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Estimating masses of selected stars in the Pulkovo program by means of astrometry methods

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Abstract Stars in the Pulkovo Observatory program are observed with a 65-cm refractor during many years to study their positions and movements. We present examples of two visual binary stars, for which orbits and masses of components were determined, and two astrometric stars, for which masses of their unseen companions were estimated. The first two stars are ADS 14636 (61 Cygni) and ADS 7251, and the others are Gliese 623 and ADS 8035 (Alpha UMa). Direct astrometric methods are used for estimation of mass-ratio and masses.

Key words: stellar masses — orbits — binaries: individual (61 Cygni, Alpha UMa, Gliese 623, ADS 7251)

1 INTRODUCTION

The mass of a star is one of its most important characteristics. The amount of matter in a star determines its temperature and pressure in the center, and also determines other characteristics of the star and thus its evolutionary path. Direct estimates of the mass of the star are made based on the law of universal gravitation. Studies of binary stars have allowed confirmation of the unity of Newton's laws in the Universe and obtaining of fundamental knowledge about the masses of stars using observations.

The classic way of determining the masses of stars is done by means of positional astrometric observations of double stars with high-precision instruments for a long time. Observations should cover a long time interval, or the period of revolution of a double star must be sufficiently short compared to the total observation period. We also need the exact value for the distance to stars, which relies on parallax for stars in the solar neighborhood. Other methods to calculate the mass are considered to be indirect; they are not built on the law of gravity, but rather on the analysis of stellar properties (luminosity, temperature, radius) which are related to the mass.

2 HISTORY AND OBSERVATIONS

At present, more than 100 000 visual double stars have been identified. The Washington Double Star Catalog (WDS) and the Catalog of Components of Double and Multiple Stars (CCDM) respectively have 78 000 and 110 000 such objects and about 3000 calculated orbits. But only for some of them, accurate calculation of the orbits allows us to determine the sum of the masses of the components. For even fewer stars, researchers have directly determined the ratio of their masses.

The sum of the masses of the components can be determined from Kepler's Third Law

$$M_1 + M_2 = \frac{a^3}{P^2}, \tag{1}$$

where a is the semimajor axes of the true orbit in a.u. and P is the orbital period in years; M_1 and M_2 are the component masses in $M_\odot$. The sum of masses may be estimated if the relative orbit of a double star and also its distance are known.

The error in masses mostly depends on uncertainties in the parallax.

$$\frac{\Delta(M_1 + M_2)}{(M_1 + M_2)} \geq 3\frac{\Delta\pi}{\pi}. \tag{2}$$

The ratio of the masses can be determined, if part of the absolute orbit of each component is known. If you define the semimajor axis of the absolute orbits a_1 and a_2, the ratio of the masses of the components can be found, as they are inversely proportional to the semimajor axis

$$\frac{M_1}{M_2} = \frac{a_2}{a_1}. \tag{3}$$

A significant part of the Pulkovo program of photographic observations with a 26-inch (65 cm) refractor was devoted to double and multiple stars. Our observations have been carried out since 1956 to determine the dynamic parameters, orbits and masses of visual binary stars. The list of these stars contains more than 400 objects and about 20 000 astronegatives, which belong to the Pulkovo glass-plate library in that there are more than 50 000 plates obtained during more than 100 years of observations. Since the beginning of the 2000s, observations with the 26-inch refractor have been continued using CCD cameras. Most of the Pulkovo program stars are visual doubles and represent wide pairs: $3'' < \rho < 30''$. As a result of observations, ρ is a defined mutual angular distance between components of a binary star AB, and θ is the position angle of the direction of arc AB on the celestial sphere.

We used the observations conducted with the long-focus visual refractor with $D = 650$ mm, and $F = 10\,413$ mm that has a small visible field of size $0.7 \times 1.0°$. The plates ORWO-NP27 and ORWO-WO1 were used. During the observations we applied Hertzsprung's technique of multiple exposures (from 6 to 20 images on the plate), which was demonstrated in observations of double stars and in creating parallactic series. In this case, we usually get a small number of stars in the narrow field of view on the plates, from 4 to 6 and as shown, see for instance, Izmailov et al. (2016); additional terms, depending on the color and magnitude, cannot be confidently determined. We tried to reduce errors occurring due to a difference between colors of the main object and the reference star, associated with particular atmospheric dispersion, by making all observations almost directly on the meridian.

Photography was also produced with a yellow filter, installed in the cassette. By applying it with an appropriate type of plate, it was possible to reduce the difference between the spectral sensitivity of the main star and star field. This was the best way to record 61 Cyg, because we have the possibility of using orthochromatic plates ORWO-WO1.

Magnitude $m_{\rm vis}$ of Gliese 623 is equal to 10.3 mag, nearly coinciding with the average value of the reference star's magnitude, which is equal to 10.6 mag. The brightest star, ADS 8035, was observed with an additional neutral filter, which attenuates the magnitude of an object 6 mag. For stars ADS 7251 and 61 Cyg, we have combined different series of observations. Part of the observations were made in the parallactic program by weakening with a grating (61 Cyg) and with a gap-loss attenuator, but most plates were obtained in the program of double stars without weakening. Accordingly, we have difficulties in selecting plates with sufficient accuracy.

Next, we present a short review of results which were obtained during observations with the Pulkovo 26-inch refractor. We have tried to estimate masses of some stars based on positional observations.

Table 1 lists the names of stars, observation intervals, as well as references to work that has already been completed and results of the present work. N is the number of plates used to obtain relative positions to determine the orbit and the sum of the masses, and n is the number of plates where the object is measured in the frame of background stars.

Below, we summarize the results of photographic observations. Several stars have been chosen for which, in principle, the masses of the components can be determined. However, the study of each object is associated with certain problems. We wanted to test how the accuracy of observations and the ratio of the observed arc to the orbit affect the results.

We have two wide pairs of binaries: ADS 14636 (61 Cyg) and ADS 7251. Also, we have a star with an optically unseen component, spectroscopic binary Gliese 623. Our new attempt estimates the mass-ratio of star ADS 8035 (Alpha UMa). We estimated the mass ratio, for Gliese 623, which represents a lower limit on mass for the unseen component. For ADS 8035 we found the value $\boldsymbol{B} - \beta$, where $\boldsymbol{B}$ is mass ratio and β depends on luminosity of the satellite.

To determine the orbits of binary stars, we apply the apparent motion parameters (AMP) method developed in Pulkovo, which was repeatedly used to determine the orbits of satellites and asteroids from observations on a relatively short arc tracing the orbit, see Kiselev & Romanenko (1996). We also applied this method to estimating the black hole mass at the center of our Galaxy in the system consisting of the black hole and star S01 revolving around the dynamic center, see

Table 1 Observational Data

No	Star	Observation	N	n	Reference
1	Gliese 623B	1979–1995	–	90	Shakht (1995, 1997)
2	61 Cyg A	1958–1997	380	153	Gorshanov et al. (2006)
		1958–2006	420	153	present work
3	61 Cyg B	1958–1997	380	153	Gorshanov et al. (2006)
		1958–2006	420	153	present work
4	ADS 7251 A	1962–2005	204	115	Shakht et al. (2010)
5	ADS 7251 B	1962–2005	204	115	Shakht et al. (2010)
6	ADS 8035 B	1975–2005	–	20	present work

Kisselev et al. (2007).

3 DATA PROCESSING

We measured our double stars to obtain relative coordinates ρ and θ or $\Delta\alpha\cos\delta = \Delta X = \rho\sin\theta$ and $\Delta\delta = \Delta Y = \rho\cos\theta$. Then we measured each component with respect to the reference star system. We used the method of six constants and the reduction to a standard plate. Here we give the basic formula for calculations. We used the methods outlined in classical works, such as the book of van de Kamp (1981). The common cases can be described for the main component A and also for component B as follows:

$$\begin{aligned} X_A &= C_x + \mu_x(t-t_0) + \pi P_x + q_x t^2 - \boldsymbol{B}\Delta X,\\ Y_A &= C_y + \mu_y(t-t_0) + \pi P_y + q_y t^2 - \boldsymbol{B}\Delta Y, \end{aligned} \tag{4}$$

$$\begin{aligned} X_B &= C_x + \mu_x(t-t_0) + \pi P_x + q_x t^2 + \boldsymbol{A}\Delta X,\\ Y_B &= C_y + \mu_y(t-t_0) + \pi P_y + q_y t^2 + \boldsymbol{A}\Delta Y, \end{aligned} \tag{5}$$

where

$$\boldsymbol{B} = M_B/(M_A + M_B),$$

$$\boldsymbol{A} = M_A/(M_A + M_A) = 1 - \boldsymbol{B}$$

and other values are known. X_A, Y_A, X_B, Y_B are positions of stars based on reference stars with respect to a selected standard plate, C_{xy} is position of the center of mass with respect to a selected component on a standard plate, and μ_{xy}, π, P_{xy} and q_{xy} are proper motion, parallax, parallactic factors and quadratic terms respectively. Here q is a perspective acceleration equal to $q = -1.24'' \times 10^{-6}\mu\pi V_r$ and it is a significant value only for the very nearest stars; $\Delta X, \Delta Y$ are relative positions of component B and component A at the time of observation.

Next, we solve the equations with respect to the constants C_{xy}, the proper motion of the center of mass and the mass ratio. If the sum of masses is known, we can calculate the mass of each component M_A and M_B.

In some cases the orbital motion, to a high degree of approximation, may be represented as a quadratic function of time. Then the mass ratio is determined from the quadratic term Q in Equation (6) after excluding perspective acceleration from it.

$$\begin{aligned} X &= C_x + \mu_x(t-t_0) + \pi P_x + Q_x t^2,\\ Y &= C_y + \mu_y(t-t_0) + \pi P_y + Q_y t^2. \end{aligned} \tag{6}$$

In practice, taking into account our geographical location, we prefer to exclude parallactic replacement by means of parallax referenced from a catalog. Our observations for selected stars have been made in limited time intervals each year and near the meridian. As a rule, we applied the following equations:

$$\begin{aligned} X'_A &= C_x + \mu_x(t-t_0) - \boldsymbol{B}\Delta X,\\ Y'_A &= C_y + \mu_y(t-t_0) - \boldsymbol{B}\Delta Y, \end{aligned} \tag{7}$$

$$\begin{aligned} X'_B &= C_x + \mu_x(t-t_0) + \boldsymbol{A}\Delta X,\\ Y'_B &= C_y + \mu_y(t-t_0) + \boldsymbol{A}\Delta Y, \end{aligned} \tag{8}$$

where X'_A, X'_B, Y'_A, Y'_B are corrected for parallax and acceleration.

4 DATA ANALYSIS

Now we would like to give some examples and describe some specific problems in treating each case.

I. First we consider two stars which are components of binaries:

61 Cyg A [ADS 14636 A, GJ 820A, HIP 104214, HD 201091, 5.21 mag, K5V, $21^{\rm h}06.9^{\rm m}$, $+38°45'$, $\pi = 0.286''$, $V_r = -64.7\,\mathrm{km\,s^{-1}}$].

61 Cyg B [ADS 14636 B, GJ 820B, HIP 104217, HD 201092, 6.03 mag, K7V, $21^{\rm h}06.9^{\rm m}$, $+38°44'$, $\pi = 0.286''$, $V_r = -65.7\,\mathrm{km\,s^{-1}}$].

For this star, we have their relative positions obtained during 1958–2006 with associated errors $0.007''$ in ρ and 0.02° in θ. Positions of each component on the basis of reference stars on the plates were obtained for the interval 1958–1997. Earlier, we estimated the orbital elements of this pair, see Gorshanov et al. (2006). Now we reexamine our results with an additional series of observations during 1997–2006. Two variants of orbits, obtained using parallax $0.2861'' \pm 0.0005''$ from the RECONS.org website, are given in Shakht et al. (2017). (*http://www.recons.org/TOP100.posted.htm*, from the REsearch Consortium On Nearby Stars (RECONS)).

We have two estimates of the sum of the masses with a control on $O - C$. We estimated the sum of masses to be $1.31\,M_\odot$ using our observations from 1958–2006, and a sum of masses of $1.4\,M_\odot$ when including all related observations from WDS.

First, we applied the classic formulas to the stars being studied. The values of mass ratio for 61 Cyg are in Table 2. The results of the decision on the two coordinates are given for components A and B. Average error unit of weight E_0 is about $0.044''$. In Table 2 the results for masses of components are listed in Columns (2)–(4).

In addition, we managed to solve the equation with respect to quadratic terms Q_x for components A and B, which turned out to be equal to $-0.00008 \pm 0.00002''/\mathrm{yr}^2$ and $0.000205 \pm 0.000007''/\mathrm{yr}^2$ for A and B, respectively. The perspective acceleration for this star, which equaled $0.000079''/\mathrm{yr}^2$, was subtracted from Q_x and corrected values for coordinates of A and B, Q'_a and Q'_b respectively, were obtained. As a result, the value $M_b/M_a = -Q'_a/Q'_b$ equaled 0.78 and mass ratio $\boldsymbol{B}$ equaled 0.43.

In Table 3, the masses of components 61 Cyg are given for three values of the sum of masses and for two values of mass ratio obtained in different ways.

The determination of the component masses for the sum of masses $1.31\,M_\odot$ and the ratio 0.43 is most satisfactory in our results, although there is a remarkable difference between the masses. Now we limit the choice of materials by errors of the unit weight E_0 in the range $0.018'' \sim 0.045''$ and in the interval 1958–1997. At present, we hope to improve the precision of this result by adding data from 1997–2006, which can provide a greater difference in epochs and also weight of parameters.

Only the mass ratio for 61 Cyg was presented in earlier work due to the absence of reliable orbits and sum of masses. Our results obtained from solving for component B with $\boldsymbol{B}$ are more according to spectral classes of components than for component A.

Table 4 shows the values of the sum, mass ratios and masses of components 61 Cyg and ADS 7251 from literary sources. In earlier works on 61 Cyg, only mass ratios are given, since there were no reliable orbits or the sum of masses. In the second and third lines from the bottom, for 61 Cyg, the values obtained from our study of components A and B, respectively, are given.

II. Then, we analyzed the astrometric and spectral components of Gliese 623, which were observed as a single star at Pulkovo in $1979 - 1995$.

Gliese 623 [GJ 623, HIP 80346, AC$+48^\circ$ 1595/89, 10.3 mag, M3.0Ve, $16^{\mathrm{h}}24.1^{\mathrm{m}}$, $+48^\circ 21'$, $\pi = 0.124''$, $V_r = -26.3\ \mathrm{km\,s^{-1}}$]. Here we had the opportunity to explore the residuals R_x, R_y obtained from the movement of the star after eliminating proper motion and parallax. P, To and e were found by means of a graphical method. Then using the values P, To and e, the coordinates of the ellipse based on the photocenter in units of semimajor axis x, y were calculated. We solved (4) for C_{xy}, μ_{xy}, π and then considered residuals: R_x, R_y. Residuals characterize the orbital motion of the photocenter of the system

$$\begin{aligned} R_x &= C_x + x(B) + y(G), \\ R_y &= C_y + x(A) + y(F), \end{aligned} \tag{9}$$

where x, y are reduced coordinates of the photocenter: $x = (r/a)\cos u$, $y = (r/a)\sin u$, where r - the radius vector, u - the true anomaly and a - the semimajor axis. (B), (G), (A), (F) are Thiele-Innes constants that contain information about the orientation of the orbit of the photocenter and its semimajor axis a_1, which is the size of the astrometric signal. We determined M_2 according to the formula

$$M_2 = \frac{a''_1}{\pi}\Big(\frac{M_1 + M_2}{P}\Big)^{2/3}, \tag{10}$$

by the method of successive approximations, assuming M_1 equals $0.31\,M_\odot$, corresponding to its spectral class, and considering the first approximation of the mass of the other component to be zero.

Various authors, including Lippincott & Borgman (1978); Marcy & Moore (1989), have estimated the mass of the invisible satellite to be in the range $0.06 \sim 0.08\,M_\odot$ and $0.067 \sim 0.087\,M_\odot$. We estimated the lower limit on mass of the satellite as $0.09 \pm 0.03 M_\odot$, taking into account the obtained trigonometric parallax

Table 2 Mass Ratio from Solving for Components A and B in 61 Cyg with Projection on Coordinates X, Y

Component	A_x	A_y	B_x	B_y
$\boldsymbol{B}$	0.38 ± 0.04	0.43 ± 0.06	0.46 ± 0.06	0.44 ± 0.10
E_0	$0.037''$	$0.041''$	$0.057''$	$0.040''$

Table 3 Sum and Mass Ratio for 61 Cyg

$\Sigma M/M_\odot$	$\boldsymbol{B}$ from A_{xy}	$M_A/M_\odot$	$M_B/M_\odot$	$\boldsymbol{B}$ from B_{xy}	$M_A/M_\odot$	$M_B/M_\odot$
1.20	0.40	0.72	0.48	0.45	0.66	0.54
1.31	0.40	0.79	0.52	0.45	0.72	0.59
1.40	0.40	0.84	0.56	0.45	0.77	0.63

Table 4 Estimations of Mass Ratio and Stellar Masses

61 Cyg				
ΣM	$\boldsymbol{B}$	M_A	M_B	References
–	0.38	–	–	van de Kamp (1939)
–	0.55	–	–	van de Kamp & Damkoehler (1953)
–	0.47	–	–	van de Kamp & Damkoehler (1953)
1.12	0.52	0.54	0.58	van de Kamp (1954)
1.26	0.47	0.67	0.59	Walker et al. (1995)
1.30	0.46	0.69	0.60	Kervella et al. (2008)
1.31	0.48	0.68	0.63	Boyajian et al. (2012)
1.31	0.40	0.79	0.52	Present work
1.31	0.45	0.72	0.59	Present work
1.0	0.5	0.5	0.5	Cester et al. (1983)
ADS 7251				
ΣM	$\boldsymbol{B}$	M_A	M_B	References
0.91	0.505	0.45	0.46	Hopmann (1954)
2.26	0.50	1.13	1.13	Güntzel-Lingner (1955)
1.14	0.64	0.41	0.73	Chang (1972)
1.22	0.49	0.62	0.60	Boyajian et al. (2012)
1.10	0.49	0.56	0.54	present work
1.10	0.48	0.57	0.53	present work

$0.131''$ from Shakht (1995, 1997) and its catalog value $0.124''$.

In 1996, the star Gliese 623 was resolved into two components using the *Hubble Space Telescope* (Barbieri et al. 1996). According to observations that apply modern techniques that incorporate adaptive optics in the 200-inch Palomar telescope (see Martinache et al. (2007)) and with the Keck II telescope in near infrared, the satellite's mass is found to be $0.115 \pm 0.002\, M_\odot$. The current improved value is $0.114 \pm 0.002\, M_\odot$ (Benedict et al. 2016). We cannot reach such accuracy now, but we continue CCD observations to detect any long-period effects in the motion of these stars.

III. Dubhe (Alpha UMa) [ADS 8035, BU1077, HD 95689, HIP 54061, 1.79 mag, G9III, $11^{\rm h}03.7^{\rm m}$, $+61°45'$, $\pi = 0.026''$, $V_r = -9.4\,{\rm km\,s^{-1}}$] is the multiple star and spectroscopic binary ADS 8035 with optically unseen component B and remote component C.

Photographic observations of ADS 8035 A with the 26-inch refractor were made in 1975–2005. It was observed as a single star because the limiting resolution of the 26-inch refractor is $1.5''$, so the satellite was not observed. However, we can trace the path of the main star with respect to background reference stars.

The period of revolution for the satellite is 44.4 years, therefore it is possible to determine the relative astrometric orbit of the photocenter or, taking the orbital elements to be known, to estimate the mass ratio possibly with greater accuracy than by observing a short arc from the relative orbit of the visual binary. For ADS 8035, we calculated the ephemeris using the relative orbit of the satellite in terms of elements given in Scardia et al. (2011). In this way, relative positions ΔX_j, ΔY_j corresponding to our times of observations were obtained.

The path over time of relative positions for the visible component A is shown in Figure 1. For this star, we have relatively few plates, so we selected 20 test plates. Then Equation (11) was solved, for which parallactic displacement was excluded previously and so only C and μ were determined.

Residuals R_x, R_y remained from this solution and consisted of orbital motion $\boldsymbol{B}\Delta X_j$ and $\boldsymbol{B}\Delta Y_j$. These terms depend on time (1900+) and are plotted in Figures 2 and 3 respectively.

$$\begin{aligned} X_j &= C_x + \mu_x(t_j - t_0) - \boldsymbol{B}\Delta X_j, \\ Y_j &= C_y + \mu_y(t_j - t_0) - \boldsymbol{B}\Delta Y_j. \end{aligned} \tag{11}$$

As a result, after solving Equation (11), the following parameters were obtained: $C_x = -0.230'' \pm 0.012''$, $\mu_x = -0.1240'' \pm 0.0007''$/yr, $C_y = 0.008'' \pm 0.016''$, $\mu_y = -0.0625'' \pm 0.0009''$/yr.

The value of $\boldsymbol{B}$ turned out to equal 0.324 ± 0.024 as a result of the solution with respect to the X-coordinate. Error in the unit of weight for the X-coordinate E_0 is equal to $0.038''$.

The accuracy for the Y-coordinate is worse than that for X; the value of $\boldsymbol{B}$ is obtained with a lower weight and mass-relation $\boldsymbol{B}$ is equal to 0.34 ± 0.13.

We did many different variants of solving, and applied the variant with minimal E_0 error 0.038 mentioned above. M_A and M_B can be estimated as $3.9\,M_\odot$ and $1.8\,M_\odot$ respectively. Here the mass ratio corresponds to its lower limit $\boldsymbol{B}_{\rm lower} = \boldsymbol{B} - \beta$, due to the fact that the satellite is located close to the main star and has its own luminosity.

With $E_0 = 0.032''$, we obtained mass ratio 0.48 ± 0.06. The errors in masses are not less than $\pm 0.1\,M_\odot$.

IV. ADS 7251 A [GJ 338A, HIP 45343, HD 79210, 7.6 mag, K7V, $09^{\rm h}14.4^{\rm m}$, $+52°41'$, $\pi = 0.164''$, V_r = 11.1 km s^{-1}].

ADS 7251 B [GJ 338B, HIP 120005, HD 79211, 7.7 mag, M0V, $09^{\rm h}14.4^{\rm m}$, $+52°41'$, $\pi = 0.164''$, V_r = 12.5 km s^{-1}].

Another example of obtaining the sum and the mass ratio is a calculation of the corresponding values after determining the relative orbit and the sum of the masses for the visual binary star ADS 7251 on the basis of observations with the Pulkovo 26-inch refractor. The results were obtained in 1962–2006, with errors in distance ρ and position angle θ equal to $0.006''$ and $0.02°$, respectively. The star has a long-period orbit lasting more than 1500 years.

Determination of the orbit and estimation of masses were made at Pulkovo, see Shakht et al. (2010), however the observational arc is quite short. In solving the classical dynamics in Equation (5), there were correlations between unknowns and they hardly separated.

For our case, we applied the method in which the mass ratios $\boldsymbol{A} = M_A/(M_A + M_B)$ and $\boldsymbol{B} = M_B/(M_A + M_B)$ were considered as free parameters that are linked together in such a way that $\boldsymbol{A} + \boldsymbol{B} = 1$ and we computed values by trying to minimize the error unit of weight E_0.

The equations were solved, where X_{A_j}, X_{B_j}, Y_{A_j}, Y_{B_j} represent positions of components A and B relative to reference stars for each j-th time, corrected for parallax and acceleration, C_{xy} are the positions of the center of mass with respect to the selected zero-point on the standard plate and μ_{xy} is the proper motion of the center of mass of the system.

$$\begin{aligned} X_{A_j}\boldsymbol{A} + X_{B_j}\boldsymbol{B} &= C_x + \mu_x(t_j - t_0), \\ Y_{A_j}\boldsymbol{A} + Y_{B_j}\boldsymbol{B} &= C_y + \mu_y(t_j - t_0), \end{aligned} \tag{12}$$

$$H_{A_j}\boldsymbol{A} + H_{B_j}\boldsymbol{B} = C_h + \mu_h(t_j - t_0). \tag{13}$$

Then we decided to improve the estimate of the mass-ratio using a projection onto a coordinate axis where the correlation with proper motion should not be significant. A system of equations was then solved where the initial relative positions A and B were calculated through projection onto an axis H perpendicular to the direction of motion of the center of mass of the system.

$$\begin{aligned} H_A &= -X_A \cos\chi + Y_A \sin\chi, \\ H_B &= -X_B \cos\chi + Y_B \sin\chi, \end{aligned} \tag{14}$$

where $\chi = \arctan(\mu_x/\mu_y)$. Here the proper motion of the center of mass is not determined, while μ_h represents the measurement errors and C_h is the summed proper

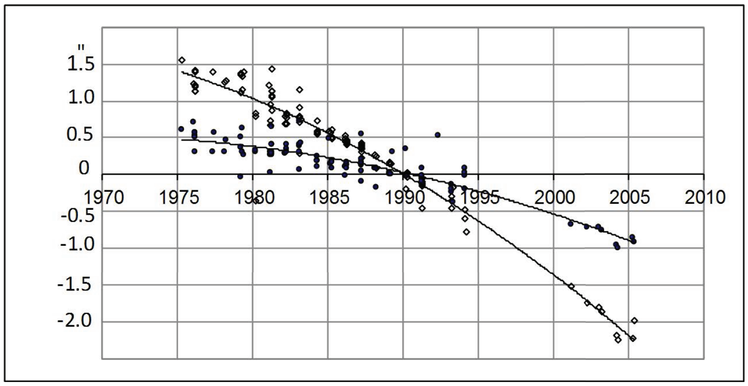

Fig. 1 The path over time of coordinates X_j,Y_j of ADS 8035 A in arcsecond.

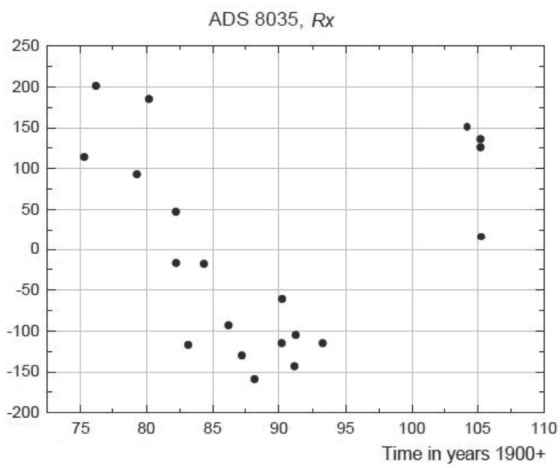

Fig. 2 Residuals R_x in mas.

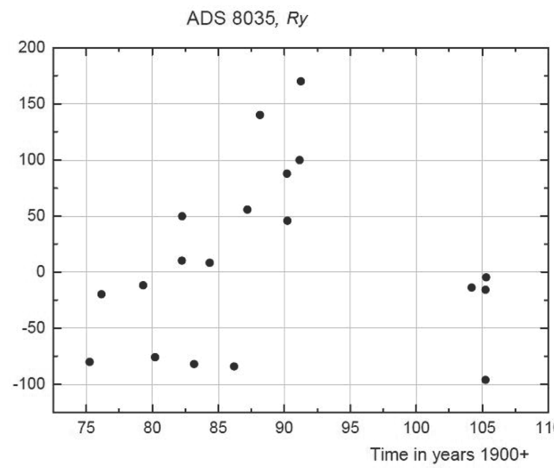

Fig. 3 Residuals R_y in mas.

Table 5 Sum and Mass Ratio for ADS 7251

$\Sigma M/M_\odot$	$\boldsymbol{B}$	$M_A/M_\odot$	$M_B/M_\odot$	n	E_0	Axis
1.1	0.46	0.59	0.51	146	0.070	X
1.1	0.49	0.56	0.54	104	0.036	X, Y
1.1	0.48	0.57	0.53	115	0.032	H

motion for reference stars projected onto the axis H. Finally with the mass ratio $\boldsymbol{B}$ value of 0.48, we obtained a minimum of E_0 equal to $0.032''$, with the masses of the components being 0.57 and 0.53 $M_\odot$, which is satisfactory with regard to spectral classes of components.

In Table 5, the sum of masses is given according to our estimate, see Shakht et al. (2010), n is the number of used plates, and X, Y are the average result from projections onto the X, Y axes, respectively.

V. Based on Pulkovo observations of homogeneous long-term series, we can estimate the minimum sum of masses for binary stars revolving in elliptical orbits, using their parameters of motion, even if the orbit itself is not defined.

$$M_1 + M_2 > \frac{\rho V^2}{8\pi^2 \pi_{tr}}, \tag{15}$$

$$M_1 + M_2 \geq \frac{\rho\mu}{4\pi^2 \rho_c \pi_{tr}^3 |\sin(\psi - \theta)|}. \tag{16}$$

In formulas (15)–(16), V is the space velocity, μ and ψ are relative proper motion of components and their position angle respectively, and ρ_c is the radius of curvature for the orbit. In some cases, an excess of mass was detected. In this way, the presence of a third companion was confirmed for some stars, such as, for example, ADS

497 and ADS 11061, see Kiselev & Romanenko (1996); Kisselev & Kiyaeva (2004) and Tokovinin & Smekhov (2002).

5 CONCLUSIONS

We have shown several examples of determining the mass using astrometric positional observations. We will make a few remarks on the process.

(1) It should be noted that the sought values were obtained within the precision of positions for the reference stars.
(2) In some cases, the mass ratio can be detected only with one coordinate (RA or Decl.), since the other coordinate was obtained with a low weight.
(3) If the observed arc is small with respect to the orbit, then correlations between unknowns can appear in results of solving Equation (4), and the required parameters are difficult to separate.
(4) There are cases where the direction of movement of the center of the system is close to the direction of orbital motion. In this case, it is preferable to solve the corresponding equations with projection in the direction of motion or perpendicular to it in order to exclude its proper motion in advance or to solve the equations with respect to the constant and only the mass ratio. Such an approach has been applied for the solution of ADS 7251.
(5) Since the stars we studied had exact trigonometric parallaxes, we did not pursue the task of determining them, but in advance excluded the effect of parallax from the initial equations.

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