mock datasets, before applying to observational data and presenting our main result in Section 4. We conclude and discuss our results in Section 5.

2 METHODOLOGY

We start from the relation between the angular diameter distance $D_A(z)$, the Hubble function $H(z)$ and the general speed of light function $c(z)$ in a flat FRW Universe¹, i.e.,

$$D_A(z) = \frac{1}{1+z} \int_0^z \frac{c(z')}{H(z')} dz'. \quad (2)$$

¹ Note that in non-flat Universes, the Friedmann equations get modified by the time derivative of $c(z)$ so Equation (2) does not hold (Barrow & Magueijo 1999). In this work, we consider a flat Universe for simplicity.

Differentiating both sides with respect to redshift z yields,

$$c(z) = \chi'(z)H(z), \quad (3)$$

where χ is the comoving distance so that $\chi(z) = (1+z)D_A(z)$. Our aim is to reconstruct the entire evolution history of $c(z)$, thus we parametrise the redshift-dependence of $\chi(z)$ and $H(z)$ as follows,

$$\frac{\chi(z)}{\chi_{\text{fid}}(z)} = \alpha_0 \left(1 + \alpha_1 x + \frac{1}{2} \alpha_2 x^2 + \frac{1}{6} \alpha_3 x^3 \right), \quad (4)$$

$$\frac{H_{\text{fid}}(z)}{H(z)} = \beta_0 + \beta_1 x + \frac{1}{2} \beta_2 x^2 + \frac{1}{6} \beta_3 x^3. \quad (5)$$

The variable $x \equiv \chi_{\text{fid}}(z)/\chi_{\text{fid}}(z_p) - 1$ where the subscript fid denotes the fiducial cosmology used, and z_p is the pivot redshift at which we apply the Taylor expansion. In this work, the fiducial cosmology is chosen to be a flat Λ CDM model with $\Omega_{M,\text{fid}} = 0.31$.

To test against the constancy of the speed of light, we define a deviation function of c as,

$$\Delta c(z) \equiv \frac{c(z)}{c_0} - 1, \quad (6)$$

where c_0 is the speed of light in the International System of Units (SI), i.e., $c_0 = 299\,792\,458 \text{ m s}^{-1}$.

Combining Equations (3), (4), (5) and (6), we have,

$$\Delta c(z) = \frac{\alpha_0 \left[1 + \alpha_1 + (2\alpha_1 + \alpha_2)x + \left(\frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_3 \right)x^2 \right]}{\beta_0 + \beta_1 x + \frac{1}{2}\beta_2 x^2} - 1. \quad (7)$$

In particular, at z_p where x vanishes,

$$\Delta c(z_p) = \frac{\alpha_0(1 + \alpha_1)}{\beta_0} - 1. \quad (8)$$

Although Δc does not explicitly depend on other cosmological parameters, its measurement does implicitly rely on how well we are able to model D_A and H using Equations (4) and (5) for general cosmologies. The expansion in Equation (4) was actually proposed by Zhu et al. (2015) for implementing the optimal redshift weighting method for BAO analyses, and it was demonstrated that expansions up to the quadratic order in x can precisely recover a wide range of cosmologies at the sub-percent level in the redshift range of $z \in [0.5, 1.5]$. Note that in Zhu et al. (2015), the speed of light was assumed to be a constant, thus $H(z)$ was derived from $\chi(z)$. In our case, as $c(z)$ is promoted to a general function of z , we have to double the number of expansion coefficients for $H(z)$. Also, we are

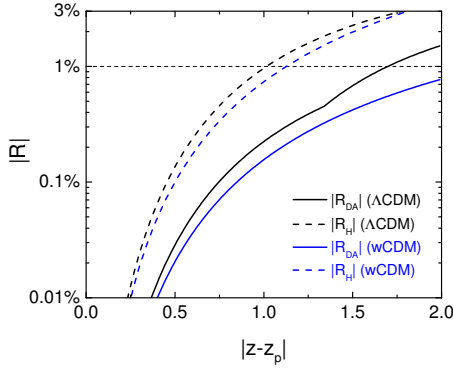


Fig. 1 The absolute residual $|R|$ defined in Eq. (11) as a function of $|z - z_p|$. As shown, the residuals are below the 1% level for $|z - z_p| < 1$, although $H(z)$ is less accurately modeled than $D_A(z)$. For larger $|z - z_p|$, say, $1 < |z - z_p| < 2$, R for $D_A(z)$ can still be controlled at the per cent level, while the residual for H approaches the 4% level.

more ambitious on the valid redshift range for these expansions, since we plan to employ all the available observations probing the background expansion of the Universe from $z = 0$ to $z = 2$. This has motivated us to utilize expansions to higher order to achieve the precision we need.

To validate our parametrisations for $\chi(z)$ and $H(z)$, we attempt to fit two cosmologies which sufficiently deviate from the fiducial cosmology we use for the expansion. Specifically,

Cosmo. Model I : a flat Λ CDM model with $\Omega_M = 0.2$;
(9)

Cosmo. Model II : a flat w CDM model with $\Omega_M = 0.31$;
 $w = -0.8$.
(10)

Both models are excluded by Planck 2018 observations, making them representative of “extreme” models that may be later sampled in the parameter space. For each model, we first choose a pivot redshift z_p , and then tune the α and β coefficients to minimise the difference between our model prediction for D_A and H calculated with Equations (4) and (5), and the exact quantities calculated using models I and II. We then compute the residual R for $D_A(z)$ and $H(z)$, i.e.,

$$R_o(z, z_p) \equiv 100 \times \left[\frac{O^{\text{model}}(z, z_p)}{O^{\text{exact}}(z, z_p)} - 1 \right] \%, \quad (11)$$

where O stands for D_A or H .

The resulting residuals are displayed in Figure 1 for various $|z - z_p|$ values, and we find that our model with third-order expansions is sufficiently accurate for our analysis: the residuals in most cases are below the 1% level, although $H(z)$ is less accurately modeled than $D_A(z)$.

Larger residuals appear when $|z - z_p|$ gets larger, which is naturally expected as the accuracy of the Taylor expansion decays with distance to the expansion point. Quantitatively, $|R| \lesssim 1\%$ for $|z - z_p| \lesssim 1$ for both models, and $|R|$ can approach 4% for H in the worst case, e.g., $|z - z_p| = 2$, but as we will explain later, this does not affect our result, if we only take the reconstructed values at z_p (so that $|z - z_p| = 0$).

Before applying our method to actual observations, there is another step missing: an actual mock test for various VSL models, which is presented in the next section.

3 MOCK TESTS

This section is devoted to a robustness test of our pipeline, before applying to actual observations, as presented in the next section.

To begin with, we choose four phenomenological VSL models for $\Delta c(z)$ to cover various possible features, including a constant, a linear function, a quadratic function and an oscillatory function, i.e.,

- VSL model 1: $\Delta c(z) = 0.05$;
- VSL model 2: $\Delta c(z) = 0.05z$;
- VSL model 3: $\Delta c(z) = 0.05z + 0.01z^2$;
- VSL model 4: $\Delta c(z) = 0.05 \sin(z\pi)$.

These VSL models are then combined with two cosmological models, corresponding to Equations (9) and (10), to make eight toy models for the mock test (see Table 1 for the labelling of the toy models).

For each toy model, we can first generate mock $H(z)$ data points using cosmological models shown in Equations (9) and (10), and then produce mock $D_A(z)$ datasets by combining $H(z)$ and $\Delta c(z)$ using Equation (2).

In practice, we produce two kinds of mock datasets, a combined BAO data sample assuming the sensitivity of the *Euclid* mission (Font-Ribera et al. 2014) and DESI survey (DESI Collaboration et al. 2016), and a combined supernova sample predicted for the Large Synoptic Survey Telescope (LSST) (LSST Science Collaboration et al. 2009) and *Euclid* (Astier et al. 2014).

For the BAO sample, we follow Font-Ribera et al. (2014) and DESI Collaboration et al. (2016) to produce D_A and H pairs at 15 and 18 effective redshifts for *Euclid* and DESI, respectively, so that our combined BAO sample consists of 33 pairs of D_A and H covering the redshift range of $z \in (0, 2.1)$. For the supernova sample, we produce luminosity distances assuming the sensitivity of

Table 1 Toy Models Used for the Mock Test

| Toy model | VSL model | Cosmological model |
|-----------|-------------|-----------------------------|
| 1 | VSL model 1 | Λ CDM; Equation (9) |
| 2 | VSL model 2 | Λ CDM; Equation (9) |
| 3 | VSL model 3 | Λ CDM; Equation (9) |
| 4 | VSL model 4 | Λ CDM; Equation (9) |
| 5 | VSL model 1 | w CDM; Equation (10) |
| 6 | VSL model 2 | w CDM; Equation (10) |
| 7 | VSL model 3 | w CDM; Equation (10) |
| 8 | VSL model 4 | w CDM; Equation (10) |

the Deep Drilling Fields and low-redshift sample to be observed by LSST, and of the Dark Energy Supernova Infra-Red Experiment (DESIRE) supernova survey with *Euclid*. The Deep Drilling Fields will observe 8800 supernovae at $z \in [0.15, 0.95]$, and an additional 8000 supernovae below redshift 0.35. This sample is complemented by the high- z sample to be collected by the DESIRE survey, which will provide 1740 supernovae at $z \in [0.75, 1.55]$.

For a given z_p , these datasets can be fitted with theoretical models listed in Equations (4) and (5) for parameters α and β respectively, using a modified version of `CosmoMC` (Lewis & Bridle 2002). We could then substitute the resultant α 's and β 's to reconstruct $\Delta c(z)$ with Equation (7). However, this result may be subject to systematic errors in $\Delta c(z)$ at redshifts that are far away from z_p , as we have seen in Figure 1. It is true that the residual $|R|$ is as low as 3% in the worst case as discussed in Section 2, but it can be further improved.

The way out is to abandon the reconstructed $\Delta c(z)$ at all redshifts except for $z = z_p$, where the Taylor expansion is error free, and repeat the fitting process for every z_p (with equal spacing for $\Delta z = 0.01$ in z) running over the entire redshift range. This is computationally expensive, and the errors in $\Delta c(z)$ at different redshifts are highly correlated, but this approach is much more accurate.

The result of this mock test is summarised in Figure 2. As depicted, the input VSL models are reconstructed perfectly for all toy models, with negligible bias compared to the statistical error budget.

4 IMPLICATION ON THE LATEST OBSERVATIONAL DATASETS

Now it is time to apply our pipeline for measuring $\Delta c(z)$ to actual observational data. We will first introduce the datasets that we utilize, and then present the result.

4.1 The Observational Dataset

The datasets employed in this analysis include,

- **The BAO measurements.** Because we are interested in reconstructing the time evolution of Δc , we use tomographic BAO measurements from the Baryonic Oscillation Spectroscopic Survey (BOSS) Data Release (DR) 12 sample, which provides measurement of D_A/r_d and Hr_d pairs at nine effective redshifts in the range of $z \in [0.31, 0.64]$ (Zhao et al. 2017). To approach the high- z end, we also utilize tomographic BAO measurement from the extended BOSS (eBOSS) DR14 quasar sample, which includes four additional D_A/r_d and Hr_d pairs at redshifts $z \in [0.98, 1.94]$ (Zhao et al. 2019). Note that as Equation (3) shows, the speed of light is a product of H and $d\chi/dz$, and r_d cancels out.
- **The observational $H(z)$ data (OHD).** The OHD, as ‘cosmic chronometers’, are measured using the ages of passively evolving galaxies, and we consider a compilation of 30 data points as listed in Table A.1.
- **The supernova data.** We also include the ‘joint light-curve analysis (JLA)’ sample (Betoule et al. 2014), which is a re-analysis of 740 data points consisting of samples from the three year SDSS-II supernova survey (Sako et al. 2018) and the Supernova Legacy Survey (SNLS) samples presented in Conley et al. (2011).

4.2 The Final Result

We apply our pipeline to the actual observational data, in the same way as we performed on the mock data, and display the result in Figure 3. Again, this is a compilation of $\Delta c(z_p)$ for numerous z_p 's that are equally spaced in z with $\Delta z = 0.01$.

As depicted, the uncertainty of $\Delta c(z_p)$ gets minimised at $z \sim 0.2$, where the Universe is best probed by current supernova and BAO experiments. The constraints on the low- z and high- z ends are looser, because of the small volume at low- z and the sparsity of galaxy distributions at high- z , which dilute the BAO constraints. Furthermore, current supernova surveys can barely access the Universe at $z \gtrsim 1$.

The result is consistent with $\Delta c(z) = 0$ given the level of uncertainty, but interesting features show up at $0.2 \lesssim z \lesssim 0.5$ and $0.8 \lesssim z \lesssim 1.3$ where $\Delta c(z) < 0$ and $\Delta c(z) > 0$, respectively, which await further investigation using future observations.

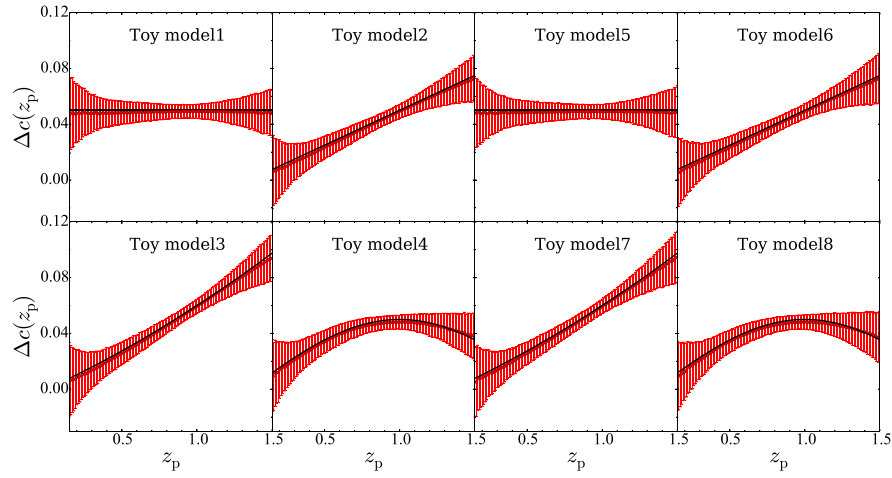


Fig. 2 The result of the mock test for eight toy models listed in Table 1. The *black solid lines* and *red dots* represent the input and reconstructed values of $\Delta c(z_p)$, respectively, and the error bars illustrate the 68% confidence limits (CLs).

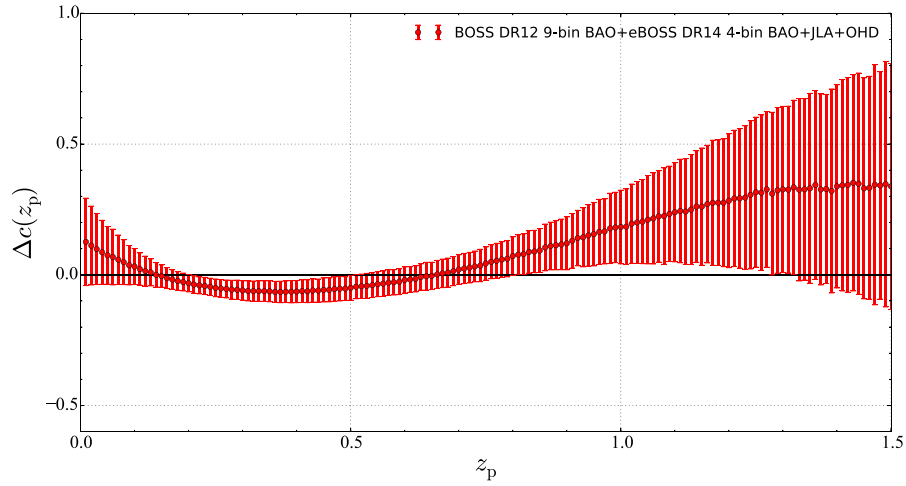


Fig. 3 A reconstruction of $\Delta c(z_p)$ derived from a compilation of current observations described in Sect. 4.1. The *black horizontal line* shows $\Delta c(z_p) = 0$ as a guide.

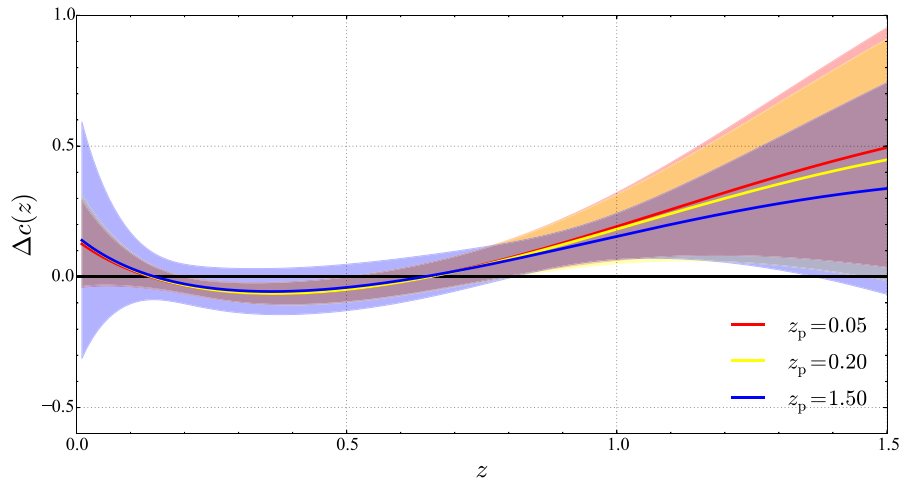


Fig. 4 The reconstructed $\Delta c(z)$ using three values of z_p , as illustrated in the legend. The *shaded bands* signify the 68% CLs.

We are interested in quantifying the signal-to-noise ratio of $\Delta c(z) \neq 0$, yet it is not straightforward because the error bars are highly correlated, but a covariance matrix is unavailable as we performed the parameter constraints at various z_p individually.

However, for a given z_p , we are able to quantify the deviation of $\Delta c(z)$ from zero with the reconstructed $\Delta c(z)$ using Equation (7), as the covariance for the α and β parameters is available. Before proceeding, we show the reconstructed $\Delta c(z)$ in Figure 4 using three values of z_p at 0.05, 0.2 and 1.5, which cover a reasonable choice of z_p (0.2) and two extreme values (0.05 and 1.5). As is apparent, these results agree reasonably well with what is displayed in Figure 3, although the result using $z_p = 1.5$ overestimates the uncertainty (but the central value remains accurate).

We then quantify the difference between $\Delta c(z)$ and zero utilizing the α 's and β 's derived by applying a specific z_p . Note that $\Delta c(z) = 0$ means the numerator is identical to the denominator of the fraction in Equation (7), which translates into,

$$\begin{aligned}\Delta_0 &\equiv \beta_0 - \alpha_0(1 + \alpha_1) = 0, \\ \Delta_1 &\equiv \beta_1 - \alpha_0(2\alpha_1 + \alpha_2) = 0, \\ \Delta_2 &\equiv \beta_2 - \alpha_0(3\alpha_2 + \alpha_3) = 0.\end{aligned}\quad (12)$$

Given the measured α 's and β 's and the corresponding covariance matrix, we can easily evaluate the following χ^2 ,

$$\chi_{z_p}^2 = \Delta^T C_{\Delta} \Delta, \quad (13)$$

where $\Delta \equiv \{\Delta_0, \Delta_1, \Delta_2\}^T$ and C_{Δ} is the derived covariance matrix for vector Δ . We average over the χ^2 for all z_p 's and compute the mean and variance, yielding,

$$\chi^2 = 3.2 \pm 1.1, \quad (14)$$

which is statistically expected for fitting a fixed value, 0, to three data points (Δ_i , $i = 1, 2, 3$). The error in χ^2 quantifies the fluctuation of the result using various z_p , thus it can be viewed as systematic. So, the conclusion is that the constant speed of light measured in SI is consistent with current cosmological observations.

5 CONCLUSIONS AND DISCUSSION

As a competitor to inflation, viable VSL theories deserve serious investigation both theoretically and observationally. The speed of light, under the assumption that c is a universal constant, was measured extremely well using laser interferometry in 1972 (Evenson *et al.* 1972), but it is

a significant extrapolation to assume that c takes the same value across cosmic time scales². With the accumulation of high quality cosmological datasets, it is time to develop theoretical and numerical tools to put constraints on the VSL scenario.

In this work, we propose a new method to constrain the speed of light using astronomical observations for the background expansion history of the Universe, including BAO, OHD and supernovae. Our method enables an accurate reconstruction of the temporal evolution of the speed of light $c(z)$ in a flat FRW universe, with no dependence on other cosmological parameters. Note that the time-evolution of $c(z)$ can be further constrained by observations probing the structure growth of the Universe including cosmic microwave background (CMB) surveys, as a variation in c means a variation in the fine-structure constant α , which leaves an imprint on the CMB. We will expand our work in the near future to combine it with the CMB constraint.

After validating our method and pipeline using mock datasets of BAO and supernovae, we apply our method to the latest astronomical observations including the anisotropic BAO measurements from BOSS (DR12) and eBOSS (DR14 quasar), and the JLA supernova sample, and reconstruct $c(z)$ in the redshift range of $z \in [0, 1.5]$. We find no evidence of a VSL, although we see some interesting features of $\Delta c(z)$, the fractional difference between $c(z)$ and c_0 (the speed of light in SI), e.g., $\Delta c(z) < 0$ and $\Delta c(z) > 0$ at $0.2 \lesssim z \lesssim 0.5$ and $0.8 \lesssim z \lesssim 1.3$, respectively, which are primarily driven by the BAO and supernovae data. Note that the significance of these features is far below statistical importance, and it may be subject to unknown observational systematics. Future observations including DESI (DESI Collaboration *et al.* 2016), LSST (LSST Science Collaboration *et al.* 2009) and *Euclid* (Font-Ribera *et al.* 2014) should be able to confirm or falsify our result in this work, up to deeper redshifts³.

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² We focus on probing the temporal evolution of c in this work, i.e., we assume that c does not depend on distance scales. In general, one could probe the spatial variation of c as well using astronomical observations, but it is beyond the scope of this paper.

³ We can hardly reconstruct c beyond $z = 1.5$ using current data due to the sparsity of data points at higher redshifts.

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Appendix A: THE OHD DATA

We summarise the OHD data considered in this work in the following table.

Table A.1 $H(z)$ Measurements (in unit of $[\text{km s}^{-1} \text{Mpc}^{-1}]$) Applied in This Work

| z | $H(z)$ | $\sigma_{H(z)}$ | Reference |
|---------|--------|-----------------|-----------------------|
| 0.07 | 69.0 | 19.6 | Zhang et al. (2014) |
| 0.09 | 69 | 12 | Simon et al. (2005) |
| 0.12 | 68.6 | 26.2 | Zhang et al. (2014) |
| 0.017 | 83 | 8 | Simon et al. (2005) |
| 0.179 | 75 | 4 | Moresco et al. (2012) |
| 0.199 | 75 | 5 | Moresco et al. (2012) |
| 0.20 | 72.9 | 29.6 | Zhang et al. (2014) |
| 0.27 | 77 | 14 | Simon et al. (2005) |
| 0.28 | 88.8 | 36.6 | Zhang et al. (2014) |
| 0.352 | 83 | 14 | Moresco et al. (2012) |
| 0.3802 | 83 | 13.5 | Moresco et al. (2016) |
| 0.4 | 95 | 17 | Simon et al. (2005) |
| 0.4004 | 77 | 10.2 | Moresco et al. (2016) |
| 0.4247 | 87.1 | 11.2 | Moresco et al. (2016) |
| 0.44497 | 92.8 | 12.0 | Moresco et al. (2016) |
| 0.4783 | 80.9 | 9 | Moresco et al. (2016) |
| 0.48 | 97 | 62 | Stern et al. (2010) |
| 0.583 | 104 | 13 | Moresco et al. (2012) |
| 0.68 | 92 | 8 | Moresco et al. (2012) |
| 0.781 | 105 | 12 | Moresco et al. (2012) |
| 0.875 | 125 | 17 | Moresco et al. (2012) |
| 0.88 | 90 | 40 | Stern et al. (2010) |
| 0.9 | 117 | 23 | Simon et al. (2005) |
| 1.037 | 154 | 20 | Moresco et al. (2012) |
| 1.3 | 168 | 17 | Simon et al. (2005) |
| 1.363 | 160 | 33.6 | Moresco (2015) |
| 1.43 | 177 | 18 | Simon et al. (2005) |
| 1.53 | 140 | 14 | Simon et al. (2005) |
| 1.75 | 202 | 40 | Simon et al. (2005) |
| 1.965 | 186.5 | 50.4 | Moresco (2015) |

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