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Effects of non-Newtonian gravity on the properties of strange stars

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Abstract The properties of strange star matter are studied in the equivparticle model with inclusion of non-Newtonian gravity. It is found that the inclusion of non-Newtonian gravity makes the equation of state stiffer if Witten's conjecture is true. Correspondingly, the maximum mass of strange stars becomes as large as two times the solar mass, and the maximum radius also becomes bigger. The coupling to boson mass ratio has been constrained within the stability range of strange quark matter.

Key words: quark matter — non-Newtonian gravity — equation of state — strange stars

1 INTRODUCTION

Strange stars are astronomical compact objects consisting of deconfined quark matter and many great efforts have been devoted to the relevant theoretical (Yu & Xu 2010; Li et al. 2010; Bordbar & Ziaei 2012; Chu & Chen 2014; Xu et al. 2015a) and experimental researches (Greiner et al. 1987; Spieles et al. 1996; Bauswein et al. 2009, Bauswein et al. 2010). Such dense stars could be formed directly after a supernova explosion (Hatsuda 1987; Sato & Suzuki 1987) or converted indirectly from neutron stars (Pagliara et al. 2013). Strange star matter contains not only up (u) and down quarks (d), but also strange quarks (s). Therefore, it is normally referred to as strange quark matter (SQM). The most striking feature is that SQM might be absolutely stable, i.e. more stable than the $^{56}\mathrm{F_e}$ isotope, as conjectured by Witten (1984) and first proved with models by Farhi & Jaffe (1984).

It has been widely accepted for a long time that universal gravitation obeys the inverse-square law. However, theoretical schemes like string/M theory and new physics beyond the Standard Model (Fischbach & Talmadge 1999; Adelberger et al. 2003) have proposed possible violations of conventional Newtonian gravity from the inverse-square law. Fujii pointed out (Fujii 1971) that the addition of a Yukawa-type potential to the conventional gravitational potential, forming modified Newtonian gravitational potential, can describe the phenomenon of non-Newtonian gravity. Also, some scientists have referred to it as a new fundamental intermediate-range force (Hubler et al. 1995), i.e., the fifth force.

In order to have a better understanding of non-Newtonian gravity, many investigations have been dedicated to study its possible existence and properties (Eckhardt et al. 1988; Kamyshkov et al. 2008; Gudkov et al. 2011; Boynton et al. 2014) and constrain its strength parameter from experimental researches (Geraci et al. 2008; Lucchesi & Peron 2010; Biedermann et al. 2015). In addition, this issue has attracted a great deal of attention in a nuclear physics context, for instance, it has strong effects on finite nuclei (Xu et al. 2013), dark matter (Schmidt 1990), nuclear matter (Wen et al. 2009; Zhang et al. 2011), and neutron star processes (Sulaksono et al. 2011; Wen & Zhou 2013).

The aim of the present paper is to study the effects of non-Newtonian gravity on the equation of state (EoS) of SQM and the structure of strange stars by taking into account the beta equilibrium and charge neutrality conditions. The rest of the paper is arranged as follows. We first present the theoretical approach in this paper with inclusion of non-Newtonian gravity in Section 2. The numerical results and discussions are given in Section 3. Finally, Section 4 is a summary.

2 NON-NEWTONIAN GRAVITY AND EQUATION OF STATE OF QUARK MATTER

The Yukawa-type non-Newtonian gravitational potential (Fujii 1971) between two objects with masses m_1 and m_2 is

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + \alpha \exp\left(-\frac{r}{\lambda}\right)\right], \tag{1}$$

where G is the universal gravitational constant, α is a dimensionless strength parameter and λ is a length scale. In the boson exchange model, the extra interaction due to boson exchange for non-Newtonian gravity may lead to finite-range Yukawa potential, i.e.,

$$V_Y(r) = \frac{g^2}{4\pi}\frac{1}{r}\exp(-m_b r), \tag{2}$$

where $m_b = 1/\lambda$ is the mass of a boson exchanged, and the Yukawa interaction coupling g is related to the strength parameter by $g = \sqrt{4\pi|\alpha|Gm_1m_2}$. At the mean-field level, the effect of non-Newtonian gravity on the energy density of dense quark matter can be calculated by averaging the corresponding Yukawa-type non-Newtonian gravitational potential

$$\begin{aligned} E_Y &= \frac{1}{2V}\int 3n_b(\boldsymbol{r}_1)V_Y(r)3n_b(\boldsymbol{r}_2)\mathrm{d}^3\boldsymbol{r}_1\mathrm{d}^3\boldsymbol{r}_2 \\ &= 18\pi n_b^2\int_0^R V_Y(r)r^2\mathrm{d}r \end{aligned} \tag{3}$$

with $r = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$ being the distance between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, R being the radius of SQM, $V = 4\pi R^3/3$ being the volume, and $n_b(\boldsymbol{r}_1)$ and $n_b(\boldsymbol{r}_2)$ being the local baryon number density at $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ respectively. Substituting Equation (2) into Equation (3) and carrying out the integration gives

$$E_Y = \frac{9}{2}\frac{g^2n_b^2}{m_b^2}\Big[1 - (1 + m_bR)\exp(-m_bR)\Big]. \tag{4}$$

Because the system we are considering is in principle very large, we take $R \to \infty$ in Equation (4) and accordingly have

$$E_Y = \frac{9}{2}\frac{g^2}{m_b^2}n_b^2. \tag{5}$$

From the fundamental differential equality of thermodynamics, one can easily get

$$P_Y = n_b^2\frac{\mathrm{d}}{\mathrm{d}n_b}\left(\frac{E_Y}{n_b}\right) = \frac{9}{2}\frac{g^2n_b^2}{m_b^2}\left(1 - \frac{2n_b}{m_b}\frac{\partial m_b}{\partial n_b}\right). \tag{6}$$

For simplicity, a constant mass independent of the density has been assumed for the boson mass (Krivoruchenko et al. 2009; Wen & Zhou 2013; Yan & Wen 2013), i.e., $\frac{\partial m_b}{\partial n_b} = 0$. Then, the corresponding pressure due to non-Newtonian gravity is given by

$$P_Y = E_Y = \frac{9}{2}\frac{g^2}{m_b^2}n_b^2. \tag{7}$$

Inside a strange star, quark matter maintains chemical equilibrium by weak processes. We therefore have the following conditions for chemical potentials μ_i ($i = u, d, s, e$)

$$\mu_s = \mu_u + \mu_e, \quad \mu_d = \mu_s \equiv \mu. \tag{8}$$

Our next step is to include the strong interactions between quarks. Since no one can model quarks in quantum chromodynamics (QCD) due to the known difficulty in the nonperturbative regime, many phenomenological models have been proposed and applied to study the properties of SQM, e.g., the MIT bag model (Farhi & Jaffe 1984; Chakrabarty 1996; Mustafa & Ansari 1996), the quasiparticle models (Gorenstein & Yang 1995; Peshier et al. 2000; Lu et al. 2016), the conventional perturbative QCD model (Fraga et al. 2001; Fraga & Romatschke 2005), the enhanced pQCD model (Xu et al. 2015a,b), etc. In the present study, we use the equivparticle model which obtains quark confinement by the density dependence of quark masses (Peng et al. 2000a,b, 2008; Xia et al. 2014a,b)

$$m_i = m_{i0} + \frac{D}{n_b^{1/3}}, \tag{9}$$

where m_{i0} is the current quark mass of flavor i, n_b is the baryon number density and D is the confinement parameter to be determined by the stability of SQM. Then the strongly interacting quark matter is viewed as an ideal gas of quarks with equivalent masses, and consequently, other thermodynamical quantities can be obtained from the thermodynamic potential of the free-particle form

$$\begin{aligned} \Omega_0 = -\sum_i \frac{d_i}{48\pi^2}\Big[&\mu_i^*\nu_i(2\mu_i^{*2} - 5m_i^2) \\ &+ 3m_i^4\ln\frac{\mu_i^* + \nu_i}{m_i}\Big], \end{aligned} \tag{10}$$

where d_i is the degeneracy factor (2 for electrons and 6 for quarks), $\nu_i = \sqrt{\mu_i^{*2} - m_i^2}$ is the Fermi momentum and μ_i^* is the effective chemical potential. It is connected to the actual chemical potential μ_i by

$$\mu_i = \mu_i^* + \sum_j \frac{\partial\Omega_0}{\partial m_j}\frac{\partial m_j}{\partial n_i}. \tag{11}$$

In this case, the quark number density n_i is

$$n_i = -\frac{\partial\Omega_0}{\partial\mu_i^*} = \frac{d_i}{6\pi^2}\nu_i^3. \tag{12}$$

So, the chemical potential equilibrium conditions in Equation (8) become

$$\mu_d^* = \mu_s^* \equiv \mu^*, \quad \mu_s^* = \mu_u^* + \mu_e, \tag{13}$$

and the corresponding energy density is

$$E_{\rm Q} = \Omega_0 + \sum_i \mu_i^* n_i \tag{14}$$

while the corresponding pressure is

$$P_{\rm Q} = -\Omega_0 + \sum_{i,j} n_i \frac{\partial \Omega_0}{\partial m_j} \frac{\partial m_j}{\partial n_i}. \tag{15}$$

As can be seen above, due to the density-dependence of the quark masses, an additional term appears in the pressure compared with the thermodynamic formula in the normal case. This approach has been verified to be fully thermodynamically consistent (Xia et al. 2014b) and used in many aspects of nuclear physics (Xia et al. 2014a; Gao et al. 2013; Isayev 2015; Cui et al. 2015; Qauli & Sulaksono 2016).

Including non-Newtonian gravity effects, the total energy density and pressure inside strange stars is

$$E = E_{\rm Q} + E_Y, \quad P = P_{\rm Q} + P_Y. \tag{16}$$

3 NUMERICAL RESULTS AND DISCUSSIONS

In a strange star, the charge neutrality condition of stable quark matter requires the presence of electrons. The electrical neutrality condition is

$$\sum_i q_i n_i = 0, \tag{17}$$

where the index i runs over all particle types, i.e., u, d, s quarks and electrons, and $q_u = 2/3$, $q_d = q_s = -1/3$ and $q_e = -1$. At the same time, baryon number conservation requires

$$n_b = \frac{1}{3}(n_u + n_d + n_s). \tag{18}$$

This equation together with the electrical neutrality condition and the thermodynamic equilibrium condition will give the effective chemical potential of type i particle μ_i^* as a function of baryon density. The total energy density and pressure of quark matter including non-Newtonian gravity effects are determined by Equation (16) for a given baryon density.

The stability window of strange star matter at zero temperature is shown in Figure 1. The bottom-left area is forbidden in which the energy per baryon of two-flavor quark matter is larger than 930 MeV. We therefore choose four typical sets of (g^2/m_b^2, $D^{1/2}$) pairs in the absolutely stable region: (g^2/m_b^2 GeV^2, $D^{1/2}$/MeV) = (0, 155), (2, 155), (5, 151) and (11, 146). These parameter sets are labeled with solid dots in Figure 1. As for the current mass of quarks, we take $m_{u0} = 5$ MeV, $m_{d0} = 10$ MeV and $m_{s0} = 95$ MeV (Particle Data Group 2014).

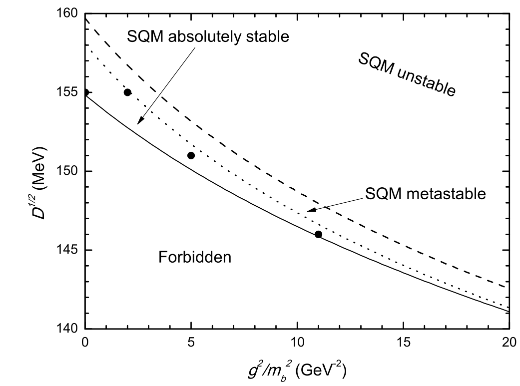

Fig. 1 The stability window of strange star matter in the g^2/m_b^2-$D^{1/2}$ plane. The *solid*, *dotted* and *dashed* curves and the *vertical axes* divide the plane into four parts (from *lower-left* to *upper-right*): the forbidden, absolutely stable, metastable and unstable regions.

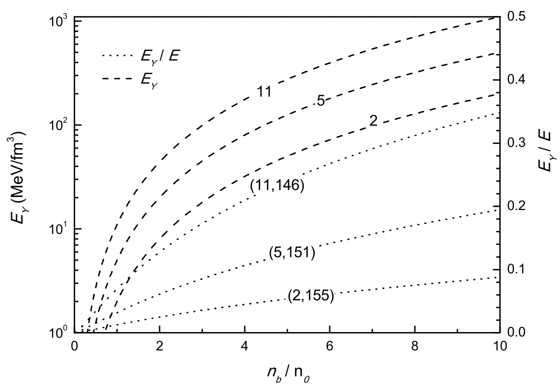

Fig. 2 The extra energy density due to the non-Newtonian component and its relative importance as a function of n_b/n_0, where $n_0 = 0.17\,\mathrm{fm}^{-3}$ is the normal nuclear saturation density.

The density behavior of energy density with inclusion of non-Newtonian gravity is shown in Figure 2. The relative importance of the contribution from the non-Newtonian component has also been plotted in the same figure on the right axis. It is obvious that the non-Newtonian energy density increases with increasing density. This is understandable in view of Equation (3) in which the non-Newtonian contributed energy density E_Y is a monotonically increasing function of baryon density. Also, the ratio of E_Y/E increases almost linearly with

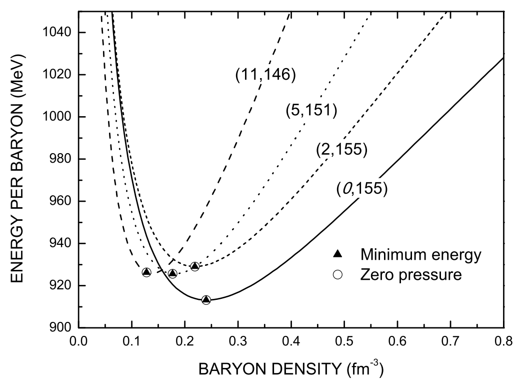

Fig. 3 Density behavior of the energy per baryon with and without the inclusion of non-Newtonian gravity effects. Zero pressure (*open circles*) exactly corresponds to the minimum energy per baryon (the *triangles*).

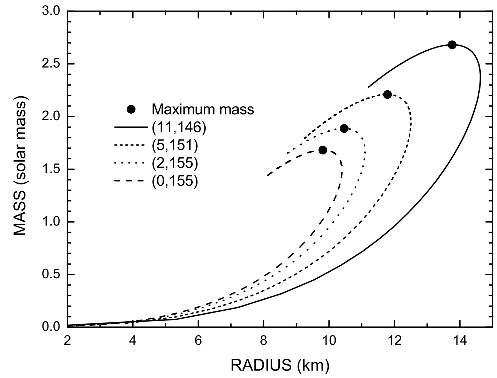

Fig. 4 The mass-radius relation of strange stars with several typical sets of model parameters.

increasing density for different non-Newtonian gravity parameters. This means that the energy density from non-Newtonian gravity plays an increasingly important role with increasing density.

In Figure 3, we show the density behavior of the energy per baryon, E/n_b, with and without non-Newtonian gravity. The open circles represent the zero pressure point which is exactly located at the minimum energy per baryon for each curve. This condition ensures that the theoretical approach used in the present paper with inclusion of non-Newtonian gravity is thermodynamically self-consistent. Actually, there is a detailed discussion on the problem of thermodynamic consistency for phenomenological models in Xia et al. (2014b). It has been shown that for any thermodynamically consistent models, the following discriminant

$$\Delta = P - n_b^2 \frac{\mathrm{d}}{\mathrm{d}n_b}\left(\frac{E}{n_b}\right) \tag{19}$$

is zero at arbitrary density, where P and E are the model-given pressure and energy density. The pressure P and energy density E derived in the present paper satisfy the zero discriminant condition at arbitrary density, i.e., $\Delta = 0$.

The mass-radius relation is one of the important features of compact stars. In the preceding section, we have obtained the EoS of dense quark matter with and without non-Newtonian gravity. Then for a given central baryon density, one can self-consistently solve the Tolman-Oppenheimer-Volkoff (TOV) equations supplemented with the boundary conditions and the EoS of quark matter to give the mass and radius of a strange star. For a detailed discussion of consistently solving the TOV equations, one can refer to Peng et al. (2000b).

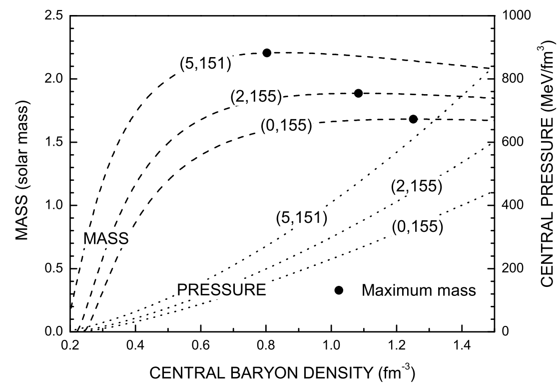

Fig. 5 Mass and central pressure of strange stars versus central baryon density with different non-Newtonian gravitational parameters.

In Figure 4, we present our results for the mass-radius relation of strange stars with several typical parameter sets. The corresponding curve for zero value of the non-Newtonian gravity parameter is also included for comparison. In both cases, it is obvious from the figure that a larger value of non-Newtonian gravitational parameter g^2/m_b^2 tends to support a larger maximum mass as well as radius of strange stars at fixed $D^{1/2}$. The corresponding radius of strange star sequences with maximum mass is in the range of 10$\sim$13.5 km. We can conclude from Figure 4 that the non-Newtonian gravitational parameter g^2/m_b^2 has a significant influence on the structure of strange stars.

We show the star mass as a function of the central baryon density for three typical parameter sets in Figure 5 with the corresponding central pressure given on the right axis. It is obvious that first the star mass increases rapidly with increasing central baryon density up

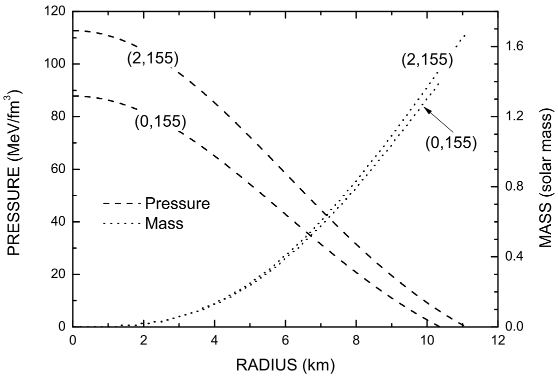

Fig. 6 The pressure and mass profiles of strange stars with typical parameter sets at fixed central baryon density $n_c = 0.6\,\mathrm{fm}^{-3}$.

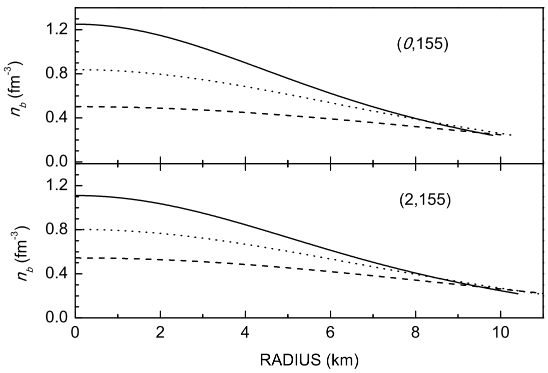

Fig. 7 The density profiles of strange stars for different values of non-Newtonian gravitational parameter g^2/m_b^2. The *solid curve* in each panel is for the highest central density corresponding to a strange star with maximum mass.

to the maximum mass which is denoted by a solid circle at a critical density $n_{\max}$, then for $n_b > n_{\max}$, the star mass decreases and the star becomes mechanically unstable. In addition, a larger value of g^2/m_b^2 makes the EoS of quark matter stiffer, so that the corresponding star mass increases faster than that of a smaller value case with variation of the central baryon density.

The pressure in a strange star is not uniformly distributed along the radial direction. We have shown the pressure and mass profiles at fixed central baryon density $n_c = 0.6\,\mathrm{fm}^{-3}$ for different sets of parameters in Figure 6. In the figure, the pressure is signified by dashed curves while the star masses are represented by dotted curves. It can be seen that the inclusion of non-Newtonian gravity tends to have larger central pressure and tends to support a larger radius as well as mass of strange stars at fixed baryon density. However, the pressure always decreases with the increase of radius till $r = R$, at which the pressure vanishes at the surface of a strange star. In other words, different from neutron stars, quark stars are self-bound.

The density profiles of strange stars for different parameter sets are shown in Figure 7. The solid curves in each panel correspond to the central baryon density of the quark stars with maximum mass. The baryon density decreases with increasing radius in each curve, which exhibits the same behavior of pressure as shown in Figure 6. However, there is a difference: the baryon density approaches a non-vanishing value at the surface of strange stars, i.e., strange stars have a sharp surface density while the pressure is zero at the star surface.

4 SUMMARY

We have studied the stability of strange star matter in beta equilibrium with inclusion of non-Newtonian gravity in the equivparticle model. It is found that SQM still has the possibility of absolute stability with inclusion of non-Newtonian gravity. However, the coupling to the boson mass ratio should be limited to a narrow range of the SQM stability window. In this case, the EoS of SQM becomes stiffer so that it can support more massive strange stars. The maximum mass of strange stars can be as big as, or even larger than, two times the solar mass. The maximum radius of strange stars also becomes bigger.

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