Determination of the free lunar libration modes from ephemeris DE430

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Abstract The Moon's physical librations have been extensively studied, and elaborate researches have been developed for the purpose of deriving accurate modes of free librations. Our motivation comes from the Planetary and Lunar Ephemeris DE430 by JPL/NASA, which was created in April 2013, and is reported to be the most accurate lunar ephemeris today using the data from Gravity Recovery and Interior Laboratory (GRAIL). Therefore, the residuals after fitting the model have reduced owing to improvement in the libration models, and the free librations embedded in the Euler angles have also improved. We use Fourier analysis to extract the approximate frequencies from DE430 and then a quadratic interpolation method is used to determine higher accuracy frequencies. With the frequencies, the linear least-squares fitting method is employed to fit the lunar physical librations to DE430. From this analysis we identified the three modes of free physical librations, and estimated the amplitudes as $1.471''$ in longitude, $0.025''$ in latitude and $8.19'' \times 3.31''$ for the wobble, with the respective periods of 1056.16, 8806.9 and 27262.99 d. Since the free librations damp with time, they require recent excitation or a continuous stimulating mechanism in order to sustain.

Key words: Moon — physical librations — Fourier analysis — least squares method

1 INTRODUCTION

There are many moons which keep facing one side of their central planet on average in solar system bodies, including Earth's Moon, most regular satellites of the giant planets and even Pluto's moon Charon. This is called synchronous rotation, which was originally described in work by Cassini in the eighteenth century. For the Moon, because of the aspherical distribution of lunar mass, the rotational dynamics are not uniform. There are two types of librations: forced libration and free libration; the former is caused by time-varying torque on the lunar figure due to attraction by the Earth, Sun and planets, and the latter is a kind of normal mode excited by internal or external activities.

There are three modes in free librations for a solid Moon, and the periods can be calculated by linear theory (Eckhardt 1965; Eckhardt 1981). The first and second modes, the longitudinal and latitudinal modes respectively, are equivalent to the length of day variation and the nutation in the Earth, and have periods of about 2.9 and 81 yr, respectively. The third mode, on the other hand, is like the Chandler wobble in the Earth and is called the wobble mode with a 75 yr period. The free librations are different from the forced librations where amplitudes, phases and periods can all be calculated theoretically from a lunar structure model. Only the periods can be calculated for the free librations, and observations are necessary for determination of the amplitudes.

Due to energy dissipation, the free librations damp out over time. If the free librations exist, there must be some sources of excitation such as impacts that recently occurred. Analyses of Lunar Laser Ranging (LLR) data for detection of free librations have already been performed by some researchers (Calame 1977; Jin & Li 1996; Chapront et al. 1999; Newhall & Williams 1996;

Rambaux & Williams 2011), and the amplitudes and phases have been determined.

The results of previous studies were reflected in the ephemerides DE403 (Standish 1995) and DE421 (Folkner et al. 2009). The present paper extracts free modes from DE430 (Folkner et al. 2014), which is reported to be the most accurate lunar ephemeris today. Compared with DE421, DE430 added new data for the Moon and planets, for example, estimations of the lunar orbit and rotation have been improved through usage of additional LLR data (about 5 yr), i.e. about more than 14% data that were used in the fit, and also an unprecedentedly improved gravity field model from the Gravity Recovery and Interior Laboratory (GRAIL) mission (Williams et al. 2014) was used to create the ephemeris. The free librations depend on the orbital and lunar physical model and also the method of data fit. With the improvement of models and fits with time, we expect estimations of the free librations embedded in the Euler angles to also improve. Therefore, as a preliminary study and first step toward a better understanding of lunar rotation, we try to extract the free libration modes from DE430.

In this paper, newly released lunar ephemeris DE430 is analyzed to extract the free libration modes, and we employ the Fourier analysis method (Newhall & Williams 1996) with our improvement to determine the periods of the librations, and further use the least-squares method to estimate amplitudes of the librations. The trace of pole vectors with direction cosines (P_1, P_2) in space is also analyzed. Discussion is presented in the last section.

2 LIBRATION MODEL

Study of the rotation of the Moon is based on the following three laws provided by Cassini (Beletskii 1972):

(1) The Moon rotates about its polar axis with constant angular velocity equal to that of its revolution around Earth.
(2) The inclination of the Moon's equator with respect to the plane of the ecliptic is constant.
(3) The poles of the Moon's axis of rotation, those of the ecliptic, and those of the lunar orbit, lie in one great circle.

When we use the selenocentric coordinate system, it is supposed that the Moon has the form of a triaxial ellipsoid, with its longest axis directed towards the mean direction of the Earth, and the smallest one is the axis of rotation. We direct OX_1 along the longest axis, OX_3 along the axis of rotation with positive being northwards and OX_2 so as to form a right-handed coordinate system. The directions of these axes with respect to the mean Earth equatorial system at J2000.0 $O\xi_{123}$ are defined by three Euler angles Θ, Ψ and Φ, where Θ is the inclination of the lunar equator with respect to the mean Earth plane, Ψ is the angle along the mean Earth equatorial plane and Φ is the rotation angle along the lunar equatorial plane (see Figure 1). However, the three Euler angles mentioned above are not on the mean ecliptic of date, thus, we need to transform the Euler angles from the J2000.0 equatorial Earth reference system to the mean ecliptic of date reference system by applying four successive rotation matrices BPNR$_1(\varepsilon)$, where $R_1(\varepsilon)$ denotes the rotation along the X-axis, and B, P and N are the frame bias, precession and nutation matrices, respectively. The four matrices can be calculated using SOFA routines (Hohenkerk 2011), then we will get the new Euler angles, θ, ϕ and ψ.

If Cassini's laws hold perfectly when applied, the Euler angles would satisfy the equations

$$\theta = I, \quad \psi = \Omega, \quad \phi = \pi + \lambda - \psi, \quad (1)$$

where I is the mean inclination of the lunar equator with respect to the ecliptic and equal to about $1.543°$, Ω is the longitude of ascending node of the lunar orbit and λ is the mean longitude of the Moon. The deviation of the angles θ, ψ and ϕ from the Cassini state, $I\sigma$, ρ and τ, are designated as the libration angles (Eckhardt 1981), where

$$\theta = I + \rho, \quad \psi = \Omega + \sigma, \quad \phi = \pi + \lambda - \psi + \tau. \quad (2)$$

Usually, σ is multiplied by I to be comparable to ρ, ρ and $I\sigma$, which are referred to as the latitude librations and τ is referred to as the longitude libration. The pole position of the ecliptic of date on the lunar equator coordinate system is denoted by (P_1, P_2) (Eckhardt 1981; Petrova 1996), which is

$$P_1 = -\sin\theta\sin\phi, \qquad P_2 = -\sin\theta\cos\phi. \quad (3)$$

To describe the pole oscillations around the equilibrium position, we use P_1, P_2 and $\eta = \cos\theta$ as the unit vector to show the pole position in space.

Time variations of the libration angles and polar coordinates are shown in Figures 2 and 3 for 1100 yr. We found that the width of the circle at the top and bottom is thinner than that at the side in the left panel of Figure 3. Although the difference is very small, we will discuss it in Section 5.
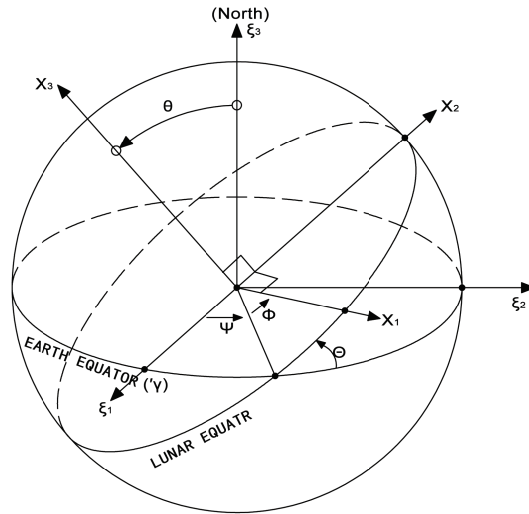
Fig. 1 Euler angles used to describe the orientation of the selenocentric principal axis system $(OX_1X_2X_3)$ relative to the mean Earth equatorial system at J2000 $(O\xi_1\xi_2\xi_3)$.
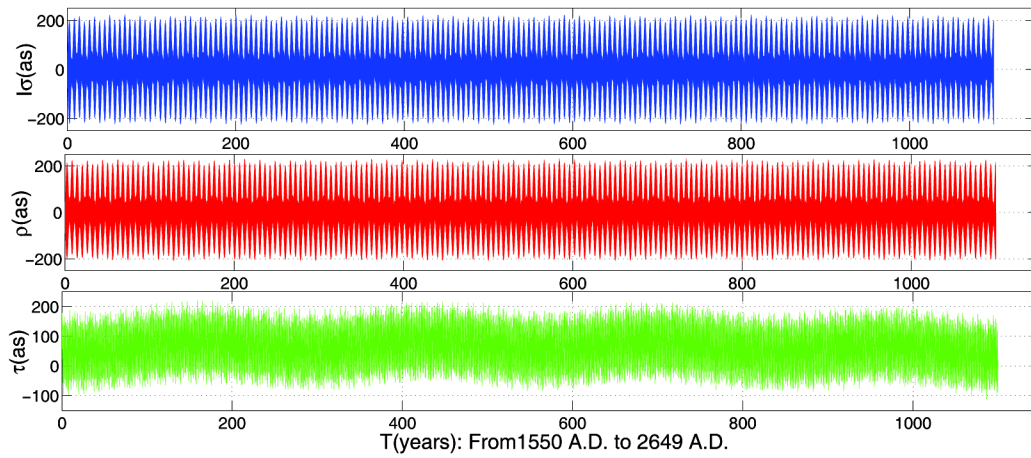

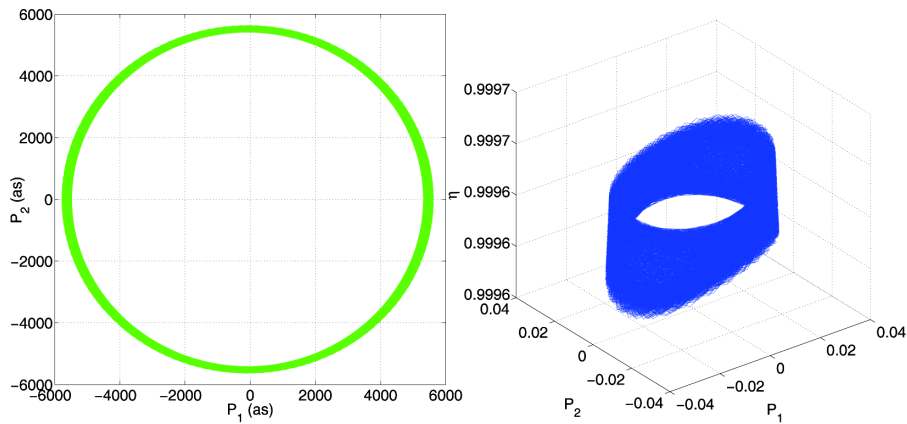**Fig. 2** Temporal evolution of the three librations angles over 1100 yr.

Fig. 3 Ecliptic pole precessional cone over 1100 yr (*left*) and pole oscillation unit vector (P_1, P_2, η) in space (*right*).

3 FREE LIBRATION OF THE MOON

The lunar physical librations have three components of free modes: two components of the pole, P_1 and P_2, and one longitudinal τ. In order to describe lunar rotation, the Moon is usually considered as a solid body, and the elastic deformation and presence of the core alter the proper periods by only small amounts (Williams et al. 2001). In this case, P_1, P_2 and τ which are functions of time can be determined from the well known Euler-Liouville equations,

$$\frac{d(\mathbf{I}\omega)}{dt} + \omega \times \mathbf{I}\omega = N, \qquad (4)$$

where $\mathbf{I}$ is the inertia tensor, N is the total of all externally applied torques and ω is the rotation vector.

We get the linear approximation of the differential equations as Rambaux & Williams (2011), and add a small correction term (Moons 1982) to the third one,

$$\frac{d^2 P_1}{dt^2} - (1-\beta)n\frac{dP_2}{dt} + 4\beta n^2 P_1 = F_1,$$
$$\frac{d^2 P_2}{dt^2} + (1-\alpha)n\frac{dP_1}{dt} + \alpha n^2 P_2 = F_2, \qquad (5)$$
$$\frac{d^2 \tau}{dt^2} + 2.93\gamma n^2 \tau = F_3,$$

where n is the lunar mean motion, $\alpha = (C-B)/A, \beta = (C-A)/B$ and $\gamma = (B-A)/C$, with $A < B < C$ the principal moments of inertia, and $F_i (i = 1, 2, 3)$ are forcing functions. In this case, the eigen-frequencies of this linear system are corresponding to the proper period of lunar librations (Li et al. 2014).

The longitude frequency is obtained as $\omega_{\mathrm{lon}} = \sqrt{2.93\gamma} n$, and the latitude and wobble are $\omega_{\mathrm{lat}} = \frac{3}{2}\beta n$ and $\omega_{\mathrm{wob}} = 2\sqrt{\alpha\beta} n$ (referring to an inertial system) respectively. Using the most recent data from GRAIL, we computed the eigen-frequencies of the free librations (Table 1).

4 FREQUENCY ANALYSIS

The forced and free lunar physical librations are included together in the ephemeris. Firstly, we obtain the Euler angles from DE430 covering 1100 yr (from 1550 A.D. to 2649 A.D.), then we transform the angles from the equatorial reference frame with equinox at J2000.0 to the ecliptic of date. We use $I = 0.02692$ radian and Ω, λ from Simon et al. (1994). To estimate the components of the parameters $I\sigma, \rho, \tau, P_1$ and P_2, a fitting model is needed. Let us assume the solution of those five quanti-

ties has the form,

$$f(t) = \sum_{i=0}^{3} a_i t^i + \sum_{j=1}^{n} (1 + \mu_j t)$$
$$\times \left[C_j \cos(\omega_j t + p_j) + S_j \sin(\omega_j t + p_j) \right]. \quad (6)$$

To estimate the polynomial coefficients a_i, Poisson term μ_j, Fourier series coefficients C_j, S_j and phase p_j by the least-squares method, we need to know the frequencies ω_j precisely. We employ Fourier analysis that is part of the improved version and the estimation procedure as follows:

Step 1 Subtract the mean value, then use the Hanning window function.

Step 2 Find an approximate frequency with the maximum power by applying a Fast Fourier Transform (FFT). Then in the vicinity of the frequency, search for the maximum amplitude of $H(\omega) = \langle f(t), e^{i\omega t} \rangle$ with a quadratic interpolation method, where $\langle, \rangle$ means scalar product.

Step 3 This will give us the best frequency ω and then compute the projection of $f(t)$. The contribution of the first frequency is then subtracted from $f(t)$.

Step 4 Iterate this process until there are enough parameters.

In order to obtain the linear Poisson term μ_j, we divide the data into subsets covering 300 yr and apply Fourier analysis to them. Then, by linear regression of the amplitudes as the first estimation, we determine whether the Poisson term is necessary or not. However, the disadvantage of this method is that any nonlinear term will influence the constant term. Then we apply the least-squares method to fit the form (6).

Table 1 Frequencies and Periods of Free Librations from Theory

Mode	Eigen-frequency (rad d^{-1})	Period (d)	Period (yr)
Longitude	5.9404×10^{-3}	1057.6976	2.8958
Latitude	2.1767×10^{-4}	28865.0161	79.0281
Wobble	2.3202×10^{-4}	27079.8675	74.1406

5 DETERMINATION OF LUNAR FREE LIBRATIONS

The amplitude and phases of free librations can be determined from the fitting process. The wobble mode (P_1, P_2) is the largest in the selenocentric reference system, and the semi-axes of the ellipse are obtained as

Table 2 Free Librations Determined from Ephemeris DE430 and Comparison with the Previous Results

	Longitude blend	Longitude mode	Latitude mode	Wobble mode
Period (d)				
This work	1056.21	1056.16	8806.9	27262.99
Rambaux and Williams (2011)	1056.21	1056.13	8822.88	27257.27
Newhall and Williams (1997)	1056.20	1056.12	8826	27257.27
Chapront et al. (1999)	1056.12	–	8804	27259.29
Amplitude (arc)				
This work	1.8002	1.471	0.025	8.19×3.31
Rambaux and Williams (2011)	1.8080	1.296	0.032	8.183×3.306
Newhall and Williams (1997)	1.8070	1.37	0.022	8.19×3.31
Chapront et al. (1999)	1.8120	–	0.022	8.182
Phase at JD 2451545 (°)				
This work	223.41	210.5	250.67*	161.64
Rambaux and Williams (2011)	223.5	207.0	160.8	161.60
Newhall and Williams (1997)	223.8	208.9	246.4	161.82
Chapront (99)	224.3	–	250.3	161.77

Notes: The difference in phase between this paper and Rambaux & Williams (2011) of the latitude mode is because we used the sine function to fit but they used the cosine function.

$8.19'' \times 3.31''$, with respect to the polar axis. With the wobble period of 74.642 yr, P_1 and P_2 are

$$P_{1\text{wob}} = -3.31'' \sin\left(360° \times \frac{t - 2451545.0}{27262.99} + 161.64°\right),$$
$$P_{2\text{wob}} = 8.19'' \sin\left(360° \times \frac{t - 2451545.0}{27262.99} + 251.67°\right), \quad (7)$$

where 2451545.0 is Julian date of the corresponding epoch, 2000 January 1, at 12^{h}.

As for the latitude mode, thanks to the long term ephemeris series DE430 spanning 1100 yr, we can fit the small mode with an amplitude of $0.025''$ and a period of 24.112 yr (8806.9 d) when referring to the precession with the node, but the motion of the rotation axis will be retrograde with an 79.02 yr period as viewed from the inertial frame (Table 1).

$$\rho_{\text{lat}} = 0.025'' \sin\left(360° \times \frac{t - 2451545.0}{8806.9} + 250.77°\right),$$
$$I\sigma_{\text{lat}} = 0.025'' \sin\left(360° \times \frac{t - 2451545.0}{8806.9} + 160.77°\right). \quad (8)$$

We need to point out that the free libration in longitude is combined with two forced libration terms (Eckhardt 1982) arising from Venus at periods of 2.8923 yr and 2.8921 yr, and the mixed term has a period of 2.8917 yr (1056.207 d) with an amplitude of $1.8002''$, and a phase of $223.41°$ at epoch J2000.0 as

$$\tau_{\text{blend}} = 1.8002''$$
$$\times \sin\left(360° \times \frac{t - 2451545.0}{1056.207} + 223.41°\right). \quad (9)$$

In this case, we use Eckhardt's theory to derive the two forced terms and then subtract them from the mixed term. By the least-squares method, we can get the amplitude and phase of the longitudinal free libration with the assumed period. The obtained free libration in longitude has a period of 1056.16 d, an amplitude of $1.471''$ and a phase of $210.5°$ at epoch J2000.0 as

$$\tau_{\text{free}} = 1.471''$$
$$\times \sin\left(360° \times \frac{t - 2451545.0}{1056.16} + 210.5°\right). \quad (10)$$

Table 2 lists the periods, amplitudes and phases of the free librations that we found and also previous results from ephemeris DE421 (Rambaux & Williams 2011) and DE403 (Newhall & Williams 1996; Chapront et al. 1999).

The ephemerides DE421 and DE430 include a fluid core with an oblate core-mantle boundary. This will lead to the fourth free mode, i.e. free core nutation (FCN). The period of FCN, as given by Rambaux & Williams (2011), is 197 yr in DE421, but in DE430 the f_{c} (core oblateness) changes by 2.46×10^{-4} and it becomes 300 yr. This difference, which comes from the period of the fourth mode, is proportional to $1/f_{\text{c}}$, so a smaller f_{c} causes a longer period.

As for P_1 and P_2, we find the second largest amplitude term has a period of 2190 d (5.99 yr) with different amplitudes of $124.5''$ and $75.4''$. This leads to the unequal width of the circle in the left panel of Figure 3, and we can see it is consistent with the right panel.

6 DISCUSSION

In this paper, based on the most accurate ephemeris DE430, we have determined the periods, amplitudes and phases of three free libration modes by FFT, which are part of the improved version, and the least-squares method. Comparisons with previous results are shown in Table 2, and they are similar but different in detail. We estimated the amplitudes as $1.471''$ in longitude, $0.025''$ in latitude and $8.19'' \times 3.31''$ for the wobble, with the respective periods of 1056.16, 8806.9 and 27262.99 d. Due to the different ephemeris using a different lunar interior model, the different ephemeris gives different values of amplitude and phase. On the other hand, the FCN frequency is very uncertain.

The amplitude of the free libration depends on excitation and damping processes. Free librations will eventually be damped to zero in the absence of excitation. The observational detection of free librations requires recent existence of excitation to counter damping. Without recent excitation, the free librations are therefore expected to be completely damped. Some possible excitation mechanisms have been proposed in the past, although they were not satisfactory. Passage through a resonance was proposed by Eckhardt (1993), and existence of a liquid core was proposed by Yoder (1981).

With the continuation of LLR, more and more distance data between the Moon and Earth have been accumulated. Moreover, during the extension of the Chinese Chang'e-3 lander mission, lunar microwave ranging technology with sub-millicycle accuracy (Ping et al. 2017) was realized and implemented from 2015 in China. The new determination of free librations modes will enable investigation of their mechanisms.

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