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On the radiative and thermodynamic properties of the cosmic radiations using *COBE* FIRAS instrument data: III. Galactic far-infrared radiation

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Abstract Using the three-component spectral model describing the FIRAS average continuum spectra, the exact analytical expressions for thermodynamic and radiative functions of Galactic far-infrared radiation are obtained. The *COBE* FIRAS instrument data in the 0.15–2.88 THz frequency interval at the mean temperatures of $T_1 = 17.72$ K, $T_2 = 14$ K and $T_3 = 6.73$ K are used for calculating the radiative and thermodynamic functions, such as the total radiation power per unit area, total energy density, total emissivity, number density of photons, Helmholtz free energy density, entropy density, heat capacity at constant volume and pressure for the warm, intermediate-temperature and very cold components of the Galactic continuum spectra. The generalized Stefan-Boltzmann law for warm, intermediate-temperature and very cold components is constructed. The temperature dependence of each component is determined by the formula $I^{\rm S-B}(T) = \sigma' T^6$. This result is important when we construct the cosmological models of radiative transfer that can be applied inside the Galaxy. Within the framework of the three-component spectral model, the total number of photons in our Galaxy and the total radiation power (total luminosity) emitted from a surface of the Galaxy are calculated. Their values are $N_{\rm Gtotal} = 1.3780 \times 10^{68}$ and $I_{\rm Gtotal}(T) = 1.0482 \times 10^{36}$ W. Other radiative and thermodynamic properties of the Galactic far-infrared radiation (photon gas) of the Galaxy are calculated. The expressions for astrophysical parameters, such as the entropy density/Boltzmann constant and number density of the Galactic far-infrared photons are obtained. We assume that the obtained analytical expressions for thermodynamic and radiative functions may be useful for describing the continuum spectra of the far-infrared radiation for other galaxies.

Key words: Galaxy: general — galaxies: general

1 INTRODUCTION

It is well-known that spectra from a galaxy, such as the continuum, absorption lines, and emission lines, provide us with information about the galaxy's velocity and mass, the average age of its stellar population, and the star-formation rate (Schneider 2006).

However, it is essential to note that the continuum spectra of the thermal radiation also contain information about the radiative and thermal properties of astronomical objects that emitted it. These properties include the total energy density, total radiation power per unit area, total emissivity, number density of photons, free energy density, entropy density, pressure, heat capacity at constant volume, etc. Based on these data, the following additional information can be obtained: (a) astrophysical parameters, such as the entropy density/Boltzmann constant and number density of photons (Groom 2013[1]); (b) generalized Stefan-Boltzmann law, which is important for the construction of cosmological models that describe radiative transfer; (c) contribution of a photon gas to the total radiation of other particles (protons, alpha and beta particles, etc.); and others. It is therefore of interest to study these properties for different cosmic radiations, such as the cosmic microwave background (CMB) radiation (Mather et al. 1990; Fixsen et al. 1994), the extragalactic far-infrared background radiation (Fixsen et al. 1998), the Galactic far-infrared radiation (Wright et al. 1991; Reach et al. 1995; Fixsen et al. 1996) and others. As a result, the astrophysical properties of these thermal radiations can be obtained.

This article is one of a set of articles associated with the study of the radiative and thermodynamic properties of the cosmic radiations using observational data from *COBE* FIRAS. In previous articles, these properties have been discussed in detail for the CMB radiation (Fisenko & Lemberg 2014a) and extragalactic far-infrared background radiation (Fisenko & Lemberg 2014b). In the first article, the analytical expressions for the temperature and redshift dependences of the radiative and thermodynamic functions have been obtained. New astrophysical parameters for the dipole spectrum, such as the entropy density/Boltzmann constant and the number density of CMB photons were constructed. The need was noted to consider the dipole contribution to the Stefan-Boltzmann law for the construction of cosmological models that describe radiative transfer. In the second article, the same properties for the extragalactic far-infrared radiation were calculated using numerical methods. As a result, the total intensity received in the 0.15–2.4 THz frequency interval was calculated and is 13.6 nW m^{-2} sr^{-1}. This value is about 19.4% of the total intensity expected from the energy released by stellar nucleosynthesis over cosmic history.

In this article, the *COBE* FIRAS observation data are used to obtain analytical expressions for the thermodynamic and radiative properties of the Galactic far-infrared radiation.

In Wright et al. (1991), the one-component model that describes the FIRAS galaxy continuous spectrum was proposed. Modeling the spectral emissivity in the form $\nu^{1.65}$, the observed data were fitted using a dust mean temperature of $T_{\rm dust}$ = 23.3 K. As a result, a map of the dust emission was obtained.

Another form for the spectral emissivity ν^2 was proposed in Fixsen et al. (1996). The one-component model was used to fit the observed data for the Galactic continuous spectrum. In this work, the Galactic radiation is considered as a contaminant in the CMB radiation.

In Reach et al. (1995), Galactic continuum spectra were observed in the wavenumber range from 5 – 96 cm^{-1} using the *COBE* FIRAS instrument. To describe the continuum spectra of the Galactic far-infrared radiation, a three-component model (warm, intermediate-temperature and very cold) was proposed. The continuous spectrum for each component was fitted using a power-law dependence of the spectral emissivity $\varepsilon(\nu) \sim \nu^2$. In this model, two free parameters (temperature T and optical depth) were used.

The present paper focuses on the study of the radiative and thermodynamic properties of Galactic far-infrared radiation. Using a three-component model with a spectral emissivity in the form $\varepsilon(\nu) = (v/\nu_0)^2$, the analytical expressions for the temperature dependences of the total energy density, the total radiation power per unit area, total emissivity, number density of photons, free energy density, entropy density, pressure, and heat capacity at constant volume for the warm, intermediate-temperature, and cold components are obtained. These radiative and thermodynamic properties were calculated in a finite range of frequencies between 5 cm^{-1} and 96 cm^{-1}, using the following mean temperatures: (a) warm temperature $T_1 = 17.72$ K, (b) intermediate-temperature $T_2 = 14$ K, and (c) very cold temperature $T_3 = 6.75$ K. The generalized Stefan-Boltzmann law for warm, intermediate-temperature, and very cold components is constructed. New astrophysical parameters, such as the entropy density/Boltzmann constant and number density of the Galactic far-infrared photons are obtained. The total radiation power (total luminosity) emitted from the surface of our Galaxy, the total number of photons in the Galaxy, and others quantities are calculated.

[1] *pdg.lbl.gov/2013/reviews/rpp2013-rev-astrophysical-constants.pdf*

2 GENERAL RELATIONSHIPS

According to Mather et al. (1994) and Reach et al. (1995), the observed continuum spectra $I(\tilde{\nu})$ of the sky radiation is modeled by four separate components

$$I(\tilde{\nu}) = B(\tilde{\nu}, T_0) + \frac{\partial B(\tilde{\nu}, T)}{\partial T} |_{T=T_0} \Delta T + G_k(l, b) g_k(\tilde{\nu}) + z(\tilde{\nu}) Z(l, b) \,. \tag{1}$$

Here, the first and second terms are the monopole and dipole components of the CMB radiation. These components represent the isotropic background. The third term is composed of one or two spatial distributions $G_k(l, b)$ with the spectra $g_k(\nu)$ derived from the all-sky data set. The final term is the zodiacal component with the spectrum described by a power law, $z(\tilde{\nu}) \propto \tilde{\nu}^3$. $T_0 = 2.72548$K is the mean temperature of the CMB radiation (Mather et al. 2013; Fixsen 2009), $\Delta T = T - T_0$ is the temperature anisotropy in a given direction of the sky, and

$$B(\tilde{\nu}, T) = 2hc^2 \frac{\tilde{\nu}^3}{e^{\frac{h\tilde{\nu}}{k_B T}} - 1} \tag{2}$$

is the Planck function at temperature T. Here h is the Planck constant, c is the speed of light and $\tilde{\nu}$ is the wavenumber (cm^{-1}).

According to Reach et al. (1995), FIRAS data for the Galactic spectra were modeled as a sum of three components (warm, intermediate-temperature and very cold) with the form

$$I_0(l, b, \tilde{\nu}, T) = G(l, b) \tau \varepsilon(\tilde{\nu}) B_{\tilde{\nu}}(T) \,, \tag{3}$$

where τ is the optical depth at $\tilde{\nu}_0 = 30\,\text{cm}^{-1}$ and $\varepsilon(\tilde{\nu}, T)$ is the spectral emissivity normalized to unity at $\tilde{\nu}_0$. In this three-component model, the spectral emissivity for each component has the following form

$$\varepsilon(\tilde{\nu}) = \left(\frac{\tilde{\nu}}{30\ \text{cm}^{-1}} \right)^2 . \tag{4}$$

Using Equation (4) and Equation (3), we have an expression for the spectral intensity as the sum of three components

$$I_{0G}(l, b, \tilde{\nu}, T) = \varepsilon(\tilde{\nu}) \sum_{i=1}^{3} G_i(l, b) \tau_i B_{\tilde{\nu}}(T_i) \,. \tag{5}$$

Here $G_i(l, b)$, τ_i and T_i are the spatial distribution, optical depth and temperature for the warm (i=1), intermediate-temperature (i=2) and very cold (i=3) components respectively.

According to Equation (5), the total intensity of the Galactic far-infrared radiation in the finite range of wavenumbers has the following form

$$I'_{0G}(\tilde{\nu}_1, \tilde{\nu}_2, T) = \sum_{i=1}^{3} G_i(l, b) \tau_i \int_{\tilde{\nu}_1}^{\tilde{\nu}_2} \varepsilon(\tilde{\nu}) B_{\tilde{\nu}}(T_i) d\tilde{\nu} \,. \tag{6}$$

Using a procedure described in Fisenko & Lemberg (2014a) for the representation of the Planck function $B_{\tilde{\nu}}(T)$ in the frequency domain, the expression for the spectral energy density has the form

$$I_G(\nu_1, \nu_2, T) = \sum_{i=1}^{3} G_i(l, b) \tau_i \int_{\nu_1}^{\nu_2} \varepsilon(\nu) B'_{\nu}(T_i) d\nu \,. \tag{7}$$

Here $\varepsilon(\nu) = \left(\frac{\nu}{0.9\ \text{THz}} \right)^2$. The Planck function in the frequency domain is defined as follows

$$B'_{\nu}(T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1} \,. \tag{8}$$

Equation (7) is general and describes the spectral energy density in the far infrared, visible, ultraviolet, X-rays and other ranges of Galactic thermal radiation.

The total emissivity in the finite range of frequencies $\nu_1 \leq \nu \leq \nu_2$ has the following structure

$$\varepsilon(T) = \sum_{i=1}^{3} \left(\frac{I_G(\nu_1, \nu_2, T_i)}{\int\limits_{\nu_1}^{\nu_2} B'_\nu(T_i) \mathrm{d}\nu} \right), \tag{9}$$

where $\int\limits_{\nu_1}^{\nu_2} B'_\nu(T)\mathrm{d}\nu$ is the total energy density of the blackbody radiation. According to Fisenko & Lemberg (2014a), the latter has the following structure

$$\int\limits_{\nu_1}^{\nu_2} B'_\nu(T_i)\mathrm{d}\nu = \frac{48\pi k_B^4}{c^3 h^3} \sum_{i=1}^{3} T_i^4 [P_3(x_{1i}) - P_3(x_{2i})] , \tag{10}$$

where $x_{1i} = \frac{h\nu_1}{k_B T_i}$ and $x_{2i} = \frac{h\nu_2}{k_B T_i}$. $P_3(x_i) = \sum\limits_{s=0}^{3} \frac{(x_i)^s}{s!} \mathrm{Li}_{4-s}(e^{-x_i})$ is defined using the polylogarithm function $\mathrm{Li}_{4-s}(e^{-x_i}) = \sum\limits_{k=1}^{\infty} \frac{e^{-kx_i}}{k^{4-s}}$ (Abramowitz & Stegun 1972).

According to Landau & Lifshitz (1980), the free energy density for the three component model can be represented in the form

$$f(\nu_1, \nu_2, T) = \frac{8\pi k_B}{c^3} \sum_{i=1}^{3} G_i(l,b)\tau_i T_i \int\limits_{\nu_1}^{\nu_2} \nu^2 \varepsilon(\nu) \ln(1 - e^{-\frac{h\nu}{k_B T_i}}) d\nu , \tag{11}$$

where $G_1(l,b)$, τ_1 and T_1 are the spatial distribution, optical depth and temperature for the warm component, $G_2(l,b)$, τ_2 and T_2 those values for the intermediate-temperature component, and $G_3(l,b)$, τ_3 and T_3 those values for the very cold component respectively.

Using Equation (11) the thermodynamic functions of the Galactic far-infrared radiation are defined as follows

(1) The entropy density

$$s = -\frac{\partial f}{\partial T} . \tag{12}$$

(2) Pressure

$$P = -f . \tag{13}$$

(3) Heat capacity at constant volume per unit volume

$$c_V = \left(\frac{\partial I_G(\nu_1, \nu_2, T)}{\partial T} \right)_V . \tag{14}$$

(4) The number density of photons $n = \frac{N}{V}$

$$n = \frac{8\pi}{c^3 \nu_0^2} G(l,b)\tau \int\limits_{\nu_1}^{\nu_2} \frac{\nu^4}{e^{\frac{h\nu}{k_B T}} - 1} d\nu . \tag{15}$$

3 RADIATIVE PROPERTIES OF THE GALACTIC FAR-INFRARED RADIATION

Let us now construct an expression for the total energy density of the Galactic far-infrared radiation. In this case, according to Equation (7), we need to calculate an integral with the following form

$$I_0(\nu_1,\nu_2,T) = G(l,b)\tau \int_{\nu_1}^{\nu_2} \varepsilon(\nu) B'_\nu(T) d\nu\,. \tag{16}$$

Using the analytical expression for spectral emissivity given by Equation (4), after integration, we obtain

$$I_0(\nu_1,\nu_2,T) = G(l,b)a\tau T^6 \Big[P_5(x_1) - P_5(x_2)\Big]\,. \tag{17}$$

Then, Equation (7) for the total energy density of the Galactic far-infrared radiation takes the form

$$I_G(\nu_1,\nu_2,T) = a \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^6 \Big[P_5(x_{1i}) - P_5(x_{2i})\Big]\,. \tag{18}$$

Here $a = \frac{960\pi k_B^6}{c^3 h^5 \nu_0^2} = 7.4890 \times 10^{-18}$ J m^{-3} K^{-6}.

In the semi-infinite range $0 \le \nu \le \infty$, since $P_5(0) = \mathrm{Li}_6(1) = \xi(6) = \frac{\pi^6}{945}$ and $P_5(\infty) = 0$, Equation (18) simplifies to

$$I_G(\nu_1,\nu_2,T) = a' \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^6\,, \tag{19}$$

where $a' = \frac{64\pi^7 k_B^6}{63 c^3 h^5 \nu_0^2} = 7.6189 \times 10^{-18}$ J m^{-3} K^{-6}.

The total radiation power per unit area is defined as

$$I'_G(\nu_1,\nu_2,T) = \frac{c}{4} I_G(\nu_1,\nu_2,T)\,, \tag{20}$$

and, in accordance with Equations (18) and (19), has the following structure:

(1) Finite range of frequency $\nu_1 \le \nu \le \nu_2$

$$I'_G(\nu_1,\nu_2,T) = b \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^6 \Big[P_5(x_{1i}) - P_5(x_{2i})\Big]\,, \tag{21}$$

where $b = \frac{240\pi k_B^6}{c^2 h^5 \nu_0^2} = 5.6129 \times 10^{-10}$ W m^{-2} K^{-6}.

(2) Semi-infinite range $0 \le \nu \le \infty$

$$I'_G(0,\infty,T) = b' \sum_{i=1}^{3} G_i(l,b)\,\tau_i T_i^6\,, \tag{22}$$

where $b' = \frac{16\pi^7 k_B^6}{63 c^2 h^5 \nu_0^2} = 5.7102 \times 10^{-10}$ W m^{-2} K^{-6}.

As seen, in accordance with Equation (21) and Equation (22), the temperature dependence of the total radiation power per unit area differs from the well-known Stefan-Boltzmann law $I(T) = \sigma T^4$ (Landau & Lifshitz 1980).

Using Equation (10) and Equation (18), for the total emissivity defined by Equation (9), we obtain

$$\varepsilon(T) = A \sum_{i=1}^{3} \frac{G_i(l,b)\tau_i T_i^2 \Big[P_5(x_{1i}) - P_5(x_{2i})\Big]}{[P_3(x_{1i}) - P_3(x_{2i})]}\,, \tag{23}$$

where $A = \frac{20k_B^2}{h^2\nu_0^2} = 0.01071\,\mathrm{K}^{-2}$.

In the semi-infinite frequency range $0 \leq \nu \leq \infty$, Equation (23) is simplified to

$$\varepsilon(T) = A' \sum_{i=1}^{3} G_i(l,b)\tau_i\, T_i^2 \tag{24}$$

with $A' = \frac{40\pi^2 k_B^2}{21h^2\nu_0^2} = 0.01007\,\mathrm{K}^{-2}$.

In Table 1 and Table 2, the values for the radiative functions for the warm, intermediate-temperature and very cold components as well as a sum of components are presented. As seen in Table 1 and Table 2, the finite frequency range from 0.15 to 2.88 THz covers a substantial portion of the total Galactic continuous spectrum.

It is well-known that when constructing cosmological models that describe radiative transfer in the inner Galaxy, the generalized Stefan-Boltzmann law is used in the following form

$$I^{\mathrm{S-B}}(T) = \varepsilon(T)\sigma T^4\,. \tag{25}$$

Here $\sigma = 5.6704 \times 10^{-8}$ J m^{-2} s^{-1} K^{-4} is the Stefan-Boltzmann constant.

Table 1 Calculated values for the radiative and thermodynamic state functions of the Galactic far-infrared radiation in the frequency interval 0.15–2.88 THz. $G(l,b) = 1$. (a) $T_1 = 17.72$ K and $\tau_1 = 1.74 \times 10^{-5}$ for the warm component; (b) $T_2 = 14$ K and $\tau_2 = 1.00 \times 10^{-4}$ for the intermediate-temperature component; (c) $T_3 = 6.75$ K and $\tau_3 = 1.23 \times 10^{-4}$ for the very cold component.

Quantity	Warm Component $\nu_1 \leq \nu \leq \nu_2$	Intermediate-Temperature Component $\nu_1 \leq \nu \leq \nu_2$	Very Cold Component $\nu_1 \leq \nu \leq \nu_2$	Sum of Components $\nu_1 \leq \nu \leq \nu_2$
$I_G\,(\nu_1, \nu_2, T)$ [J m^{-3}]	3.2536×10^{-15}	5.3285×10^{-15}	8.8509×10^{-17}	8.6706×10^{-15}
$I'_G\,(\nu_1, \nu_2, T)$ [W m^{-2}]	2.4385×10^{-7}	3.9936×10^{-7}	6.6336×10^{-9}	6.4984×10^{-7}
$\varepsilon(T)$	4.5812×10^{-5}	1.8636×10^{-4}	5.8750×10^{-5}	2.9092×10^{-4}
f [J m^{-3}]	-7.3035×10^{-16}	-1.1110×10^{-15}	-1.7618×10^{-17}	-1.8590×10^{-15}
s [Jm^{-3} K^{-1}]	2.2483×10^{-16}	4.5996×10^{-16}	1.5723×10^{-17}	7.0051×10^{-16}
P [Jm^{-3}]	7.3035×10^{-16}	1.1110×10^{-15}	1.7618×10^{-17}	1.8590×10^{-15}
c_V [J m^{-3} K^{-1}]	9.2633×10^{-16}	2.1230×10^{-15}	7.8748×10^{-17}	3.1281×10^{-15}
n [m^{-3}]	3.0494×10^{6}	5.8605×10^{6}	1.9229×10^{5}	9.1022×10^{6}

However, the temperature dependence of $\varepsilon(T)$ should be known. The three-component model allows one to obtain analytical expressions for the total emissivity $\varepsilon(T)$. In accordance with Equation (24), and assuming $G_i = 1$, we obtain:

Table 2 Calculated values for the radiative and thermodynamic state functions of the Galactic far-infrared radiation in the semi-infinite frequency interval. $G_i(l,b) = 1$. (a) $T_1 = 17.72$ K and $\tau_1 = 1.74 \times 10^{-5}$ for the warm component; (b) $T_2 = 14$ K and $\tau_2 = 1.00 \times 10^{-4}$ for the intermediate-temperature component; (c) $T_3 = 6.75$ K and $\tau_3 = 1.23 \times 10^{-4}$ for the very cold component.

Quantity	Warm Component $0 \le \nu \le \infty$	Intermediate-Temperature Component $0 \le \nu \le \infty$	Very Cold Component $0 \le \nu \le \infty$	Sum of Components $0 \le \nu \le \infty$
$I_G\,(0,\infty,T)$ $[\mathrm{J\,m^{-3}}]$	4.1041×10^{-15}	5.7367×10^{-15}	8.8638×10^{-17}	9.9294×10^{-15}
$I'_G\,(0,\infty,T)$ $[\mathrm{W\,m^{-2}}]$	3.0760×10^{-7}	4.2995×10^{-7}	6.6433×10^{-9}	7.4419×10^{-7}
$\varepsilon(T)$	5.5020×10^{-5}	1.9738×10^{-4}	5.6436×10^{-5}	3.0884×10^{-4}
f $[\mathrm{J\,m^{-3}}]$	-8.2083×10^{-16}	-1.1473×10^{-15}	-1.7728×10^{-17}	-1.9858×10^{-15}
s $[\mathrm{J\,m^{-3}\,K^{-1}}]$	2.7793×10^{-16}	4.9172×10^{-16}	1.5758×10^{-17}	7.8541×10^{-16}
P $[\mathrm{J\,m^{-3}}]$	8.2083×10^{-16}	1.1473×10^{-15}	1.7728×10^{-17}	1.9858×10^{-15}
c_V $[\mathrm{Jm^{-3}\,K^{-1}}]$	1.3897×10^{-15}	2.4586×10^{-15}	7.8789×10^{-17}	3.9271×10^{-15}
$n\,[\mathrm{m^{-3}}]$	3.3551×10^{6}	5.9358×10^{6}	1.9022×10^{5}	9.4811×10^{6}

(a) Warm dust component

$$\varepsilon(T) = cT^2\,, \tag{26}$$

where $c = 1.7522 \times 10^{-7}\ \mathrm{K^{-2}}$.

(b) Intermediate-temperature component

$$\varepsilon(T) = dT^2\,, \tag{27}$$

where $d = 1.007 \times 10^{-6}\ \mathrm{K^{-2}}$.

(c) Very cold component

$$\varepsilon(T) = eT^2\,, \tag{28}$$

where $e = 1.2386 \times 10^{-6}\ \mathrm{K^{-2}}$.

In accordance with Equations (26)–(28), the generalized Stefan-Boltzmann law, which can be used for modeling the radiative transfer in the warm dust region, intermediate-temperature component dust region, and very cold dust region in our Galaxy, has the following structure:

(a) Warm dust component

$$I^{\mathrm{S-B}}(T) = \sigma' T^6\,, \tag{29}$$

where $\sigma' = 9.8831 \times 10^{-15}\ \mathrm{J\,m^{-2}\,s^{-1}\,K^{-6}}$. σ' can be called the generalized Stefan-Boltzmann constant for the warm component.

(b) Intermediate-temperature dust component

$$I^{\mathrm{S-B}}(T) = \sigma'' T^6\,, \tag{30}$$

where $\sigma'' = 5.6799 \times 10^{-14}\ \mathrm{J\,m^{-2}\,s^{-1}\,K^{-6}}$. σ'' can be named the generalized Stefan-Boltzmann constant for the intermediate-temperature component.

(c) Very cold dust component

$$I^{\mathrm{S-B}}(T) = \sigma''' T^6\,, \tag{31}$$

where $\sigma''' = 6.99862 \times 10^{-14}$ J m^{-2} s^{-1} K^{-6}. σ''' can be called the generalized Stefan-Boltzmann constant for the very cold component.

In conclusion, let us apply the above results to calculate the radiative parameters of the Galactic far-infrared radiation (photon gas) of our Galaxy. The total radiation power emitted from the surface S' of the Galaxy has the form

$$I_{\mathrm{Gtotal}}(T) = S' I'_G(\nu_1, \nu_2, T)\,. \tag{32}$$

The continuous spectrum measured by the FIRAS *COBE* instrument allows us to calculate the total radiation power (total luminosity) emitted from the surface of our Galaxy. Indeed, the calculated total radiation power $I'_G(\nu_1, \nu_2, T)$ emitted from a unit area is presented in Table 1 and Table 2. According to Equation (25), we need to know a value of the surface area S' of our Galaxy. It is well-known that the Milky Way Galaxy is a stellar disk with a diameter d of about 30 kpc and a width h of about 0.7 kpc (Martinez & Saar 2010). The disk has a spiral structure and is populated by interstellar dust, interstellar gas and metal-rich stars. The surface area of the Galaxy can be determined as

$$S' = 2\pi\left(\frac{d}{2}\right)^2 + \pi h d\,. \tag{33}$$

Now let us assume that all astronomical objects that produce thermal emissions (stars etc.) are located in a thin shell on the surface of our Galaxy, and they are uniformly distributed. The astronomical objects in the shell emit the thermal radiation isotropically in all directions. The warm dust, intermediate-temperature dust and very cold dust are uniformly distributed over the volume of our Galaxy. Then, in accordance with Equation (32), we have

(1) Finite range 0.15 THz $\leq \nu \leq 2.88$ THz

$$I_{\mathrm{Gtotal}} = 2\pi\left(\frac{d}{2}\right) h\left[1 + \frac{d}{2h}\right] I'_G(\nu_1, \nu_2, T)\,. \tag{34}$$

(2) Semi-infinite range $0 \leq \nu \leq \infty$

$$I_{\mathrm{Gtotal}} = 2\pi\left(\frac{d}{2}\right) h\left[1 + \frac{d}{2h}\right] I'_G(0, \infty, T)\,. \tag{35}$$

Using the relationship 1 kpc $\cong 3.086 \times 10^{19}$m, the calculated value for S' in Equation (33) is

$$S' = 1.4085 \times 10^{42}\ \mathrm{m}^2\,. \tag{36}$$

Then, for the total radiation power (total luminosity) emitted from the surface area S', we obtain the following values:

(1) Finite range 0.15 THz $\leq \nu \leq 2.88$ THz

$$I'_{\mathrm{Gtotal}}(T) = 9.1530 \times 10^{35}\ \mathrm{W}\,. \tag{37}$$

(2) Semi-infinite range $0 \leq \nu \leq \infty$

$$I'_{\mathrm{Gtotal}}(T) = 1.0482 \times 10^{36}\ \mathrm{W}\,. \tag{38}$$

These values are different from the value of $I'_{\rm Gtotal}(T) \approx 1.0\times10^{37}$ W given in the literature (Groom 2013[2]). This difference can be explained by using the spectral emissivity $\varepsilon(\nu)$ in our calculations. The volume of the Milky Way Galaxy can be calculated using the following expression

$$V_{\rm Galaxy} = \pi h\left(\frac{d}{2}\right)^2 = 1.4534 \times 10^{61}\ {\rm m}^3\,. \tag{39}$$

Then, in accordance with Equation (18), Equation (19), Tables 1 and 2, the total energy density of photons produced by our Galaxy has the following values:

(1) Finite range 0.15 THz $\le \nu \le 2.88$ THz

$$I_{\rm Gtotal}(T) = V_{\rm Galaxy} I_G(T) = 1.2062 \times 10^{47}\ {\rm J}\,. \tag{40}$$

(2) Semi-infinite range $0 \le \nu \le \infty$,

$$I_{\rm Gtotal}(T) = V_{\rm Galaxy} I_G(T) = 1.4431 \times 10^{47}\ {\rm J}\,. \tag{41}$$

4 THERMODYNAMICS OF THE GALACTIC FAR-INFRARED RADIATION

To construct the thermodynamics of the Galactic far-infrared radiation let us start with the calculation of free energy. The free energy density is given by Equation (11)

$$f(\nu_1,\nu_2,T) = \frac{8\pi k_B}{c^3\nu_0^2}\sum_{i=1}^{3} G_i(l,b)\tau_i T_i \int_{\nu_1}^{\nu_2} \nu^4 \ln(1-e^{-\frac{h\nu}{k_B T_i}})d\nu\,. \tag{42}$$

Introducing the variable $x_i = \frac{h\nu}{k_B T_i}$ and integrating by parts, we obtain

$$\begin{aligned} f(\nu_1,\nu_2,T) \;=\; & -B\sum_{i=1}^{3} G_i(l,b)\tau_i T_i^6\Big\{\Big[P_5(x_{1i}) - P_5(x_{2i})\Big] \\ & -\frac{1}{120}\Big[x_{1i}^5\ {\rm Li}_1(e^{-x_{1i}}) - x_{2i}^5\ {\rm Li}_1(e^{-x_{2i}})\Big]\Big\}\,, \end{aligned} \tag{43}$$

where $B = \frac{192\pi k_B^6}{c^3h^5\nu_0^2} = 1.4978 \times 10^{-18}$ J m^{-3} K^{-6}.

In the frequency range $0 \le \nu \le \infty$, Equation (43) simplifies to

$$f(\nu_1,\nu_2,T) = -B'\sum_{i=1}^{3} G_i(l,b)\tau_i T_i^6\,. \tag{44}$$

Here $B' = \frac{64\pi^7 k_B^6}{315c^3h^5\nu_0^2} = 1.5238 \times 10^{-18}$ J m^{-3} K^{-6}.

The entropy density $s = -\frac{\partial f}{\partial T}$ in the finite frequency range has the following structure

$$s = C\sum_{i=1}^{3} G_i(l,b)\tau_i T_i^5\Big\{\Big[P_5(x_{1i}) - P_5(x_{2i})\Big] - \frac{1}{720}\Big[x_{1i}^5\ {\rm Li}_1(e^{-x_{1i}}) - x_{2i}^5\ {\rm Li}_1(e^{-x_{2i}})\Big]\Big\}, \tag{45}$$

where $C = \frac{1152\pi k_B^6}{c^3h^5\nu_0^2} = 8.9868 \times 10^{-18}$ J m^{-3} K^{-5}.

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[2] *pdg.lbl.gov/2013/reviews/rpp2013-rev-astrophysical-constants.pdf*

In the semi-infinite frequency range, Equation (45) transforms to

$$s = C' \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^5 \,. \tag{46}$$

Here $C' = \frac{128\pi^7 k_B^6}{105 c^3 h^5 \nu_0^2} = 9.1427 \times 10^{-18}$ J m^{-3} K^{-5}.

Pressure is defined by the formula

$$P = -f \;=\; D \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^6 \Big\{ \Big[P_5(x_{1i}) - P_5(x_{2i}) \Big] - \frac{1}{120} \Big[x_{1i}^5 \mathrm{Li}_1(e^{-x_{1i}}) - x_{2i}^5 \,\mathrm{Li}_1(e^{-x_{2i}}) \Big] \Big\} \,, \tag{47}$$

where $D = \frac{192\pi k_B^6}{c^3 h^5 \nu_0^2} = 1.4978 \times 10^{-18}$ J m^{-3} K^{-6}.

In the semi-infinite range, Equation (47) is presented as follows

$$P = D' \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^6 \,. \tag{48}$$

Here $D' = \frac{64\pi^7 k_B^6}{315 c^3 h^5 \nu_0^2} = 1.5238 \times 10^{-18}$ J m^{-3} K^{-6}.

Knowledge of the total energy density Equation (18) allows us to construct the heat capacity at constant volume per unit volume $c_V = (\frac{\partial I_0(\nu_2,\nu_2,T)}{\partial T})_V$ in the form

$$c_V \;=\; E \sum_{i=1}^{3} G_i(l,b)\tau_i \, T_i^5 \Big\{ \Big[P_5(x_{1i}) - P_5(x_{2i}) \Big] + \frac{1}{720} \Big[x_{1i}^6 \,\mathrm{Li}_0(e^{-x_{1i}}) - x_{2i}^6 \,\mathrm{Li}_{01}(e^{-x_{2i}}) \Big] \Big\} \,, \tag{49}$$

where $E = \frac{5760\pi k_B^6}{c^3 h^5 \nu_0^2} = 4.4934 \times 10^{-17}$ J m^{-3} K^{-5}.

In the semi-infinite frequency range, Equation (49) simplifies to

$$c_V = E' \sum_{i=1}^{3} G_i(l,b)\tau_i \, T_i^5 \,. \tag{50}$$

Here $E' = \frac{128\pi^7 k_B^6}{21 c^3 h^5 \nu_0^2} = 4.5713 \times 10^{-18}$ J m^{-3} K^{-5}.

Using the formula for the number density of photons

$$n = \frac{8\pi}{c^3 \nu_0^2} \sum_{i=1}^{3} G_i(l,b)\tau_i \int_{\nu_1}^{\nu_2} \frac{\nu^4}{e^{\frac{h\nu}{k_B T_i}} - 1} d\nu \,, \tag{51}$$

after integration, we obtain

$$n = F \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^5 \Big[P_4(x_{1i}) - P_4(x_{2i}) \Big] \,, \tag{52}$$

where $F = \frac{192\pi k_B^5}{c^3 h^5 \nu_0^2} = 1.0848 \times 10^5$ m^{-3} K^{-5}.

In the range $0 \le \nu \le \infty$, Equation (52) takes the form

$$n = F' \sum_{i=1}^{3} G_i(l,b)\tau_i T_i^5 \,. \tag{53}$$

Here $F' = \frac{64\pi^6 k_B^5}{315 c^3 h^5 \nu_0^2} = 1.1037 \times 10^5$ m^{-3} K^{-5}.

By definition (Landau & Lifshitz 1980), the chemical potential density $\mu = (\frac{\partial f}{\partial n})_{T,V}$, as clearly seen from Equation (35), is equal to zero.

The obtained expressions are used in computer calculations for the three-component Galactic model. In Table 1 and Table 2, the calculated values for the radiative and thermodynamic functions of the warm, intermediate-temperature and cold components, and also a sum of the three-components, are represented in the finite and semi-infinite frequency ranges. To calculate the values for the radiative and thermodynamic functions we used the following data: (a) warm component - the mean temperature $T_1 = 17.72$ K and optical depth $\tau_1 = 1.74 \times 10^{-5}$; (b) the intermediate-temperature component - the mean temperature $T_2 = 14$ K and mean optical depth $\tau_2 = 1.00 \times 10^{-4}$; (c) very cold component - the mean temperature $T_3 = 6.75$ K and optical depth $\tau_3 = 1.23 \times 10^{-4}$.

In conclusion, let us calculate the thermodynamic properties of the Galactic far-infrared radiation (photon gas) for our Galaxy.

Using Equation (39) and the values in Tables 1 and 2, for our Galaxy, we obtain

(1) The total number of photons $N_{\rm Gtotal}$ in our Galaxy
 (a) Finite range 0.15 THz $\le \nu \le 2.88$ THz

$$N_{\rm Gtotal} = V_{\rm Galaxy} n = 1.3229 \times 10^{68} \,. \tag{54}$$

 (b) Semi-infinite range $0 \le \nu \le \infty$

$$N_{\rm Gtotal} = V_{\rm Galaxy} n = 1.3780 \times 10^{68} \,. \tag{55}$$

(2) The total entropy $S_{\rm Gtotal}$ of a photon gas
 (a) Finite range 0.15 THz $\le \nu \le 2.88$ THz

$$S_{\rm Gtotal} = s V_{\rm Galaxy} = 1.0181 \times 10^{46} \text{ J K}^{-1} \,. \tag{56}$$

 (b) Semi-infinite range $0 \le \nu \le \infty$

$$S_{\rm Gtotal} = s V_{\rm Galaxy} = 1.1415 \times 10^{46} \text{ J K}^{-1} \,. \tag{57}$$

(3) The total free energy $F_{\rm Gtotal}$ of a photon gas
 (a) Finite range 0.15 THz $\le \nu \le 2.88$ THz

$$F_{\rm Gtotal} = f V_{\rm Galaxy} = -2.7019 \times 10^{46} \text{ J} \,. \tag{58}$$

 (b) Semi-infinite range $0 \le \nu \le \infty$

$$F_{\rm Gtotal} = f V_{\rm Galaxy} = -2.8862 \times 10^{46} \text{ J} \,. \tag{59}$$

(4) The total heat capacity at constant value $C_{V\,\rm Gtotal}$ of a photon gas
 (a) Finite range 0.15 THz $\le \nu \le 2.88$ THz

$$C_{V\,\rm Gtotal} = c_V V_{\rm Galaxy} = 4.5464 \times 10^{46} \text{ J K}^{-1} \,. \tag{60}$$

 (b) Semi-infinite range $0 \le \nu \le \infty$

$$C_{V\,\rm Gtotal} = c_V V_{\rm Galaxy} = 5.7076 \times 10^{46} \text{ J K}^{-1} \,. \tag{61}$$

(5) Total pressure $P_{\rm Gtotal}$ of a photon gas

(a) Finite range 0.15 THz $\leq \nu \leq 2.88$ THz

$$P_{\rm Gtotal} = pV_{\rm Galaxy} = 2.7019 \times 10^{46} \text{ J} \,. \tag{62}$$

(b) Semi-infinite range $0 \leq \nu \leq \infty$

$$P_{\rm Gtotal} = pV_{\rm Galaxy} = 2.8862 \times 10^{46} \text{ J} \,. \tag{63}$$

In Table 3, the radiative and thermodynamic properties of the Galactic far-infrared radiation (photon gas) in the finite and semi-infinite frequency ranges for the Milky Way Galaxy are presented.

In conclusion, it is important to note the following. The obtained results can be used to describe the radiative and thermodynamic properties for any other galaxies in the universe. In favor of what has been said it is worth noting the work of Spinoglio et al. (2005). In this work, a two-component model with an emissivity law in the form $\varepsilon \sim \nu^2$ is proposed to fit the observed thermal continuum spectra for the Seyfert 2 galaxy NGC 1068. As a result, the expressions obtained in this paper are applicable for calculating the radiative and thermodynamic properties of this galaxy. This topic will be a point of discussion in a subsequent publication.

Table 3 Radiative and Thermodynamic Properties of a Photon Gas in Our Galaxy $G_i(l,b) = 1$

Quantity	Three-component Model 0.15 THz $\leq \nu \leq 2.88$ THz	Three-component Model $0 \leq \nu \leq \infty$
$I_{\rm Gtotal}$ [J]	1.2062×10^{47}	1.4431×10^{47}
$I'_{\rm Gtotal}$ [W]	9.1530×10^{35}	1.0482×10^{36}
$F_{\rm Gtotal}$ [J]	-2.7019×10^{46}	-2.8862×10^{46}
$S_{\rm Gtotal}$ [J K^{-1}]	1.0181×10^{46}	1.1415×10^{46}
$P_{\rm Gtotal}$ [J]	2.7019×10^{46}	2.8862×10^{46}
$C_{V\,\rm Gtotal}$ [J K^{-1}]	4.5464×10^{46}	5.7076×10^{46}
$N_{\rm Gtotal}$	1.3229×10^{68}	1.3780×10^{68}

5 ASTROPHYSICAL PARAMETERS

Now let us calculate astrophysical parameters for Galactic far-infrared photons. In accordance with the table of astrophysical constants and parameters (Groom 2013), three fundamental constants and parameters for a monopole spectrum, such as the present day CMB temperature, entropy density/Boltzmann constant and number density of CMB photons are presented. The parameter of the entropy density/Boltzmann constant, for example, has the form

$$\frac{s}{k_B} = 2.8912\left(\frac{T}{T_0}\right)^3 \text{ cm}^{-3} \,, \tag{64}$$

where $T_0 = 2.72548$ K is the present day CMB temperature.

Let us assume that the optical depth varies with temperature. Then, as in the case of a monopole spectrum, the expressions for the entropy density/Boltzmann constant and the number density of Galactic far-infrared photons can be considered as additional parameters to the astrophysical ones presented in Groom (2013). In accordance with Equation (46) and Equation (53), the analytical expressions for the parameter of entropy density/Boltzmann constant and number density of Galactic far-infrared photons can be represented in the form

(1) Entropy density/Boltzmann constant

(a) Warm dust component

$$\frac{s}{k_B} = 20.1304\left(\frac{\tau}{\tau_{01}}\right)\left(\frac{T}{T_{01}}\right)^5 \text{ cm}^{-3} \,. \tag{65}$$

(b) Intermediate-temperature component

$$\frac{s}{k_B} = 35.6151\left(\frac{\tau}{\tau_{02}}\right)\left(\frac{T}{T_{02}}\right)^5 \text{ cm}^{-3} . \tag{66}$$

(c) Very cold component

$$\frac{s}{k_B} = 1.1413\left(\frac{\tau}{\tau_{03}}\right) \left(\frac{T}{T_{03}}\right)^5 \text{ cm}^{-3} . \tag{67}$$

(2) Number density of Galactic far-infrared photons

(a) Warm dust component

$$n = 3.3551\left(\frac{\tau}{\tau_{01}}\right)\left(\frac{T}{T_{01}}\right)^5 \text{ cm}^{-3} . \tag{68}$$

(b) Intermediate-temperature component

$$n = 5.9358\left(\frac{\tau}{\tau_{02}}\right)\left(\frac{T}{T_{02}}\right)^5 \text{ cm}^{-3} . \tag{69}$$

(c) Very cold component

$$n = 0.1902\left(\frac{\tau}{\tau_{03}}\right)\left(\frac{T}{T_{03}}\right)^5 \text{ cm}^{-3} . \tag{70}$$

Here τ_{0i} and T_{0i} are the present day optical depths and temperatures respectively. Their values are: (a) $\tau_{01} = 1.74 \times 10^{-5}$ and $T_{01} = 17.72$ K for the warm component; (b) $\tau_{02} = 1.00 \times 10^{-4}$ and $T_{02} = 14$ K for the intermediate-temperature component; (c) $\tau_{03} = 1.23 \times 10^{-4}$ and $T_{03} = 6.75$ K for the very cold component.

6 CONCLUSIONS

In this paper, using the three-component model to describe the thermal continuum spectra of Galactic far-infrared radiation, the exact analytical expressions for the temperature dependences of the radiative and thermodynamic functions, such as the total radiation power per unit area, total energy density, total emissivity, number density of photons, Helmholtz free energy density, entropy density, heat capacity at constant volume and pressure are obtained. These expressions allow us to calculate the astrophysical properties of our Galaxy. A more detailed description of the obtained results is given below:

(1) The radiative and thermodynamic properties of our Galaxy in the finite frequency range from 0.15 THz to 2.88 THz and semi-infinite range are calculated. In this case, the following parameters were used for the warm, intermediate-temperature and cold components: (a) warm component – the mean temperature $T_1 = 17.72$ K and mean optical depth $\tau_1 = 1.74 \times 10^{-5}$; (b) intermediate-temperature component – the mean temperature $T_2 = 14$ K and mean optical depth $\tau_2 = 1.00 \times 10^{-4}$; and c) very cold component – the mean temperature $T_3 = 6.75$ K and mean optical depth $\tau_3 = 1.23 \times 10^{-4}$. The calculated values are presented in Tables 1 and 2.

(2) Under the assumption that all the astronomical objects that emit radiation (stars etc.) are uniformly distributed in a thin shell on the surface of our Galaxy, the total radiation power received from the surface of the Galaxy in the finite frequency range from 0.15 THz to 2.88 THz and semi-infinite range are calculated. Their values are $I_{\text{Gtotal}}(T) = 3.3757 \times 10^{35}$ W and $I_{\text{Gtotal}}(T) = 4.2286 \times 10^{35}$ W, accordingly. The calculated values for other radiative and thermodynamic properties for the Milky Way Galaxy are presented in Table 3.

(3) The total numbers of photons in finite and semi-infinite ranges in our Galaxy are obtained. For the finite frequency range 0.15 THz $\leq \nu \leq 2.88$ THz, we have $N_{\text{Gtotal}} = 1.3229 \times 10^{68}$. For the semi-infinite range of frequencies, the total number of photons is $N_{\text{Gtotal}} = 1.3780 \times 10^{68}$.

(4) The thermodynamic properties of Galactic far-infrared photons of the Galaxy, such as the total Helmholtz free energy, total entropy, total heat capacity at constant volume and total pressure are calculated. The value for the total entropy in the finite frequency range 0.15 THz $\leq \nu \leq 2.88$ THz is $S_{\mathrm{Gtotal}} = 1.0181 \times 10^{46}$ J K^{-1}. For the semi-finite frequency range, we have $S_{\mathrm{Gtotal}} = 1.1415 \times 10^{46}$ J K^{-1}. As for the total pressure, their values are: $P_{\mathrm{Gtotal}} = 2.7019 \times 10^{46}$ J- finite frequency range; $P_{\mathrm{Gtotal}} = 2.8862 \times 10^{46}$ J- semi-infinite range. Other thermodynamic values of our Galaxy are presented in Table 3.
(5) The generalized Stefan-Boltzmann law for the warm, intermediate-temperature and cold components is constructed and presented by Equations (29)–(31). The temperature dependences have the same structure $I^{\mathrm{S-B}}(T) = \sigma' T^6$ with different values of the generalized Stefan-Boltzmann constant. These results are important for the construction of cosmological models of radiative transfer in our Galaxy.
(6) Knowing the dependence of the temperature T and the frequency ν on redshift z (Sunyaev & Zeldovich 1980) allows us to study what the the thermodynamic and radiative state of our Galaxy was like many years ago with the help of analytical expressions derived in this paper.
(7) The results of the present paper allow us to estimate the contribution of the radiative and thermodynamic properties of a photon gas to the total radiation emitted by other particles (protons, alpha and beta particles, etc.).

In conclusion, it is important to note that the analytical expressions obtained in this study may be useful for calculating the radiative and thermodynamic properties of any galaxy for which a two-component model is a good approximation. The Seyfert 2 galaxy NGC 1068 is one example where the two-component model can be applied. These topics will be discussed in forthcoming publications.

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