RAA 2016 Vol. **16** No. **5**, 85 (10pp) doi: 10.1088/1674–4527/16/5/085
http://www.raa-journal.org *http://iopscience.iop.org/raa*

Research in Astronomy and Astrophysics

The distance duality relation test from the ACT cluster and type Ia supernova data

Lin-Hai Hu[1], Pu-Xun Wu[1,2,3] and Hong-Wei Yu[1,2]

[1] Center of Nonlinear Science and Department of Physics, Ningbo University, Ningbo 315211, China; *wupuxun@nbu.edu.cn*
[2] Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, China
[3] Center for High Energy Physics, Peking University, Beijing 100080, China

Received 2015 December 1; accepted 2016 January 4

Abstract The validity of the cosmic distance-duality (DD) relation is investigated by using 91 measurements of the gas mass fraction of galaxy clusters recently reported by the Atacama Cosmology Telescope (ACT) and the luminosity distance from the Union2.1 type Ia supernova (SNIa) sample independent of any cosmological models and the value of the Hubble constant. We consider four different approaches to derive the gas mass function and two different parameterizations of the DD relation, and find that they have very slight influences on the DD relation test and the relation is valid at the 1σ confidence level. We also discuss the constraints on α and β, which represent the effects of the shapes and colors of the light curves of SNIa, respectively. Our results on α and β are different from those obtained from the ΛCDM model and the galaxy cluster plus SNIa data.

Key words: cosmology: theory — galaxies — clusters: general-distance scale

1 INTRODUCTION

About eighty years ago, Etherington (1933) proved, with two basic assumptions that photons travel along a null geodesic and the photon number is conserved, the famous distance reciprocity law

$$\frac{D_L}{D_A}(1+z)^{-2} = 1\,, \tag{1}$$

which relates the luminosity distance (LD) D_L to the angular diameter distance (ADD) D_A, and is thus also called the distance duality (DD) relation. This reciprocity law is valid for all cosmological models based on Riemannian geometry and is a fundamental keystone for the interpretation of observational data in cosmology.

However, it is plausible that one of the conditions underlying the DD relation may be violated (Csáki et al. 2002; Brax et al. 2013; Avgoustidis et al. 2010, 2012; Bassett & Kunz 2004; Uzan et al. 2004). Therefore, it is necessary and important to check the validity of the DD relation directly from astronomical observations. Based on the ΛCDM model, Uzan et al. (2004) first used the 18 ADD galaxy cluster samples (Reese et al. 2002) to perform this check and found that the DD relation is consistent with observations at the 1σ confidence level (CL). This result was further confirmed by de Bernardis et al. (2006) with a bigger galaxy cluster sample (Bonamente et al. 2006). Meanwhile, if the galaxy cluster data from different models used to describe the galaxy clusters, such as the elliptical and spherical β models (Bonamente et al. 2006; De Filippis et al. 2005), are considered, then it has been found that the consistency between the validity of the DD relation and the elliptical and spherical β models occurs at the 1σ and 3σ CLs, respectively (Holanda et al. 2011). In addition, the DD relation was found to be valid at the 2σ CL from the type Ia supernova (SNIa) standard candles and the standard rulers from the cosmic microwave background (CMB) and baryon acoustic oscillation (BAO) measurements (Lazkoz et al. 2008). Once current CMB observational data were used, Ellis et al. (2013) discovered that the DD relation cannot be violated by more than 0.01% between decoupling and today. However, all of these results are model-dependent since they are usually obtained by comparing the observed values with the corresponding theoretical ones in an assumed cosmological model.

For a model-independent check on the DD relation, Equation (1) shows that the observed LD and ADD of some objects at the same redshift are needed. Current observations indicate that the LD can be estimated by means of a standard candle, such as an SNIa, and the values of ADD can be obtained from the X-ray plus Sunyaev-Zel'dovich (SZ) effect exhibited by galaxy clusters, the gas mass fraction measurement in galaxy clusters, or BAO observations. However, the redshifts of the observed LD and ADD do not usually match. Recently, Holanda et al. (2010) proposed a method to circumvent this problem, in which for a given galaxy cluster data point, one SNIa whose redshift is the closest to the cluster's within the range $\Delta z = |z - z_{\rm SNIa}| < 0.005$ is selected, and found that the DD relation is valid at the 2σ CL for the Constitution SNIa (Hicken et al. 2009) and the elliptical galaxy cluster sample (Bonamente et al. 2006), but is violated for the Constitution SNIa and the spherical galaxy cluster sample (De Filippis et al. 2005) beyond the 3σ CL. Replacing the Constitution SNIa data set with the Union2 data set (Amanullah et al. 2010), the consistency between the DD relation and observations was improved to be 1σ CL for the elliptical model, and 3σ CL for the spherical model (Li et al. 2011). Similar results have also been obtained in (Holanda et al. 2012b; Nair et al. 2011; Fu et al. 2011; Cao & Liang 2011; Holanda 2012; Holanda et al. 2012a; Lima et al. 2011; Cardone et al. 2012; Li et al. 2013; Holanda & Busti 2014; Jhingan et al. 2014; Meng et al. 2012; Liang et al. 2013; Gonçalves et al. 2015a, 2012) with different methods to match the redshifts of the observed LD and ADD data. In addition, the Gaussian process has been used to test the DD relation (Hees et al. 2014; Zhang 2014).

When testing the DD relation, the observed LD is provided by the SNIa data released with the distance modulus μ, which depends on four nuisance parameters: H_0, M, α and β, where H_0 is the Hubble constant, M is the SNIa's peak absolute magnitude, and α and β represent the effects of the shapes and colors respectively of the light curve of SNIa. Usually, $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$ is taken, and M, α and β are determined in a given cosmological model. Using the BAOs to provide the ADD and the Union2.1 SNeIa (Suzuki et al. 2012), Wu et al. (2015) analyzed the effect of the uncertainty of H_0 on the DD relation test by letting H_0 be a free parameter, and found that the value of H_0 significantly affects the result. In addition, Yang et al. (2013) proposed an improved method to overcome the defect that the distance moduli of SNeIa are dependent on a given cosmological model and Hubble constant, and found that the DD relation is consistent with galaxy clusters and SNeIa at the 1σ CL.

Recently, the Atacama Cosmology Telescope (ACT) survey reported the latest cluster mass data sample from 91 galaxy clusters (Hasselfield et al. 2013), and proposed four models to estimate the corresponding cluster mass M_{500}, which determines the value of $f_{\rm gas}$ by a semi-empirical relation (Vikhlinin et al. 2009). Here, M_{500} is defined as the mass measured within the radius R_{500}, at which the enclosed mean density is 500 times the critical density at the cluster redshift. By combining the 91 measurements of the gas mass fraction of the galaxy clusters from ACT and the Union2.1 SNIa compilation, Gonçalves et al. (2015b) tested the validity of the DD relation and found that it can be verified within the 1σ CL. However, the distance moduli of Union2.1 data that are used to derive the LD in Gonçalves et al. (2015b) are given with $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$ and M, α and β are determined in a flat ΛCDM model, which indicates that the result remains model-dependent. In this paper, we plan to reanalyze the validity of the DD relation from the 91 data points related to gas mass function of the galaxy clusters from ACT and the SNIa data with the method in which M, α and β are allowed to be free parameters. For the SNIa data, we also use the Union2.1 complication (Suzuki et al. 2012).

2 OBSERVATIONAL DATA

We use two observational data sets to check the validity of the DD relation. One is the gas mass fraction ($f_{\rm gas}$), obtained from the M_{500} data sets provided by the ACT galaxy cluster survey. The other is the LD (D_L) derived from the Union2.1 SNIa sample.

2.1 The Gas Mass Fraction $f_{\rm gas}$

The gas mass fraction is defined as $f = M_{\rm gas}/M_{\rm tot}$ (Sasaki 1996), where $M_{\rm gas}$ is the gas mass and $M_{\rm tot}$ is the total mass (including baryonic mass and dark matter) of a galaxy cluster. The ratio $f_{\rm gas}$ is constant over cosmic history since $M_{\rm gas}$ depends on the gravitational potential of a galaxy cluster. The gas mass $M_{\rm gas}(< R)$ within a radius R can be derived by X-ray observations and is related to the distance through $M_{\rm gas}(< R) \propto D_L D_A^{3/2}$ (Sasaki 1996). However, the total mass $M_{\rm tot}$ within a given radius R can be written as $M_{\rm tot}(< R) \propto D_A$. Thus, from the definition of the gas mass fraction, we have that

$$f = M_{\rm gas}/M_{\rm tot} \propto D_L D_A^{1/2} \,. \tag{2}$$

If the DD relation is valid, the usual expression in the literature $f_{\rm gas} \propto D_A^{3/2}$ can be obtained.

Following Gonçalves et al. (2012), we assume that the general expression of the gas mass function has the form

$$f_{\rm gas}(z) = N \frac{D_A^{*1/2} D_L^*}{D_A^{1/2} D_L} \,. \tag{3}$$

Here, the symbol * denotes the corresponding quantities in the fiducial model, which is the ΛCDM with $\Omega_{M0} = 0.3$ in our analysis. N is a normalization factor, which contains all the information involved in the astrophysical modeling of the cluster. Since it is a nuisance quantity in our analysis, we marginalize over it.

The observed value of $f_{\rm gas}^{\rm obs}$ can be obtained from the total mass by using the following semi-empirical relation (Vikhlinin et al. 2009)

$$\begin{aligned} f_{\rm gas}^{\rm obs} &\equiv (0.7/h)^{-1.5} \tilde{f}_{\rm gas}^{\rm obs} \\ &= (0.7/h)^{-1.5}[0.130 + 0.039 \log_{10} M_{15}] \,, \end{aligned} \tag{4}$$

where M_{15} is the total mass of the cluster M_{500} in the units of $10^{15} h^{-1} M_\odot$. Apparently, the key point is to obtain a reliable and precise value of the total mass of the cluster M_{500}. The ACT team (Hasselfield et al. 2013) used four different approaches to estimate M_{500} from the cluster SZ signal strength. The first method proposed by the ACT team adopts a one-parameter family of possible solutions for the Universal Pressure Profile (UPP) with associated scaling laws, derived from X-ray measurements of nearby clusters, as a baseline model. The cluster mass obtained using this method is termed $M_{500}^{\rm UPP}$. The second one is based on the structure formation simulations of Bode et al. (2012), where the density and temperature of the intracluster medium are modeled as a virialized ideal gas (M_{500}^{B12}). The third one considers non-thermal pressure and an adiabatic model described in Trac et al. (2011) ($M_{500}^{\rm non}$). The last method uses the galaxy velocity dispersions to estimate dynamically the cluster mass (Sifón et al. 2013) ($M_{500}^{\rm dyn}$).

2.2 SNIa

The SNIa is regarded as a standard candle in our universe, which provides a direct measurement of the cosmic expansion history. Usually, SNIa data are released in the form of the distance modulus μ, which is relative to the LD through the expression

$$\mu = 5\log_{10}\frac{D_L}{\rm Mpc} + 25\ . \tag{5}$$

In astronomy, the peak absolute magnitudes M of SNeIa are nearly identical. If the rest-frame peak magnitude $m^{\rm max}$ is known, then the distance modulus (μ) can be obtained by the formula $\mu = m^{\rm max} - M$. However, the peak luminosities of different SNeIa are not exactly the same and are related to the shapes and colors of the light curves of the SNeIa (Guy et al. 2005). So, the formula for the distance modulus needs to be corrected, which depends on the light curve fitting method, such as MLCSC2K2 (Jha et al. 2007) or SALT2 (Guy et al. 2007), proposed to parameterize the light curves of SNIa. For the SALT2 light curve fitter, which has been used in the Union2 and Union2.1 SNIa samples, the modified distance modulus formula has the form

$$\mu_B(\alpha,\beta,M_B)\ =\ m_B^{\rm max} - M_B + \alpha x - \beta c \equiv \tilde{\mu}_B(\alpha,\beta) - M_B\ , \tag{6}$$

where M_B and $m_B^{\rm max}$ correspond to the absolute and observed peak magnitudes in the rest frame B band, respectively. x is the stretch factor, which describes the effects of shapes of light curves on μ, and c is the color parameter, which denotes the influences of the intrinsic color and reddening by dust.

In our analysis, the data set used for SNeIa is the Union2.1 compilation (Suzuki et al. 2012) which contains 580 data points distributed in the redshift range ($0.01 < z < 1.41$). In obtaining the observed μ, α, β and M_B are fitted by minimizing the residuals in the Hubble diagram in the framework of the ΛCDM model with $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$. Here, we allow M_B, α and β to be free parameters and treat them as nuisance ones.

3 THE METHOD AND RESULTS

To check the validity of the DD relation from observations, we parametrize it as

$$\frac{D_L}{D_A}(1+z)^{-2} = \eta(z)\ , \tag{7}$$

and consider two different parameterizations

$$\eta(z) = 1 + \eta_1 z\ , \quad \eta(z) = 1 + \eta_2\frac{z}{1+z}\ , \tag{8}$$

with η_1 and η_2 being two constants that need to be constrained. If the observational data allow $\eta_1 = 0$ and $\eta_2 = 0$, the DD relation is valid. Clearly, the observed D_L and D_A at the same redshift are needed. However, the observed LD from SNeIa and ADD from the gas mass function of galaxy clusters usually do not share the same redshift. Here, we use the following method to match them. For a data point representing a given galaxy cluster, we select the corresponding SNIa point with its redshift being the closest to the cluster's within the range $\Delta z = |z_{\rm SNIa} - z_{\rm cluster}| \leq \Delta$.

In order to reduce the systematic error from the difference in redshift between the SNIa and the gas mass function of a galaxy cluster, a very small value of Δ ($\Delta = 0.001$) is considered. Only 55 galaxy cluster data satisfy this selection criterion ($\Delta z \leq 0.001$). Thus, to keep almost all data from the gas mass function of galaxy, we consider another larger value of Δ ($\Delta = 0.01$) to select the SNIa. In this case there are 90 galaxy cluster data satisfying the selection criterion except for the cluster ACT@CCL J0342.0+0105 (Hasselfield et al. 2013), which gives $\Delta z = 0.013$.

Combining Equations (3), (5) and (7), one can obtain

$$\begin{aligned}\mu_{\rm cluster}(\eta, N_h, z) &= \frac{5}{3}\log_{10}\frac{N^2(1+z)^6 D_A^{*3}\,\eta(z)}{f_{\rm gas}^2} + 25 \\ &= \frac{5}{3}\log_{10}\frac{(1+z)^6 D_A^{*3}\,\eta(z)}{\tilde{f}_{\rm gas}^2} + \frac{10}{3}\log_{10}(N_h) + 25 \\ &\equiv \tilde{\mu}_{\rm cluster}(z,\eta) + \frac{10}{3}\log_{10}(N_h)\ ,\end{aligned} \tag{9}$$

which is the distance modulus of a galaxy cluster data point built from the measurement of $\tilde{f}_{\rm gas}$ and the DD relation. Here, $N_h = N(h/0.7)^{3/2}$.

Now, we use the χ^2 statistic to constrain η_1 and η_2,

$$\begin{aligned}\chi^2(\alpha,\beta,M_{BN},\eta) &= \sum_i \frac{\left[\mu_B(\alpha,\beta,M_B;z_i) - \mu_{\rm cluster}(\eta,N_h;z_i)\right]^2}{\sigma_{\rm total}^2(z_i)} \\ &= \sum_i \frac{\left[\tilde{\mu}_B(\alpha,\beta;z_i) - \tilde{\mu}_{\rm cluster}(\eta;z_i) - M_{BN}\right]^2}{\sigma_{\rm total}^2(z_i)}\ ,\end{aligned} \tag{10}$$

where

$$M_{BN} = M_B + \frac{10}{3}\log_{10}(N_h) \tag{11}$$

and the uncertainty $\sigma_{\rm total}^2(z_i)$ is given by

$$\sigma_{\rm total}^2(z) = \sigma_m^2(z) + \alpha^2\sigma_x^2(z) + \beta^2\sigma_c^2(z) + \left[\frac{10\sigma_{\tilde{f}_{\rm gas}}(z)}{3\ln 10\tilde{f}_{\rm gas}(z)}\right]^2. \tag{12}$$

Here σ_m, σ_x, σ_c and $\sigma_{f_{\rm gas}}$ are the errors of $m_B^{\rm max}$, x, c and $f_{\rm gas}$ respectively.

The joint probability density of these parameters can be obtained from Equation (10),

$$P(\alpha,\beta,M_{BN},\eta) = A\exp\left(-\frac{1}{2}\chi^2(\alpha,\beta,M_{BN},\eta)\right), \tag{13}$$

where A is a normalization coefficient, which makes $\int P\, d\alpha d\beta dM_{BN} d\eta = 1$. Since α, β and M_{BN} are nuisance parameters, we integrate over them to obtain the probability distribution of η

$$P(\eta) = \int_{-\infty}^{+\infty} P(\alpha,\beta,M_{BN},\eta)\, d\alpha d\beta dM_{BN}. \tag{14}$$

Actually, it is unnecessary to integrate α, β and M_{BN} from $-\infty$ to $+\infty$. For instance, the value of A for these parameters in a 4σ interval is the same as that in a 6σ interval when the result is accurate to 10^{-6}. Thus we just calculate the values of $\chi^2(\alpha,\beta,M_{BN},\eta)$ for these parameters in a suitable interval instead of an infinite interval. The last step is to calculate 1σ and 2σ CLs of the parameter, which satisfy $\chi^2 - \chi^2_{\rm min} \le 1$ and $\chi^2 - \chi^2_{\rm min} \le 4$, respectively (Press et al. 1992).

Using the above procedure, the best-fitting values of η, α, β and M_{BN} are obtained when χ^2 take its minimum value. The probability distributions of η_1 and η_2 are derived from Equation (14), which are shown in Figures 1–4.

Figures 1 and 2 present the results of η_1 with $\Delta z \le 0.01$ and $\Delta z \le 0.001$, respectively. Figures 3 and 4 are the results of η_2 with $\Delta z \le 0.01$ and $\Delta z \le 0.001$, respectively. We can see that the DD relation is consistent with the observations at the 1σ CL except for the case of the constraint on η_2 from the gas mass function obtained by the UPP method which supports this relation at the 2σ CL. The same as what was obtained by Yang et al. (2013), the best-fitting values of η_1 and η_2 are positive when α, β and M_{BN} are allowed to be free parameters. However, this is different from what is given in Gonçalves et al. (2015b) where the best-fitting values of η_1 and η_2 are negative, which are determined with $M_B = -19.321$, $\alpha = 0.121$ and $\beta = 2.47$ obtained from the ΛCDM model with $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$. Thus, different values of α, β and M_B have apparent effects on the DD relation test, which is similar to what was obtained by Wu et al. (2015) where it was found that the validity of the DD relation sensitively depends on the value of H_0.

In Tables 1–4, the best-fitting values of α, β and M_{BN} and their 1σ CLs are shown. Figures 5–10 show the allowed regions in planes (η_i, α), (η_i, β) and (η_i, M_{BN}) with

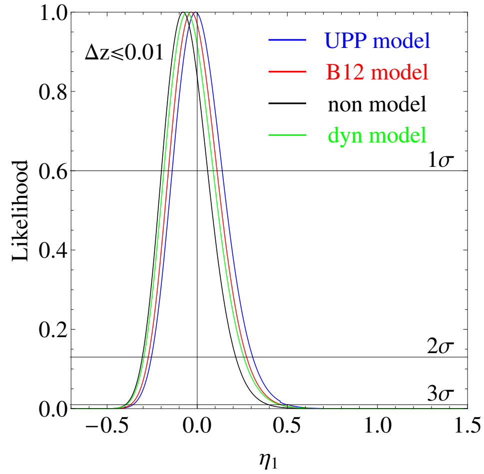

Fig. 1 Likelihood distribution of η_1 when $\Delta z \leq 0.01$.

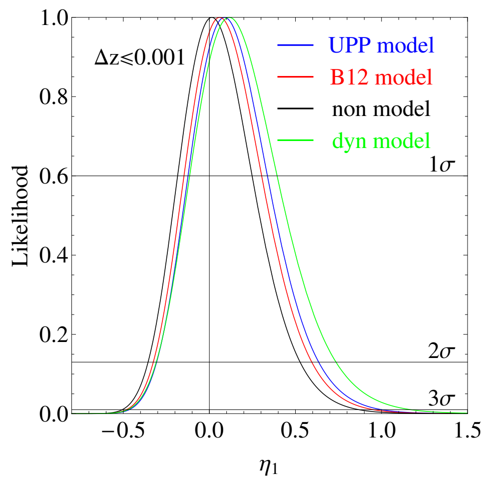

Fig. 2 Same as Fig. 1, but for $\Delta z \leq 0.001$.

Table 1 Constraints on η_1, α, β and M_{BN} with $\Delta z \leq 0.01$

data set		η_1	α	β	M_{BN}
$f_{\rm gas}^{\rm UPP}$ + SNIa	the best-fitting values	0.111	0.161	2.756	–22.462
	1σ	$0.111^{+0.129}_{-0.115}$	$0.163^{+0.026}_{-0.025}$	$2.784^{+0.220}_{-0.205}$	$-22.466^{+0.038}_{-0.040}$
$f_{\rm gas}^{\rm non}$ + SNIa	the best-fitting values	0.027	0.161	2.776	–22.345
	1σ	$0.027^{+0.119}_{-0.105}$	$0.162^{+0.026}_{-0.025}$	$2.804^{+0.220}_{-0.205}$	$-22.349^{+0.037}_{-0.038}$
$f_{\rm gas}^{\rm dyn}$ + SNIa	the best-fitting values	0.019	0.155	2.777	–22.378
	1σ	$0.020^{+0.128}_{-0.092}$	$0.157^{+0.027}_{-0.026}$	$2.810^{+0.241}_{-0.225}$	$-22.382^{+0.040}_{-0.044}$
$f_{\rm gas}^{B12}$ + SNIa	the best-fitting values	0.079	0.159	2.752	–22.392
	1σ	$0.079^{+0.125}_{-0.111}$	$0.161^{+0.026}_{-0.024}$	$2.780^{+0.220}_{-0.205}$	$-22.396^{+0.038}_{-0.040}$

$i = 1$ or 2. One can see that the constraints on M_{BN} are independent of the parameterizations and the values of $\Delta(z)$, but they are affected by different methods used to derive the gas mass function, and tension appears at the 1σ CL between the UPP model (Hasselfield et al. 2013) and the one given in (Trac et al. 2011). Different parameterizations and different methods used to obtain the gas mass function have slight effects on the values of α and β, but they are strongly affected by the values of $\Delta(z)$. $\Delta z \leq 0.01$ favors large values of α and β. It is just opposite for $\Delta z \leq 0.001$.

In the Union2.1 SNIa sample (Suzuki et al. 2012), the distance modulus is given with $\alpha = 0.121 \pm 0.007$ and $\beta = 2.47 \pm 0.06$ at the 1σ CL obtained from the flat ΛCDM model. The value of α is less than our result. The value of β is also less than ours from $\Delta z \leq 0.01$, but it is larger when $\Delta z \leq 0.001$. In addition, Yang et al. (2013) found that at the 1σ CL $\alpha = 0.34^{+0.08}_{-0.06}$ and $\beta = 4.19^{+0.58}_{-0.62}$ for the galaxy cluster data from the spherical model, and $\alpha = -0.04^{+0.10}_{-0.09}$ and $\beta = 4.35^{+1.20}_{-1.73}$ in the case of the elliptical model, which are apparently different from our results.

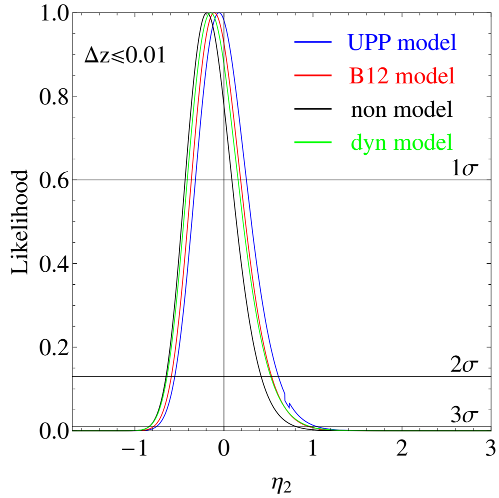

Fig. 3 Likelihood distribution of η_2 when $\Delta z \leq 0.01$.

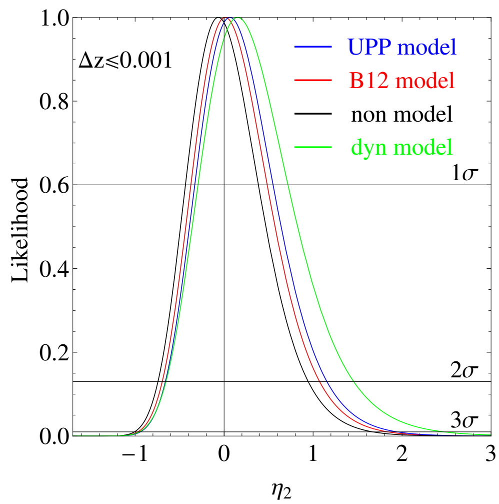

Fig. 4 Same as Fig. 3, but for $\Delta z \leq 0.001$.

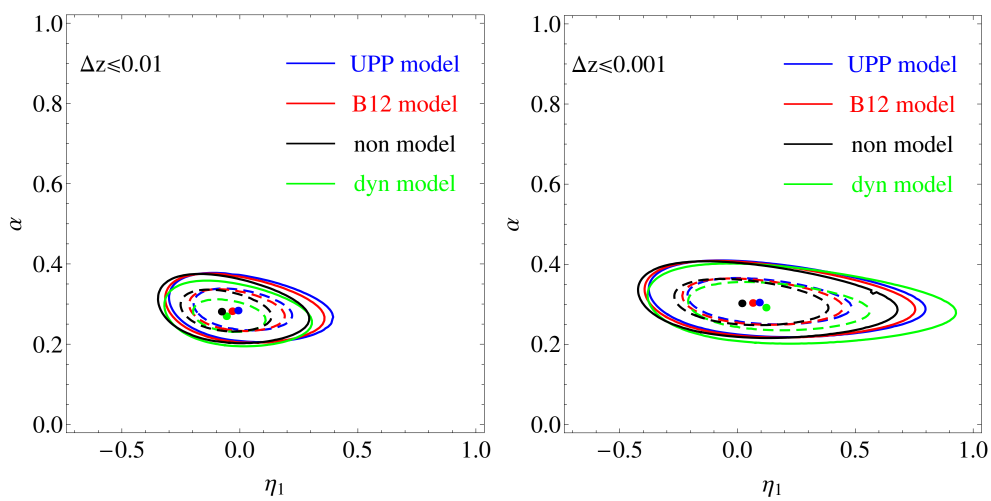

Fig. 5 Confidence regions at the 1σ and 2σ levels in the plane (η_1, α).

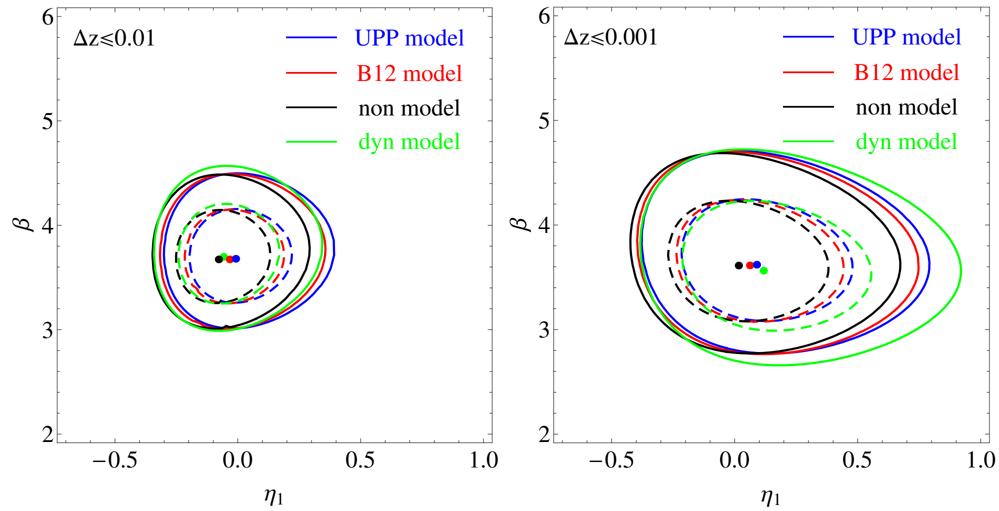

Fig. 6 Same as Fig. 5, but for the plane (η_1, β).

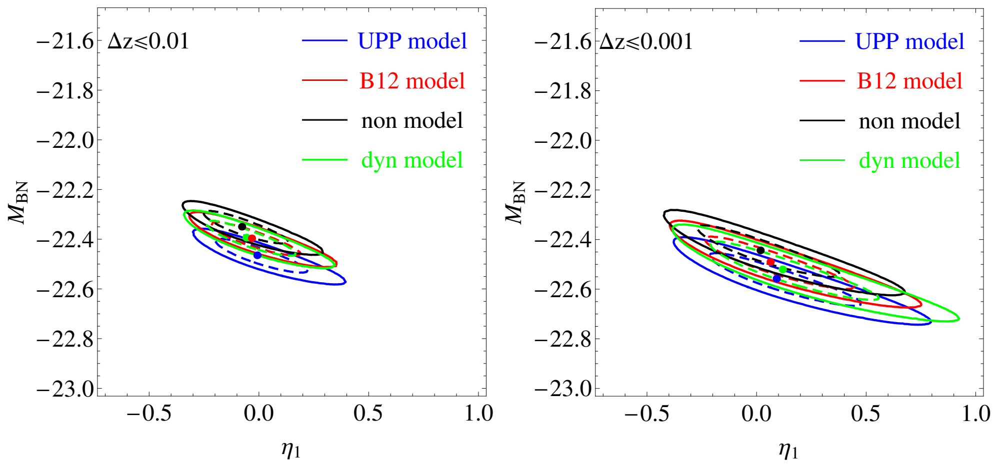

Fig. 7 Same as Fig. 5, but for the plane (η_1, M_{BN}).

Table 2 Constraints on η_1, α, β and M_{BN} with $\Delta z \leq 0.001$

data set		η_1	α	β	M_{BN}
	the best-fitting values	0.155	0.137	2.392	–22.484
$f_{\rm gas}^{\rm UPP}$+SNIa	1σ	$0.155^{+0.194}_{-0.166}$	$0.140^{+0.027}_{-0.027}$	$2.431^{+0.252}_{-0.231}$	$-22.491^{+0.054}_{-0.059}$
	the best-fitting values	0.061	0.138	2.422	–22.365
$f_{\rm gas}^{\rm non}$+SNIa	1σ	$0.061^{+0.181}_{-0.155}$	$0.140^{+0.028}_{-0.026}$	$2.461^{+0.254}_{-0.233}$	$-22.372^{+0.054}_{-0.058}$
	the best-fitting values	0.0537	0.128	2.328	–22.399
$f_{\rm gas}^{\rm dyn}$+SNIa	1σ	$0.057^{+0.201}_{-0.169}$	$0.131^{+0.029}_{-0.026}$	$2.375^{+0.277}_{-0.253}$	$-22.409^{+0.061}_{-0.066}$
	the best-fitting values	0.118	0.136	2.392	–22.414
$f_{\rm gas}^{B12}$+SNIa	1σ	$0.118^{+0.188}_{-0.162}$	$0.139^{+0.028}_{-0.026}$	$2.431^{+0.252}_{-0.231}$	$-22.421^{+0.084}_{-0.058}$

4 DISCUSSIONS AND CONCLUSIONS

The distance reciprocity law is a relation between the LD and ADD, and plays a very important role in astrophysics and cosmology. To check the validity of the DD relation, the observed D_L and D_A at the same redshift are needed. The observed D_L can be provided by the SNIa sample, and the observed D_A can come from the galaxy clusters or BAO observations. Recently, the ACT survey reported the largest gas mass function data set of galaxy clusters. Combining this sample with the Union2.1 SNIa data, Gonçalves et al. (2015b) tested the DD relation and found that it is accommodated at the 1σ CL. However, in Gonçalves et al. (2015b), the observed luminosity distance is determined in the ΛCDM model with $H_0 = 70\,\rm km\,s^{-1}\,Mpc^{-1}$, and it has been found that the

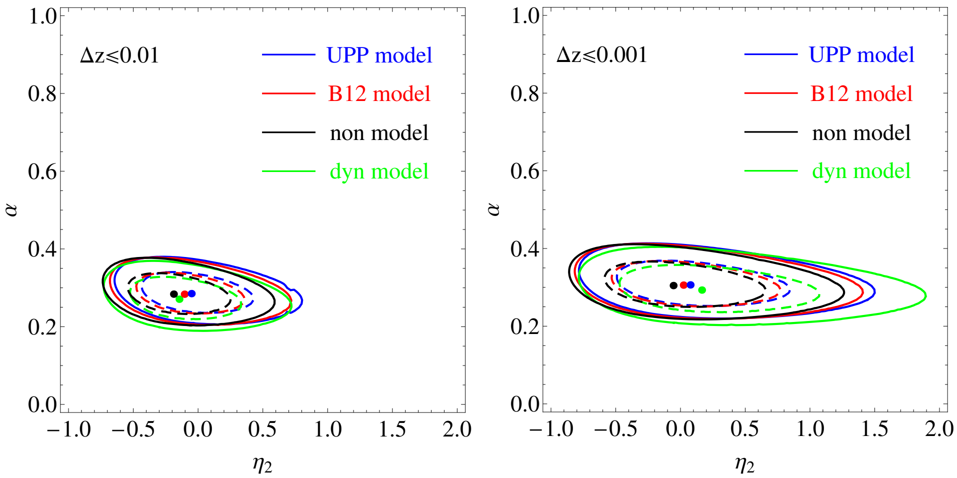

Fig. 8 Confidence regions at the 1σ and 2σ levels in the plane (η_2, α).

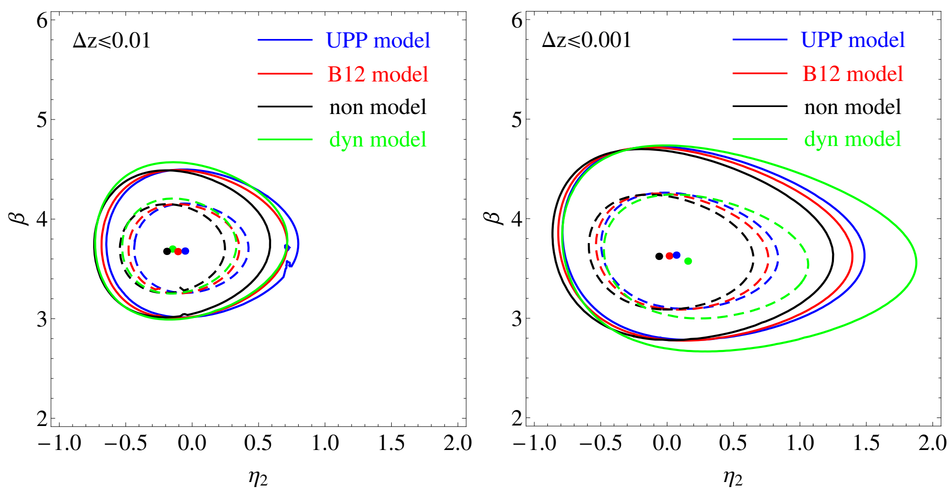

Fig. 9 Same as Fig. 8, but for the plane (η_2, β).

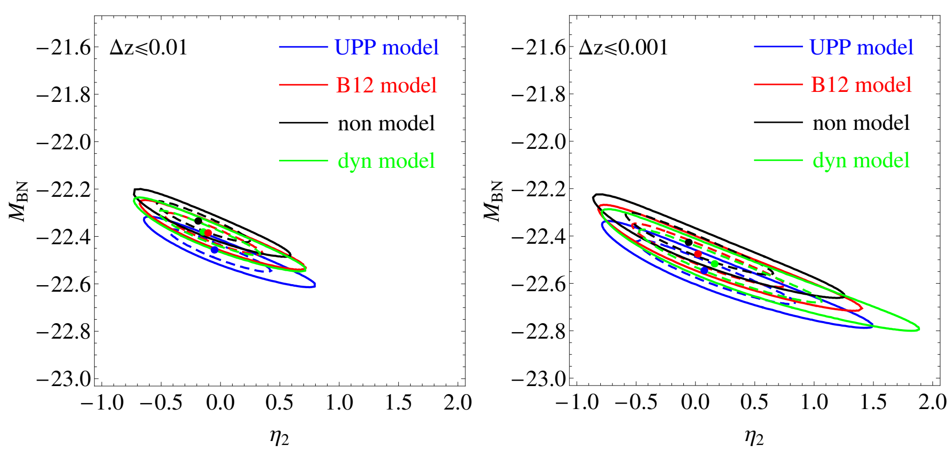

Fig. 10 Same as Fig. 8, but for the plane (η_2, M_{BN}).

test of the DD relation is sensitively dependent on the value of H_0 (Wu et al. 2015).

In this paper, we generalize the Gonçalves et al. (2015b) work by using the observed D_L from the Union2.1 SNIa sample independent of any cosmological models and the value of H_0. We consider four different approaches to obtain the gas mass function and two different parameterizations of the DD relation given in Equation (8). We find that the DD relation is valid at the 1σ CL except for the case of the constraint on η_2 from the gas mass function ob-

Table 3 Constraints on η_2, α, β and M_{BN} with $\Delta z \leq 0.01$

data set		η_2	α	β	M_{BN}
$f_{\rm gas}^{\rm UPP}$+SNIa	the best-fitting values	0.284	0.159	2.751	–22.484
	1σ	$0.281^{+0.289}_{-0.252}$	$0.160^{+0.026}_{-0.024}$	$2.778^{+0.219}_{-0.204}$	$-22.488^{+0.050}_{-0.055}$
$f_{\rm gas}^{\rm non}$+SNIa	the best-fitting values	0.102	0.160	2.778	–22.356
	1σ	$0.100^{+0.263}_{-0.230}$	$0.161^{+0.026}_{-0.025}$	$2.806^{+0.218}_{-0.205}$	$-22.361^{+0.048}_{-0.052}$
$f_{\rm gas}^{\rm dyn}$+SNIa	the best-fitting values	0.081	0.154	2.780	–22.388
	1σ	$0.081^{+0.287}_{-0.246}$	$0.156^{+0.027}_{-0.025}$	$2.812^{+0.240}_{-0.224}$	$-22.394^{+0.054}_{-0.059}$
$f_{\rm gas}^{B12}$+SNIa	the best-fitting values	0.209	0.158	2.750	–22.409
	1σ	$0.207^{+0.279}_{-0.243}$	$0.159^{+0.026}_{-0.024}$	$2.777^{+0.218}_{-0.204}$	$-22.413^{+0.049}_{-0.054}$

Table 4 Constraints on η_2, α, β and M_{BN} with $\Delta z \leq 0.001$

data set		η_2	α	β	M_{BN}
$f_{\rm gas}^{\rm UPP}$+SNIa	the best-fitting values	0.395	0.135	2.389	–22.515
	1σ	$0.392^{+0.427}_{-0.353}$	$0.138^{+0.027}_{-0.026}$	$2.426^{+0.252}_{-0.229}$	$-22.524^{+0.071}_{-0.080}$
$f_{\rm gas}^{\rm non}$+SNIa	the best-fitting values	0.197	0.137	2.423	–22.386
	1σ	$0.195^{+0.390}_{-0.324}$	$0.140^{+0.027}_{-0.027}$	$2.462^{+0.252}_{-0.232}$	$-22.396^{+0.071}_{-0.076}$
$f_{\rm gas}^{\rm dyn}$+SNIa	the best-fitting values	0.193	0.128	2.333	–22.422
	1σ	$0.198^{+0.454}_{-0.362}$	$0.131^{+0.029}_{-0.026}$	$2.380^{+0.277}_{-0.253}$	$-22.422^{+0.068}_{-0.106}$
$f_{\rm gas}^{B12}$+SNIa	the best-fitting values	0.313	0.135	2.391	–22.440
	1σ	$0.310^{+0.412}_{-0.342}$	$0.138^{+0.027}_{-0.026}$	$2.429^{+0.250}_{-0.230}$	$-22.450^{+0.071}_{-0.078}$

tained by the UPP method which supports this relation at the 2σ CL. We also discuss the constraints on M_{BN}, α and β. The constraints on M_{BN} are mainly affected by different methods used to derive the gas mass function, and disagreement appears at the 1σ CL between the UPP model (Hasselfield et al. 2013) and the one given in Trac et al. (2011). However, these different methods have slight effects on the values of α and β, which are apparently influenced by the values of $\Delta(z)$. Our results on α and β are different from those obtained from the ΛCDM model (Suzuki et al. 2012) and the galaxy cluster plus SNIa data (Yang et al. 2013).

Acknowledgements This work was supported by the National Natural Science Foundation of China under Grant Nos. 11175093, 11222545, 11435006 and 11375092; and by the Specialized Research Fund for the Doctoral Program of Higher Education under Grant No. 20124306110001.

References

Amanullah, R., Lidman, C., Rubin, D., et al. 2010, ApJ, 716, 712
Avgoustidis, A., Burrage, C., Redondo, J., Verde, L., & Jimenez, R. 2010, J. Cosmol. Astropart. Phys., 10, 024
Avgoustidis, A., Luzzi, G., Martins, C. J. A. P., & Monteiro, A. M. R. V. L. 2012, J. Cosmol. Astropart. Phys., 2, 013
Bassett, B. A., & Kunz, M. 2004, Phys. Rev. D, 69, 101305
Bode, P., Ostriker, J. P., Cen, R., & Trac, H. 2012, arXiv:1204.1762
Bonamente, M., Joy, M. K., LaRoque, S. J., et al. 2006, ApJ, 647, 25
Brax, P., Burrage, C., Davis, A.-C., & Gubitosi, G. 2013, J. Cosmol. Astropart. Phys., 11, 001
Cao, S., & Liang, N. 2011, RAA (Research in Astronomy and Astrophysics), 11, 1199
Cardone, V. F., Spiro, S., Hook, I., & Scaramella, R. 2012, Phys. Rev. D, 85, 123510
Csáki, C., Kaloper, N., & Terning, J. 2002, Physical Review Letters, 88, 161302
de Bernardis, F., Giusarma, E., & Melchiorri, A. 2006, International Journal of Modern Physics D, 15, 759
De Filippis, E., Sereno, M., Bautz, M. W., & Longo, G. 2005, ApJ, 625, 108
Ellis, G. F. R., Poltis, R., Uzan, J.-P., & Weltman, A. 2013, Phys. Rev. D, 87, 103530
Etherington, I. M. H. 1933, Philosophical Magazine, 15, 761
Fu, X.-Y., Wu, P.-X., Yu, H.-W., & Li, Z.-X. 2011, RAA (Research in Astronomy and Astrophysics), 11, 895
Gonçalves, R. S., Alcaniz, J. S., Carvalho, J. C., & Holanda, R. F. L. 2015a, Phys. Rev. D, 91, 027302
Gonçalves, R. S., Bernui, A., Holanda, R. F. L., & Alcaniz, J. S. 2015b, A&A, 573, A88
Gonçalves, R. S., Holanda, R. F. L., & Alcaniz, J. S. 2012, MNRAS, 420, L43
Guy, J., Astier, P., Nobili, S., Regnault, N., & Pain, R. 2005, A&A, 443, 781
Guy, J., Astier, P., Baumont, S., et al. 2007, A&A, 466, 11
Hasselfield, M., Hilton, M., Marriage, T. A., et al. 2013, J. Cosmol. Astropart. Phys., 7, 008
Hees, A., Minazzoli, O., & Larena, J. 2014, Phys. Rev. D, 90, 124064
Hicken, M., Wood-Vasey, W. M., Blondin, S., et al. 2009, ApJ, 700, 1097

Holanda, R. F. L. 2012, International Journal of Modern Physics D, 21, 1250008
Holanda, R. F. L., & Busti, V. C. 2014, Phys. Rev. D, 89, 103517
Holanda, R. F. L., Gonçalves, R. S., & Alcaniz, J. S. 2012a, J. Cosmol. Astropart. Phys., 6, 022
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B. 2010, ApJ, 722, L233
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B. 2011, A&A, 528, L14
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B. 2012b, A&A, 538, A131
Jha, S., Riess, A. G., & Kirshner, R. P. 2007, ApJ, 659, 122
Jhingan, S., Jain, D., & Nair, R. 2014, Journal of Physics Conference Series, 484, 012035
Lazkoz, R., Nesseris, S., & Perivolaropoulos, L. 2008, J. Cosmol. Astropart. Phys., 7, 012
Li, Z., Wu, P., & Yu, H. 2011, ApJ, 729, L14
Li, Z., Wu, P., Yu, H., & Zhu, Z.-H. 2013, Phys. Rev. D, 87, 103013
Liang, N., Li, Z., Wu, P., et al. 2013, MNRAS, 436, 1017
Lima, J. A. S., Cunha, J. V., & Zanchin, V. T. 2011, ApJ, 742, L26
Meng, X.-L., Zhang, T.-J., Zhan, H., & Wang, X. 2012, ApJ, 745, 98
Nair, R., Jhingan, S., & Jain, D. 2011, J. Cosmol. Astropart. Phys., 5, 023
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in FORTRAN. The Art of Scientific Computing (2nd ed.; Cambridge: Cambridge Univ. Press)
Reese, E. D., Carlstrom, J. E., Joy, M., et al. 2002, ApJ, 581, 53
Sasaki, S. 1996, PASJ, 48, L119
Sifón, C., Menanteau, F., Hasselfield, M., et al. 2013, ApJ, 772, 25
Suzuki, N., Rubin, D., Lidman, C., et al. 2012, ApJ, 746, 85
Trac, H., Bode, P., & Ostriker, J. P. 2011, ApJ, 727, 94
Uzan, J.-P., Aghanim, N., & Mellier, Y. 2004, Phys. Rev. D, 70, 083533
Vikhlinin, A., Burenin, R. A., Ebeling, H., et al. 2009, ApJ, 692, 1033
Wu, P., Li, Z., Liu, X., & Yu, H. 2015, Phys. Rev. D, 92, 023520
Yang, X., Yu, H.-R., Zhang, Z.-S., & Zhang, T.-J. 2013, ApJ, 777, L24
Zhang, Y. 2014, arXiv:1408.3897