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Delay of planet formation at large radius and the outward decrease in mass and gas content of Jovian planets

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Abstract A prominent observation of the solar system is that the mass and gas content of Jovian planets decrease outward with orbital radius, except that, in terms of these properties, Neptune is almost the same as Uranus. In previous studies, the solar nebula was assumed to preexist and the formation process of the solar nebula was not considered. It was therefore assumed that planet formation at different radii started at the same time in the solar nebula. We show that planet formation at different radii does not start at the same time and is delayed at large radii. We suggest that this delay might be one of the factors that causes the outward decrease in the masses of Jovian planets. The nebula starts to form from its inner part because of the inside-out collapse of its progenitorial molecular cloud core. The nebula then expands outward due to viscosity. Material first reaches a small radius and then reaches a larger radius, so planet formation is delayed at the large radius. The later the material reaches a planet's location, the less time it has to gain mass and gas content. Hence, the delay tends to cause the outward decrease in mass and gas content of Jovian planets. Our nebula model shows that the material reaches Jupiter, Saturn, Uranus and Neptune at $t = 0.40$, 0.57, 1.50 and 6.29×10^6 yr, respectively. We discuss the effects of time delay on the masses of Jovian planets in the framework of the core accretion model of planet formation. Saturn's formation is not delayed by much time relative to Jupiter so that they both reach the rapid gas accretion phase and become gas giants. However, the delay in formation of Uranus and Neptune is long and might be one of the factors that cause them not to reach the rapid gas accretion phase before the gas nebula is dispersed. Saturn has less time to go through the rapid gas accretion, so Saturn's mass and gas content are significantly less than those of Jupiter.

Key words: planetary systems — planets and satellites: formation — planets and satellites: gaseous planets — planets and satellites: individual (Jovian planets) — protoplanetary disks

1 INTRODUCTION

In modern science, the model for the formation of the solar system is constructed under the framework of the nebular hypothesis (e.g., Lissauer 1993). Although the nebular hypothesis is successful in general, describing detailed physical processes in the history of the solar system and interpreting all the related observations are still challenging for researchers. Two outstanding observations that any theory must explain are planet masses and compositions. Terrestrial planets (Mercury, Venus, Earth and Mars) have small masses and are composed of rocky material while Jovian planets (Jupiter, Saturn, Uranus and Neptune) have large masses and contain both heavy elements and H and He. It seems that the difference between terrestrial planets and Jovian planets is understood. An observational fact is that the mass and gas content of Jovian planets show an outward decrease with their orbital radius from Jupiter, except that, in terms of these properties, Neptune is almost the same as Uranus. The masses of Jupiter, Saturn, Uranus and Neptune are 318, 95, 15 and 17 $M_{\oplus}$, respectively. Models for Jovian planets suggest that their masses of H and He are 276–307, 64–76, 0.5–5.0 and 0.5–4.7 $M_{\oplus}$, respectively (Guillot 1999; Podolak et al. 2000; Guillot 2005).

In previous studies, the solar nebula is assumed to preexist and the formation process of the solar nebula is not considered. It is therefore assumed that planet formation at different radii starts at the same time in the solar nebula. In this paper, we show that planet formation does not start at the same time at different radii in the solar nebula and is delayed at a large radius. The length of the delay increases with radius. We tentatively suggest that this delay might play a role in the outward decrease in mass and gas content of Jovian planets. The physical cause of this delay is the evolutionary expansion of the solar nebula. The nebula gains mass from the gravitational collapse of its progenitorial molecular cloud core. Because of the inside-out collapse, the nebula starts to form from the inner part and then expands outward due to the action of viscosity. This expansion of the nebula brings material to the outer part. Material in the nebula progressively reaches larger radii and material arrives later at these larger radii. A planet begins its formation process when material reaches its location. Therefore, planet formation is delayed at large radii and the length of the delay increases with its radius from the center of the nebula. Since the region of Jupiter gets material first, it begins its formation process first, and then Saturn, Uranus and Neptune form in turn. A planet has to gain mass before the nebula is dispersed. The later the material reaches a planet's location, the less time it has to gain mass and gas content. Hence, the delay tends to cause the outward decrease in mass and gas content of Jovian planets.

This paper is organized as follows. In Section 2, we illustrate the expansion of the solar nebula, explain the physics of this expansion, and show the delay of planet formation at large radius. In Section 3, we discuss the possible effects of this delay on the mass and gas content of Jovian planets. In Section 4, we summarize our results.

2 THE EVOLUTIONARY EXPANSION OF THE SOLAR NEBULA AND DELAY IN PLANET FORMATION AT LARGE RADIUS

2.1 Nebula Model

We use the standard protostar+disk formation model (e.g., Shu et al. 1987; McKee & Ostriker 2007). According to the standard model, a protostar+disk system forms from the collapse of a molecular cloud core. Because of the slight rotation of the cloud core, not all of the material falls directly toward the center to form a protostar. To conserve angular momentum, a protostar+disk system forms from the collapsing cloud core. For our solar system, this disk is called "the solar nebula." According to the current theory of planet formation, a planet forms in such a nebula (Lissauer 1993). The disk gains mass from the collapse of the core. The protostar gains mass from both the collapse and accretion from the disk. At the beginning of the collapse ($t = 0$), the masses of both the protostar and nebula are zero. In our model, the initial state is a cloud core without either the protostar or the nebula. At $t = 0$, the nebula does not exist. The material that falls onto the midplane from the gravitational collapse of the molecular cloud core forms the nebula. Because of the inside-out collapse, the nebula starts to form from the inner part and then it expands outward.

To explain the evolutionary expansion of the solar nebula, we use the nebula model of Jin & Sui (2010). In the following, we briefly review the parts of the model to explain the expansion of the nebula and the delay in planet formation at large radii. For the details of the model, see Jin & Sui (2010). In our model, the calculation starts at the onset of the core collapse. From previous work (Shu 1977; Cassen & Moosman 1981; Nakamoto & Nakagawa 1994), the mass influx onto the nebula is given by

$$S(R,t) = \begin{cases} \dfrac{\dot{M}_{\rm core}}{4\pi R R_d(t)} \left[1 - \dfrac{R}{R_d(t)}\right]^{-1/2} & \text{if } \dfrac{R}{R_d(t)} < 1; \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where t is the time, R is the cylindrical radius, and $\dot{M}_{\rm core}$ is the accretion rate from the cloud core to the protosun+nebula system. The centrifugal radius $R_d(t)$ is expressed as

$$R_d(t) = \frac{1}{16} a\omega^2 t^3 = 31 \left(\frac{\omega}{10^{-14}\ \ {\rm s}^{-1}}\right)^2 \left(\frac{T_{\rm core}}{10\ \ {\rm K}}\right)^{1/2} \left(\frac{t}{5\times 10^5\ \ {\rm yr}}\right)^3\ {\rm AU}, \tag{2}$$

where ω is the angular velocity of the core, a is the isothermal sound speed of the core and $T_{\rm core}$ is the core temperature. The accretion rate is given by $\dot{M}_{\rm core} = 0.975a^3/G$ (Shu 1977), where G is the gravitational constant. The time $t = 0$ is chosen to be the starting time of the collapse of the cloud core. At $t = 0$, the masses of both the protosun and the solar nebula are zero. The initial density profile of this core is $\rho(r) = a^2 r^{-2}/(2\pi G)$, where r is the radial distance to the center of the cloud core (e.g., Shu 1977; Shu et al. 1987; Evans 1999; McKee & Ostriker 2007). The size of a core is about 0.1 pc (e.g., McKee & Ostriker 2007). The typical temperature of a cloud core is $T_{\rm core} = 10$ K and the typical value of the core mass, $M_{\rm core}$, is $1\ M_\odot$. The collapse lasts until the core mass is consumed. The duration of the collapse is $M_{\rm core}/\dot{M}_{\rm core}$.

The equation describing the evolution of surface density of the solar nebula is given by

$$\begin{aligned} \frac{\partial\Sigma}{\partial t} \ &=\ \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\Sigma\nu R^{1/2})\right] + S(R,t) \\ &+S(R,t)\left\{2 - 3\left[\frac{R}{R_d(t)}\right]^{1/2} + \frac{R/R_d(t)}{1+[R/R_d(t)]^{1/2}}\right\}, \end{aligned} \tag{3}$$

where ν is the kinematic viscosity and Σ is the surface density. On the right hand side of Equation (3), the first term represents the viscous diffusion term and the second term is the mass influx from the collapse. The last term is due to the difference in specific angular momentum between the material in the nebula and that from the collapse.

In the calculation of viscosity, the alpha prescription $\nu = \alpha c_s H$ (Shakura & Sunyaev 1973) is adopted, where H is the half thickness of the nebula, c_s is the local sound speed and α is a dimensionless parameter. The α values adopted in our disk model can be expressed as,

$$\alpha = \begin{cases} \alpha_{\rm GI} = 0.02 & \text{if } Q_{\rm min} < Q_{\rm crit}; \\ \alpha_{\rm MRI} & \text{if } Q_{\rm min} > Q_{\rm crit} \text{ and } \alpha_{\rm MRI} > \alpha_{\rm min}; \\ \alpha_{\rm min} & \text{otherwise} \ \ , \end{cases} \tag{4}$$

where $\alpha_{\rm GI}$ is the viscosity caused by gravitational instability, $Q_{\rm min}$ is the minimum of the Toomre parameter, Q (Toomre 1964), in a disk, $Q_{\rm crit}$ stands for the critical value of Q, $\alpha_{\rm MRI}$ is the viscosity caused by the magnetohydrodynamic turbulence due to magnetorotational instability (MRI; Balbus & Hawley 1991; Fleming & Stone 2003) and $\alpha_{\rm min}$ is the minimum of α adopted in the region where the computed $\alpha_{\rm MRI}$ is too low for a gravitationally stable disk. For a gravitationally unstable disk, $\alpha_{\rm GI}$ is the dominant viscosity. For $\alpha_{\rm GI}$ (the first line of Eq. (4)), we use the treatment of section 3.6.4 from Hueso & Guillot (2005). The criterion for gravitational instability of a disk is Toomre parameter,

$$Q = \frac{c_s\Omega}{\pi G\Sigma}, \tag{5}$$

where Ω is the Keplerian angular velocity. If Q is smaller than $Q_{\rm crit}$, then a disk is unstable. In our calculation, we adopt $Q_{\rm crit} = 1$. For a gravitationally stable disk, $\alpha_{\rm MRI}$ is considered to be the dominant viscosity (Balbus & Hawley 1991). We compute $\alpha_{\rm MRI}$ according to the numerical results of Fleming & Stone (2003). For a gravitationally stable disk, in the region where $\alpha_{\rm MRI}$ is too small, we use the minimum of α, $\alpha_{\rm min}$, to drive the evolution of the disk. In the region where the MRI does not operate (too small $\alpha_{\rm MRI}$), the viscosity could be caused by hydrodynamic processes. We adopt $\alpha_{\rm min} = 10^{-4}$ which is a middle value for the order of magnitude from Chambers (2006), Dubrulle (1993), Klahr & Bodenheimer (2003), Richard (2003), Dubrulle et al. (2005) and Fleming & Stone (2003).

2.2 Expansion of the Nebula and Delay in Planet Formation at Large Radius

The initial conditions of the protosun+nebula system are determined by the properties of its progenitor molecular cloud core: ω, $T_{\rm core}$ and $M_{\rm core}$. The properties of the nebula are related to these parameters. We run calculations of the solar nebula with various ω for $M_{\rm core} = 1\ M_\odot$ and $T_{\rm core} = 10$ K (typical value). A suitable value of ω is inferred to be roughly $0.3 \times 10^{-14}\ {\rm s}^{-1}$ for our solar system by using the constraint that ω cannot be so low that mass cannot reach Neptune and so high that too much mass is spread beyond Neptune. The range of observed angular velocity of cloud cores is $0.1 - 13 \times 10^{-14}\ {\rm s}^{-1}$ (e.g., Goodman et al. 1993).

The physical reason for the expansion of the solar nebula is as follows. The expansion can be understood by analyzing Equation (3). The second term on the right-hand side of Equation (3) is the mass influx onto the nebula and represents the mass supply from the inside-out collapse of the molecular cloud core (Shu 1977). Equation (1) shows that the influx depends on $R_d(t)$. For radius inside $R_d(t)$, the nebula directly acquires mass from the collapse. For radius outside $R_d(t)$, the mass influx is zero. From Equation (2), $R_d(t)$ starts from zero and increases with t. The nebula starts to form from the inner part. The area of the nebula with nonzero influx expands with t. $R_d(t)$ reaches its maximum when the collapse stops. The maximum is 5.7 AU for the solar nebula. The maximum of $R_d(t)$ is small in comparison to the orbital radii of Jovian planets except Jupiter. By the expansion of $R_d(t)$, material does not reach Jovian planets except for Jupiter.

However, the nebula can expand to a radius beyond $R_d(t)$ by viscous stress. It is mainly viscous stress that makes the nebula expand to the location of each planet. This can be understood by considering the viscous term, the first term on the right-hand side of Equation (3). The timescale for the nebula to expand to a radius R_0 is the viscous diffusion timescale, which is given by

$$t_\nu \approx \frac{R_0}{v_R} \sim \frac{R_0^2}{3\nu} = \frac{1}{6\pi}\alpha^{-1}\left(\frac{H}{R}\right)^{-2}\left(\frac{R_0}{1\ \ {\rm AU}}\right)^{3/2}\left(\frac{M_*}{M_\odot}\right)^{-1/2}\ {\rm yr}\,, \tag{6}$$

where v_R is the radial drift velocity and M_* is the mass of the protosun. If we take $\alpha = 10^{-4}$, $H/R = 0.1$ and $M_* = 1\ M_\odot$, then the times for the nebula to expand to 5.2 AU (Jupiter), 9.5 AU (Saturn), 19 AU (Uranus) and 30 AU (Neptune) are 0.63, 1.55, 4.39 and 8.71×10^6 yr, respectively. The solar nebula ($M_{\rm core} = 1\ M_\odot$, $T_{\rm core} = 10$ K and $\omega = 0.3 \times 10^{-14}\ {\rm s}^{-1}$) is gravitationally stable and the viscosity due to gravitational instability does not operate. The region where $\alpha_{\rm MRI}$ is too low is from ∼1 to ∼30 AU where Jovian planets reside. Therefore, in the above estimates, $\alpha_{\rm min}$ is used as the value of α (see Eq. (4)).

The above calculations of times for the nebula to expand to a location of Jovian planets are simple estimates. In our numerical calculations of the nebula model, we need to rigorously define $t_{\rm e}(R_0)$, the time when the nebula expands to a radius R_0. We choose the surface density of the minimum mass solar nebula (MMSN) model (Hayashi 1981) as a standard value. The nebula is considered to expand to R_0 when the local surface density $\Sigma(R_0)$ reaches the surface density given by the MMSN model, $\Sigma_M = 1700(R/1\ {\rm AU})^{-3/2}\ {\rm g\ cm^{-2}}$ (0.35 AU $< R <$ 36 AU). The MMSN model gives the minimum value of surface density needed to form a planet. This, defined as $t_{\rm e}(R_0)$, is considered to be the time when material reaches R_0 and is the starting time of planet formation at R_0.

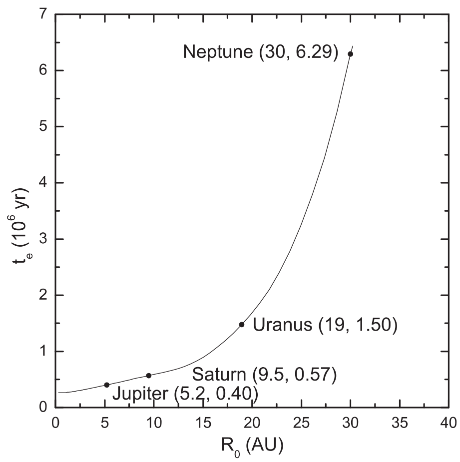

Fig. 1 The time when the nebula expands to a radius R_0, $t_\mathrm{e}(R_0)$, as a function of R_0. The time $t = 0$ is chosen to be the time when the molecular cloud core begins to collapse. For each Jovian planet, we show its radius (in AU) and $t_\mathrm{e}(R_0)$ (in 10^6 yr) in parentheses.

The evolution of the solar nebula is numerically calculated by solving Equation (3). Figure 1 shows $t_\mathrm{e}(R_0)$ as a function of R_0. For each Jovian planet, we show its radius and $t_\mathrm{e}(R_0)$. The nebula expands to the radii of Jupiter, Saturn, Uranus and Neptune at $t = 0.40$, 0.57, 1.50 and 6.29×10^6 yr, respectively, with $t = 0$ being the beginning of the collapse of the molecular cloud core. These times are considered to be the starting times of the formation of Jupiter, Saturn, Uranus and Neptune, respectively. The planet formation process is delayed at large radii and the length of the delay increases with radius.

3 THE POSSIBLE EFFECTS OF DELAY IN PLANET FORMATION ON MASS AND GAS CONTENT OF JOVIAN PLANETS

In this section, we discuss the possible effects of a delay in planet formation on the mass and gas content of Jovian planets and show that this delay might be related to the observation that the mass and gas content of Jovian planets decrease outward with orbital radius from Jupiter, aside from Neptune being almost the same, in terms of these properties, as Uranus.

As we illustrated above, the nebula starts to form from the inner part and expands outward. As the nebula expands, the material reaches the region of Jupiter first and it then reaches Saturn, Uranus and Neptune in turn. Hence, Jupiter begins its formation process first and then Saturn, Uranus and Neptune form in turn. There is a time delay in formation among Jovian planets. A planet ceases to gain gas when the gas nebula is dispersed. The earlier that the material reaches a planet's location, the more time it has to gain mass and gas. Hence, the delay in planet formation at large radius tends to cause the outward decrease in mass and gas content of Jovian planets.

In the core accretion model of planet formation, the formation of a giant planet is divided into three major phases (e.g., Pollack et al. 1996). In the first phase, a solid core forms, which occurs in a few 10^5 yr to a few 10^6 yr. As the solid core becomes massive enough, it accretes its surrounding gas. Pollack et al. (1996) showed that after formation of the solid core, the gas accretion of a giant planet goes through a slow gas accretion phase (the second phase). The slow gas accretion phase takes a few 10^6 yr. Then, the rapid gas accretion phase occurs (the third phase), when a planet gains most of its mass, which takes $\sim 10^5$ yr.

Here we discuss the possible effects of the delay in planet formation on the mass and gas content of Jovian planets in the framework of the core accretion model. From the calculation in the last section, Jupiter starts to form at 0.40×10^6 yr. Afterwards, there is a large amount of material in the region of Jupiter for a long time (several 10^6 yr). There is enough time and material for Jupiter to gain a large mass and a large amount of gas. Hence, Jupiter contains the largest mass and abundances of H and He among the Jovian planets. Saturn starts to form at 0.57×10^6 yr, later than Jupiter but not much later. Both Jupiter and Saturn reach their rapid gas accretion phase within the lifetime of the solar nebula. They significantly increase their mass during this phase, so both Jupiter and Saturn become gas giants with large mass. The delay time of Saturn relative to Jupiter is comparable to the rapid gas accretion time. Saturn has less time to go through the rapid gas accretion phase, so Saturn's mass and gas content are significantly less than those of Jupiter. Uranus and Neptune start to form at 1.50×10^6 yr and 6.29×10^6 yr, respectively, which are much later than Jupiter and Saturn. The delay times of Uranus and Neptune relative to Jupiter are 1.10×10^6 yr and 5.90×10^6 yr, respectively. The delay in Uranus' (and Neptune's) formation is long and might be one of the factors that cause them not to reach the rapid gas accretion phase before the gas nebula is dispersed, so they are ice giants with a much smaller mass than Jupiter and Saturn.

The estimated timescales from our theory are comparable to those of the core accretion model. Jupiter and Saturn start to form at $t = 0.40 \times 10^6$ yr and 0.57×10^6 yr, respectively. In our theory, they both reach the rapid gas accretion phase within the lifetime of the solar nebula, $\tau_{\rm disk}$. If we define $t_{\rm p}$ to be the core formation time plus the slow gas accretion time, this will require $0.57 \times 10^6 \ {\rm yr} + t_{\rm p} < \tau_{\rm disk}$. Uranus and Neptune start to form at $t = 1.50 \times 10^6$ yr and 6.29×10^6 yr, respectively. They do not reach the rapid gas accretion phase within the lifetime of the solar nebula. Their formation must be delayed long enough. This requires $1.50 \times 10^6 \ {\rm yr} + t_{\rm p} > \tau_{\rm disk}$. Hence, our theory requires $0.57 \times 10^6 \ {\rm yr} + t_{\rm p} < \tau_{\rm disk} < 1.50 \times 10^6 \ {\rm yr} + t_{\rm p}$. From observations, the lifetime of a protoplanetary disk is $(1 - 10) \times 10^6$ yr (e.g., Williams & Cieza 2011). This requires that $t_{\rm p}$ is on the order of 10^6 yr. The core accretion model suggests that $t_{\rm p}$ is on the order of 10^6 yr, so our theory is compatible with the core accretion model. Observationally, there are both gas giants (Jupiter and Saturn) with a lot of gas and large mass, and ice giants (Uranus and Neptune) with comparatively little gas and small mass. This suggests that some planets reach the rapid gas accretion phase before the gas nebula is dispersed and some do not, and indicates that $t_{\rm p}$ is comparable to $\tau_{\rm disk}$. The time delay of Uranus and Neptune is a significant fraction of $\tau_{\rm disk}$ and is comparable to $t_{\rm p}$. This time delay is long enough that it is more likely that they do not reach the rapid gas accretion phase.

Saturn's formation is delayed by 1.7×10^5 yr relative to Jupiter. Saturn's mass and gas content are significantly lower than those of Jupiter. Therefore, the timescale of the rapid gas accretion phase, $t_{\rm gas}$, should be on the order of 10^5 yr. This timescale from our theory is compatible with the core accretion model.

An alternative planet formation theory is the gravitational instability model. In this model, a gas clump produced by gravitational instability in a protoplanetary disk can directly contract to form a giant planet (e.g., Kuiper 1951; Cameron 1978; Boss 1997, 1998). A giant planet can form on a very short timescale, $\sim 10^3$ yr (e.g., Boss 1998, 2000). Uranus' formation is delayed by 1.10×10^6 yr relative to Jupiter. When we put our theory into the context of the gravitational instability model, Uranus should finish the entire planet formation process like Jupiter and Saturn because a giant planet forms on a short timescale. Uranus would have a large mass and gas content. Therefore in this scenario, the time delay would not cause the outward decrease in mass and gas content of Jovian planets.

4 DISCUSSION AND CONCLUSIONS

The delay in planet formation at large radii would not change the general conclusions about migration. The predicted timescale of type I migration is very short. This short timescale is a threat to the survival of protoplanetary cores (e.g., Papaloizou et al. 2007). When migration is considered, the effects due to migration become dominant.

In summary, we show that planet formation is delayed at large radii and the length of the delay increases with radius because the solar nebula starts to form from the inner part and then expands outward due to the action of viscosity. We illustrate that the outward decrease in mass and gas content of Jovian planets might be related to this delay in the framework of the core accretion model of planet formation. Our theory infers that the timescale of rapid gas accretion should be on the order of 10^5 yr and the core formation time plus the slow gas accretion time is on the order of 10^6 yr. These timescales are compatible with the core accretion model. In the framework of the gravitational instability model of planet formation, the time delay might not cause the outward decrease in mass and gas content of Jovian planets.

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