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Anisotropic stellar structure equations for magnetized strange stars

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Abstract The fact that a fermion system in an external magnetic field breaks the spherical symmetry suggests that its intrinsic geometry is axisymmetric rather than spherical. In this work we analyze the impact of anisotropic pressures, due to the presence of a magnetic field, in the structure equations of a magnetized quark star. We assume a cylindrical metric and an anisotropic energy momentum tensor for the source. We found that there is a maximum magnetic field that a strange star can sustain, closely related to the violation of the virial relations.

Key words: magnetic fields — stars: neutron — equation of state

1 INTRODUCTION

The presence of huge magnetic fields in compact objects (COs) is an established fact. Typical measured surface magnetic fields are about 10^{12} G, although in the case of the *magnetar* subclass they can be as high as 10^{15} G (Woods & Thompson 2006). However, there are no observations capable of measuring the magnetic fields in the inner regions of stars, and theoretical arguments must be used to estimate the maximum values of the field with an eye on possible modifications to the stellar structure. Based on the scalar virial theorem (Lai & Shapiro 1991), it can be estimated that the maximum magnetic field that a CO can sustain, as $(4\pi R^3/3)(B_{\rm max}^2/8\pi) \sim GM^2/R \;\Rightarrow\; B_{\rm max} \sim 2\times 10^8 (M/M_\odot)(R/R_\odot)^{-2}$ G, is $B_{\rm max} \sim 10^{18}$ G for a neutron star with a mass $M = 1.4 M_\odot$ and a radius $R \sim 10^6 {\rm cm} = 10^{-4} R_\odot$, where $M_\odot$ and $R_\odot$ are the mass and the radius of the Sun respectively. For a self-bound star a higher maximum value inside the core $B_{\rm max} \sim 10^{20}$ G has been obtained (Ferrer et al. 2010). These numbers suggest the idea that a realistic model of a CO must consider that matter is magnetized, and a few attempts have been done in order to construct models that describe the microphysics of magnetized fermion systems (González Felipe et al. 2005; Chakrabarty 1996).

In the simplest case in which the source of the magnetic field is not addressed, i.e. fermions in an external field, it is quite clear that in fermion systems the former breaks the spherical symmetry and produces an anisotropy in the pressure. Depending on its actual numerical value, this anisotropy could induce a deformation of the CO, and eventually leads to an anisotropic collapse of the object if ultrastrong magnetic fields are indeed present (Martínez et al. 2003). The limits of the absolute maximum field and the issue of spherical symmetry in the equation of state (EoS) are the subjects of the present paper.

It is important to stress that the anisotropy of the pressures is a main consequence of our assumptions of a constant external magnetic field. In Blandford & Hernquist (1982) it is argued that the pressures become isotropic because of the work done against the Lorentz force density ($(\nabla \times M) \times B$) when the gas is compressed in the perpendicular direction to B. This argument does not hold within our assumptions of a constant magnetic field because we have $\nabla \times M = 0$. Moreover, a more realistic scenario is presented in Canuto (1971) in which one could find isotropic pressures in a magnetized fermion system, but no external magnetic field is required. There are many works that recognize the existence of the anisotropic pressures as a consequence of the spherical symmetry breaking due to the preferred direction fixed by the magnetic field (Chandrasekhar & Fermi 1953; Haskell et al. 2008; Lattimer & Prakash 2007). In Chaichian et al. (2000); Perez Martinez et al. (2000); Martínez et al. (2003) it is discussed that a fermion gas in a magnetic field could collapse due to the anisotropies of the pressures.

In the following, we will describe a system with the most simple metric that is able to deal with the cylindrical geometry hypothesis. The prescription for the pressures in the energy momentum tensor are thus very well defined and do not have ambiguities.

We must emphasize that a constant magnetic field is expected for a highly conductive material without the Meissner effect. Hence, the constant interior magnetic field may be a better approximation than would be naively expected.

Anisotropic systems have been studied in the context of stars, but the standard approach has been to take spherical symmetry for granted (Dev & Gleiser 2000; Mak & Harko 2003). In the presence of pressure anisotropies due to a magnetic field, other symmetry choices (i.e. cylindrical symmetry) could give a more complete description of the physics; this feature has been pointed out by several authors (Paulucci et al. 2011; Dexheimer et al. 2012). Recently some works dealing with the problem of an axisymmetric metric (Herrera et al. 2013; Quevedo 2012) have been presented, although they remained within a theoretical perspective without application to actual systems.

Given these arguments, the introduction of a *cylindrical* symmetric metric in the Einstein equation, together with the construction of an anisotropic energy momentum tensor for the magnetized matter, seems a more "natural" choice. This anisotropic hydrostatic equilibrium equation could shed some light on how the magnetic field affects the sphericity of the CO, and yield upper limits for the values of the magnetic field that this object can sustain.

In a first approximation to investigate this problem we will use a general cylindrical symmetric metric, with coordinates (t, r, ϕ, z), following the procedures of Trendafilova & Fulling (2011) to solve the Einstein equations for an axisymmetric model of a CO to take into account the anisotropy induced by the external magnetic field. In the presence of a constant, external magnetic field there are two main directions in space, parallel and perpendicular to the magnetic field. One of the main approximations that we shall make is that all the functions and variables of our model will only depend on the radial (r) variable, so that we can simply describe the perpendicular (equatorial) direction of the CO with respect to the magnetic field. We also consider that the magnetic field is constant and in the z direction. This model can give information on the effects of the magnetic field in terms of the shape (oblateness) of the CO and yield upper limits for the values of the magnetic field that this object can sustain. More realistic models will be studied in the future.

For the microscopic description of magnetized matter, we use the results obtained in Martínez et al. (2010); Felipe & Martínez (2009) for magnetized strange quark matter (MSQM), which can represent the composition of a strange quark star in its self-bound version or the central nucleus of a hybrid neutron star if it is only considered at high pressure. Both possibilities can be achieved by selecting different sets of the available parameters, as discussed in Dexheimer et al. (2012); Paulucci et al. (2011).

In Section 2 we review the main thermodynamical properties of a magnetized quark gas used to describe the matter inside the star. In Section 3 we first obtain the mass-radius relation for MSQM in a standard spherical symmetry and show the problems related to the existence of two pressures; next Einstein equations are solved in cylindrical symmetry to obtain the structure equations that describe the equilibrium of the stars taking into account both pressures. Finally in Section 5 we present the conclusions of this study.

2 MAGNETIZED FERMION SYSTEM

The thermodynamical potential for a magnetized fermion gas in the framework of the MIT Bag model is given by

$$\Omega_f(\mu_f, B, T) = -\frac{d_f e_f B}{\beta}\left[\sum_{l=0}^{\infty}\sum_{p_4}\int_{-\infty}^{\infty}\frac{dp_3}{(2\pi)^2}\ln\det G_f^{-1}(\overline{p}^*)\right], \tag{1}$$

with $\overline{p}^* = (ip^4 - \mu_f, 0, \sqrt{2e_f Bl}, p^3)$ for $l = 0, 1, 2, ...$; β is the inverse absolute temperature, μ_f is the fermionic chemical potential and $G_f^{-1} = \det[\overline{p}^* \cdot \gamma - m_f]$.

We label the electrons with f and the quark flavors are u, d and s. After performing the Matsubara sum we obtain

$$\Omega_f(B, \mu_f, T) = -\frac{d_f e_f B}{2\pi^2}\left[\sum_{l=0}^{\infty}\alpha_l\int_0^{\infty}dp_3\left(\mathcal{E}_f + \frac{1}{\beta}\ln(1+e^{-(\mathcal{E}_f-\mu_f)})(1+e^{-(\mathcal{E}_f+\mu_f)})\right)\right]. \tag{2}$$

Taking the zero temperature limit (COs are considered highly degenerate ($\mu \gg T$), therefore the thermal effects can be neglected) we can write the thermodynamic potential as a sum of the vacuum and statistical contributions

$$\Omega_f = \Omega_f(B, 0, 0) + \Omega_f(B, \mu, 0)\,. \tag{3}$$

with

$$\Omega_f(B, 0, 0) = -\frac{e_f B}{4\pi^2}\int dp_3|\frac{e_f B}{4\pi^2}\sum_{l=0}^{\infty}\int dp_3|\mathcal{E}_f|\,, \tag{4}$$

where $\mathcal{E}_f = \sqrt{p_3^2 + m_f^2 + 2|e_f B|l}$. The $\Omega_f(B, 0, 0)$ is the vacuum contribution and the renormalized form was found in Berestetskii et al. (1980). In what follows we neglect it, since we are interested here in a region of fields $eB \leq \mu^2$ where the statistical contribution $\Omega_f(B, \mu_f, 0)$ to the physical quantities is more important than the vacuum one.

The statistical contribution $\Omega_f(B, \mu_f, 0)$ has the form

$$\Omega_f(B, \mu_f, 0) = -\frac{d_f e_f B}{2\pi^2}\sum_{l=0}^{l_{\max}}\alpha_l\int_0^{\sqrt{\mu_f^2-\varepsilon_f^2}}dp_3\mu_f\sqrt{p_{3,f}^2 + \varepsilon_f^2} \tag{5}$$

$$= -\frac{d_f e_f B}{4\pi^2}\left[\sum_{l=0}^{l_{\max}}\alpha_l\left(\mu_f\, p_F - \varepsilon_f^2\ln\frac{\mu_f + p_F}{\varepsilon_f}\right)\right], \tag{6}$$

where $l_{\max} = [\frac{\mu_f^2 - m_f^2}{2eB}]$, $I[z]$ denotes the integer part of z, $\alpha_l = 2 - \delta_{l0}$ is the spin degeneracy of the l-Landau level and $d_e = 1$ and $d_{u,d,s} = 3$ are degeneracy factors. The Fermi momentum is $p_F = \sqrt{{\mu_f}^2 - \varepsilon_f^2}$ and the rest energy is given as

$$\varepsilon_f = \sqrt{m_f^2 + 2|e_f B|l}\,, \tag{7}$$

For degenerate magnetized strange quark matter, the number density and magnetization are given by the expressions

$$N_f = -(\partial\Omega_f/\partial\mu_f) = \frac{d_f m^2}{2\pi^2}\frac{B}{B_f^c}\sum_{l=0}^{l_{\max}}\alpha_l p_F, \tag{8}$$

$$\mathcal{M}_f = -(\partial\Omega_f/\partial B) = \frac{d_f e_f m_f}{4\pi^2}\left(\sum_{l=0}^{l_{\max}}\alpha_l\left[\mu_f p_F - \left[\varepsilon_f^2 + 2\varepsilon_f C_f\right]\ln\frac{\mu_f + p_F}{\varepsilon_f}\right]\right), \tag{9}$$

where $B_f^c = m_f^2/|e_f|$ is the critical magnetic field and $C_f = \frac{\frac{B}{B_f^c}l}{\sqrt{2l\frac{B}{B_f^c}+m_f^2}}$.

Energy density and pressures are

$$\epsilon = \Omega_f + \mu_f N_f, \tag{10a}$$

$$\mathcal{P}_\parallel = -\sum_f \Omega_f, \tag{10b}$$

$$\mathcal{P}_\perp = \sum_f(-\Omega_f - B\mathcal{M}_f)\,. \tag{10c}$$

In order to study the matter inside the star we use the MIT Bag model, thus the EoS is obtained from Equations (10a), (10b) and (10c) adding the bag (vacuum energy) parameter and the classical magnetic energy,

$$E = \varepsilon + \frac{B^2}{8\pi} + B_{\rm bag}\,, \tag{11}$$

$$P_\parallel = \mathcal{P}_\parallel - \frac{B^2}{8\pi} - B_{\rm bag}\,, \tag{12}$$

$$P_\perp = \mathcal{P}_\perp + \frac{B^2}{8\pi} - B_{\rm bag}\,. \tag{13}$$

The stellar chemical equilibrium conditions are obtained by solving the system of equations

$$\mu_u + \mu_e - \mu_d = 0, \quad \mu_d - \mu_s = 0 \quad \beta\,\text{equilibrium}\,, \tag{14a}$$

$$2N_u - N_d - N_s - 3N_e = 0 \quad \text{charge neutrality}\,, \tag{14b}$$

$$N_u + N_d + N_s - 3n_B = 0 \quad \text{baryon number conservation}\,. \tag{14c}$$

Once the system Equation (14) is solved, we can find the thermodynamical properties of the MSQM in stellar chemical equilibrium conditions and study how the magnetic field modified them.

In Figure 1 we show the EoS of the magnetized gas, stressing the fact that when we increase the magnetic field the anisotropy becomes relevant. An even more illustrative graphic is the dependence of pressures on the magnetic field which we show in Figure 2. It is noted that when the magnetic field increases, the splitting of the pressures becomes greater, as expected. There is a regime where the pressures are nearly equal (isotropic regime), but for fields around $B \sim 10^{18}$ G the pressure anisotropy becomes very large. A quantitative parameter to measure the importance of the pressure anisotropy (the *splitting coefficient*) can be defined as

$$\Delta = \frac{|P_\perp - P_\parallel|}{P(B\to 0)}\,. \tag{15}$$

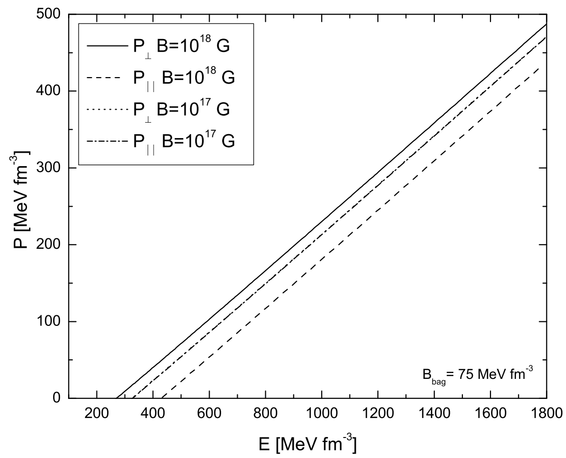

Fig. 1 Equations of state for magnetized strange quark matter.

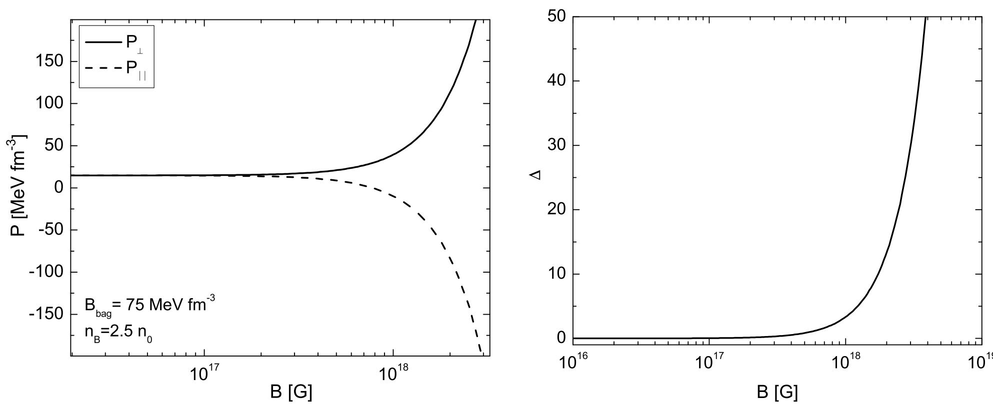

Fig. 2 Dependence of the pressures and the splitting coefficient on the magnetic field.

In the right panel of Figure 2 we can see how this coefficient depends on the magnetic field. A criterion to discriminate between isotropy and anisotropy regimes is that $\Delta \simeq \mathcal{O}(1)$ (Ferrer et al. 2010). In our case $\Delta = 1$ for a magnetic field $B = 5 \times 10^{17}$ G, while for $B = 10^{18}$ G, $\Delta \simeq 3.3$. In our numerical computations we will first use magnetic field values well within the isotropic region $B = 10^{17}$ G, and after that in the anisotropic region $B = 10^{18}$ G to compare their effects on the stellar structure.

3 TOV EQUATIONS FOR MSQM

In order to set up the problem posed by the magnetized matter EoS in the study of the structure of COs, we will analyze the usual spherical case, solving the resulting Tolman-Oppenheimer-Volkoff (TOV) equations (Misner et al. 1973). To find the static structure of a relativistic spherical star, we start from Einstein's equation

$$G^{\mu}{}_{\nu} \equiv R^{\mu}{}_{\nu} - \frac{1}{2} R g^{\mu}{}_{\nu} = 8\pi G \mathcal{T}^{\mu}{}_{\nu}\,, \tag{16}$$

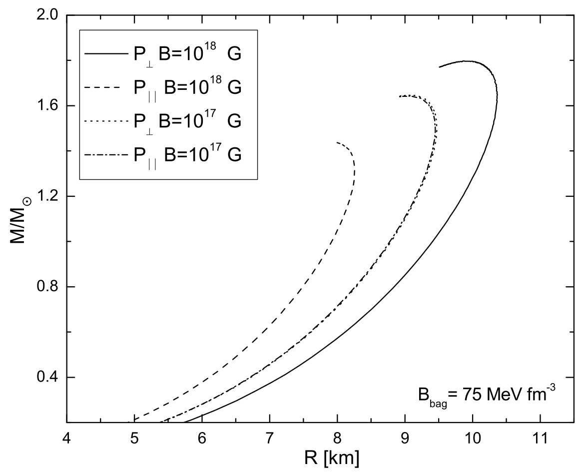

Fig. 3 Mass-Radius diagram comparing the effects of taking parallel or perpendicular pressure.

$(\mu, \nu = 0, 1, 2, 3)$, imposing the Schwarzschild metric

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2 d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2\phi d\phi^2 \,, \tag{17}$$

and the energy momentum tensor

$$\mathcal{T}^{\mu}_{\ \nu} = (E + P)u^{\mu}u_{\nu} + Pg^{\mu}_{\ \nu} \,. \tag{18}$$

We obtain the TOV equations

$$\frac{dM}{dr} \;=\; 4\pi G\, E, \tag{19}$$

$$\frac{dP}{dr} \;=\; -G\frac{(E+P)(M+4\pi P r^3)}{r^2 - 2rM} \,, \tag{20}$$

with boundary conditions $P(R) = 0$, $M(0) = 0$ and the EoS $E \to f(P)$.

When we look at Equation (20) the problem of which pressure must be used in the case of a magnetized EoS (like the one obtained in the preceding section) arises. One option is to work within the isotropic regime $(\Delta < 1)$ where $P_{\perp} = P_{\parallel}$. However, if we want to explore the anisotropic regime and address the issue of what is the maximum field, the problem is unavoidable.

In Figure 3 the mass–radius diagram is shown for MSQM for two values of the magnetic field, which compares the effects of both pressures in Equation (20). For $B = 10^{17}$ G the differences are not visible as expected because the pressures are nearly equal and we are in the isotropic regime $(\Delta = 0.03)$. For $B = 10^{18}$ G the differences between using the perpendicular or the parallel pressure are quite large. In this case one must establish a criterion to employ one of them or improve the structure equations to take into account the anisotropies.

4 ANISOTROPIC STRUCTURE EQUATIONS

In order to improve the structure equations in the presence of anisotropic pressures, we propose that a more "natural" geometry of a magnetized fermion system is an axisymmetric geometry. Thus, to obtain the structure equations, we start with the cylindrically symmetric metric

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2 d\phi^2 + e^{2\Psi}dz^2 \,, \tag{21}$$

where Φ, Λ, Ω and Ψ are only functions of r.

For this metric, the nonzero Einstein tensor components are

$$\begin{aligned}
G_t^t &= e^{-2\Lambda}(\Psi'' + \Psi'^2 - \Psi'\Lambda' - \frac{1}{r}\Lambda' + \frac{1}{r}\Psi')\,, \\
G_r^r &= e^{-2\Lambda}(\Psi'\Phi' + \frac{1}{r}\Phi' + \frac{1}{r}\Psi')\,, \\
G_\phi^\phi &= e^{-2\Lambda}(\Phi'' + \Phi'^2 - \Phi'\Lambda' + \Psi'' + \Psi'^2 - \Psi'\Lambda' + \Psi'\Phi'), \\
G_z^z &= e^{-2\Lambda}(\Phi'' + \Phi'^2 - \Phi'\Lambda' - \frac{1}{r}\Lambda' + \frac{1}{r}\Phi')\,.
\end{aligned}$$

With the energy momentum tensor for magnetized matter given by (González Felipe et al. 2005)

$$\mathcal{T}^\mu{}_\nu = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & P_\perp & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\parallel \end{pmatrix}\,, \tag{22}$$

where E, $P_\parallel$ and $P_\perp$ are given by the EoS (11), (12) and (13) respectively.

From the Einstein field equations in natural units we then obtain the following four differential equations:

$$\begin{aligned}
4\pi E &= -e^{-2\Lambda}(\Psi'' + \Psi'^2 - \Psi'\Lambda' - \frac{1}{r}\Lambda' + \frac{1}{r}\Psi')\,, \\
4\pi P_\perp &= e^{-2\Lambda}(\Psi'\Phi' + \frac{1}{r}\Phi' + \frac{1}{r}\Psi')\,, \\
4\pi P_\perp &= e^{-2\Lambda}(\Phi'' + \Phi'^2 - \Phi'\Lambda' + \Psi'' + \Psi'^2 - \Psi'\Lambda' + \Psi'\Phi')\,, \\
4\pi P_\parallel &= e^{-2\Lambda}(\Phi'' + \Phi'^2 - \Phi'\Lambda' - \frac{1}{r}\Lambda' + \frac{1}{r}\Phi')\,.
\end{aligned}$$

Performing some algebra with the previous system of equations, and using the energy momentum conservation $(\mathcal{T}^\mu{}_{\nu;\mu})$ we finally obtain

$$P_\perp' = -\Phi'(E + P_\perp) - \Psi'(P_\perp - P_\parallel)\,, \tag{23a}$$

$$4\pi e^{2\Lambda}(E + P_\parallel + 2P_\perp) = \Phi'' + \Phi'(\Psi' + \Phi' - \Lambda') + \frac{\Phi'}{r}\,, \tag{23b}$$

$$4\pi e^{2\Lambda}(E + P_\parallel - 2P_\perp) = -\Psi'' - \Psi'(\Psi' + \Phi' - \Lambda') - \frac{\Psi'}{r}\,, \tag{23c}$$

$$4\pi e^{2\Lambda}(P_\parallel - E) = \frac{1}{r}(\Psi' + \Phi' - \Lambda')\,. \tag{23d}$$

This form, together with the EoS $E \to f(P_\perp),\ P_\parallel \to f(E)$, is a system of differential equations in the variables

$$P_\perp,\ P_\parallel,\ E,\ \Phi,\ \Lambda,\ \Psi\,. \tag{24}$$

Since the differential equations involve factors of $1/r$, we will start with a power series expansion of $P_\perp$, Φ, Ψ and Λ around $r = 0$ to find the initial conditions suitable for numerical calculations

$$P_\perp = P_{\perp 0} + P_{\perp 1} r, \tag{25}$$

$$\Lambda = \Lambda_0 + \Lambda_1 r, \tag{26}$$

$$\Phi = \Phi_0 + \Phi_1 r + \Phi_2 r^2, \tag{27}$$

$$\Psi = \Psi_0 + \Psi_1 r + \Psi_2 r^2\,. \tag{28}$$

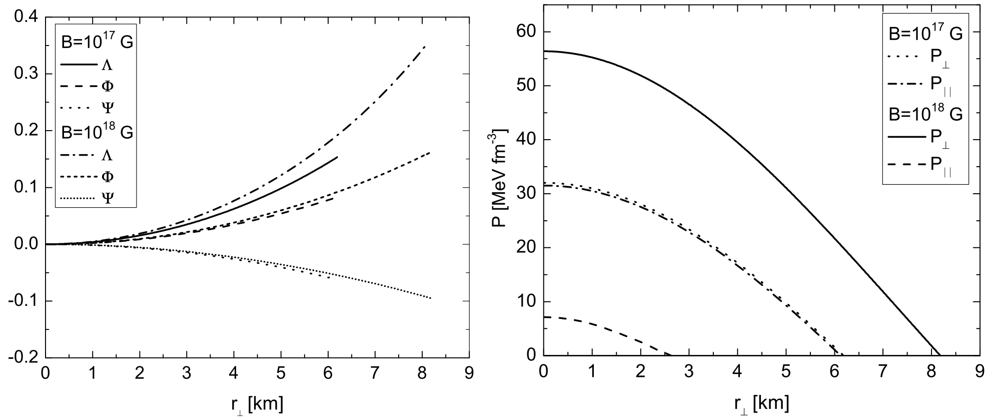

Fig. 4 Metric coefficients and pressures inside the star for two values of the magnetic field.

We take also $\Psi = \Phi = \Lambda = 0$ at $r = 0$ so that the corresponding metric coefficients are equal to 1 at that point and $\Psi' = \Phi' = 0$ to select smooth solutions on the z-axis.

By substitution of these conditions in the system of differential equations we find

$$P_\perp(0) = P_{\perp 0}\,, \tag{29a}$$
$$\Lambda(0) = 0\,, \tag{29b}$$
$$\Phi(0) = \frac{1}{2}(P_{\|0} + 2P_{\perp 0} + E_0)(r_0^2 - 2r_0)\,, \tag{29c}$$
$$\Psi(0) = \frac{1}{2}(-P_{\|0} + 2P_{\perp 0} - E_0)(r_0^2 - 2r_0)\,, \tag{29d}$$
$$\Phi'(0) = 0\,, \tag{29e}$$
$$\Psi'(0) = 0\,. \tag{29f}$$

We also impose

$$P_\perp(R_\perp) = 0\,,$$

which determines the radius of the star, in the equatorial (perpendicular) direction.

The solutions of the system given by Equation (23) with initial conditions Equation (29) are shown in Figure 4. In the left panel of Figure 4 the behavior of the metric coefficients is shown for two values of the magnetic field. As we can note, the increase of the magnetic field produces an increase of the star radius in the perpendicular direction. In the right panel of Figure 4 the pressures inside the star are depicted for a selected central density. All these quantities exhibit a regular physical behavior.

As we have pointed out, by hypothesis all our variables just depend on the perpendicular radial direction. Therefore, in the case of spherical symmetry, we cannot simply compute all the quantities needed for the mass radius diagram like in Figure 3. Instead we will compute the Tolman (Tolman 1934) generalization for the mass per unit length of a source

$$M_T = \int \sqrt{-g}(T_0^0 - T_1^1 - T_2^2 - T_3^3)dV. \tag{30}$$

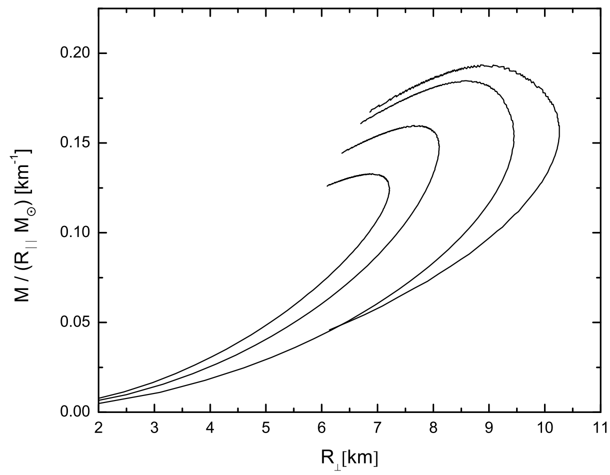

Fig. 5 Mass per unit parallel length $(M/R_{\|})$ in solar masses, versus perpendicular radius. As the magnetic field increases the perpendicular radius increases up to a critical field. The curves are organized in order of increasing values of the magnetic fields: $B = 10^{17}$ G, $B = 10^{18}$ G, $B = 1.5 \times 10^{18}$ G and $B = 1.7 \times 10^{18}$ G.

For the cylindrical metric (21) we have

$$M_T = \int re^{\Phi+\Psi+\Lambda}(E - 2P_{\perp} - P_{\|})dV \tag{31}$$

$$= \int_0^{2\pi} \int_{-R_{\|}}^{R_{\|}} \int_0^{R_{\perp}} re^{\Phi+\Psi+\Lambda}(E - 2P_{\perp} - P_{\|})d\phi\, dz\, dr \tag{32}$$

$$= 4\pi R_{\|} \int_0^{R_{\perp}} re^{\Phi+\Psi+\Lambda}(E - 2P_{\perp} - P_{\|})dr\,. \tag{33}$$

Therefore, we cannot compute the mass of the star but rather the mass per unit length $(M_T/R_{\|})$

$$\frac{M_T}{R_{\|}} = 4\pi \int_0^{R_{\perp}} re^{\Phi+\Psi+\Lambda}(\epsilon - 2P_{\perp} - P_{\|})dr\,. \tag{34}$$

In Figure 5 the mass per unit length versus perpendicular radius is shown. When the magnetic field increases, the perpendicular radius and the mass per unit length of the star also increases. We found that there is a maximum field ($B \simeq 1.8 \times 10^{18}$ G) beyond which the metric coefficients exhibit a divergent behavior. This value of the magnetic field almost coincides with the threshold for which the pressure difference has become important, and results in $B = 1.8 \times 10^{18}$ G, (for which $\Delta = 10.5$). Therefore, we infer that no stable solutions of the system are possible beyond this point and this indicates the end of the theoretical stellar sequences within the adopted assumptions.

Even though within our model we cannot compute the mass radius relation, the information given in Figure 5 is important for constraining the maximum magnetic field allowed for magnetized COs.

5 CONCLUSIONS

We have pointed out and worked on the problem of anisotropic pressures in the description of the structure of a CO in this paper. The suggestion is that when the splitting coefficient of the pressures

Δ becomes > 1, the differences in the pressures cannot be neglected and a different approach must be used to study the structure of the star.

In the constant magnetic field approximation used in our calculations, the feature of the anisotropic pressures is always present, but other possibilities, such as different geometrical configurations of the magnetic field, are not excluded.

An anisotropic metric has been employed to account for the stellar structure. The choice of a constant magnetic field allowed a simple solution containing the anisotropic features in the high magnetic field regime.

We have obtained a regular behavior of the metric coefficients inside the star and a physically consistent dependence of the pressures on the radii.

More importantly, the existence of a critical field ($B_{\rm c} \sim 1.8 \times 10^{18}$ G) beyond which there are no equilibrium configurations has been found. This critical field is essentially (up to a small numerical factor) related to the scalar virial theorem $B_{\rm max} \sim 10^{18}$ G; it suggests that a magnetic instability ends the stable sequence of stars for values of the field above the critical one.

Our main simplification in this model is that we have taken all the variables as being dependent on just the perpendicular (equatorial) radius. This allowed us to have a more tractable system of differential equations, although as a result we cannot accurately compute the physical quantities unless the dependence on both (r, z) for the variables of the problem is kept. Nevertheless, the model confirms the intuitive idea of the existence of a maximum magnetic field for which the star may undergo an anisotropic collapse due to a magnetic instability. We conjecture that a more accurate scheme should render slightly different values, but with the same qualitative behavior found above.

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