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FLRW non-singular cosmological model in general relativity

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Abstract A singularity free cosmological model is obtained in a homogeneous and isotropic background with a specific form of the Hubble parameter in the presence of an interacting dark energy represented by a time-varying cosmological constant in general relativity. Different cases that arose have been extensively studied for different values of the curvature parameter. Some interesting results have been found with this form of the Hubble parameter to meet the possible negative value of the deceleration parameter $\left(-\frac{1}{3} \leqslant q < 0\right)$ as the current observations reveal. For some particular values of these parameters, the model reduces to Berman's model.

Key words: singularity — cosmological constant — dark energy — parameter

1 INTRODUCTION

Even after the tremendous success of standard cosmology, it suffers from the problem of initial singularity (or the Big Bang), where the physical theory breaks down. If we consider the homogeneous and isotropic Robertson-Walker space-time

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{1}$$

together with the perfect fluid distribution of matter represented by the energy-momentum tensor

$$T_{ij}^{\rm M} = (\rho + p)\, U_i U_j + p\, g_{ij}, \tag{2}$$

where 'R' represents the scale factor, 'ρ' the energy density of cosmic matter present in the Universe and 'p' its isotropic pressure, then the Einstein field equations

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} = -T_{ij}^{\rm M} \quad \text{(with } 8\pi G = 1\text{)}, \tag{3}$$

yield two independent equations as follows

$$\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \tag{4}$$

$$p = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}. \tag{5}$$

Here an overdot $(\cdot)$ represents an ordinary derivative with respect to only cosmic time 't'. Equations (4) and (5) are two equations with three unknown functions R, ρ and p. If we assume the perfect fluid equation of state

$$p = w\rho, \;\; 0 \leq w \leq 1, \qquad w \text{ is a constant,} \tag{6}$$

then the system becomes closed and completely determines the dynamics of the Universe. If the matter content in the Universe is considered to be normal matter ($\rho > 0$ and $p > 0$), then we find that, from Equations (4) and (5) together with Equation (6), the scale factor R becomes $zero$ at some finite time in the past, where the space-time becomes singular as $\rho \to \infty$ and $p \to \infty$.

While considering the issue of the very early Universe (near the singular point), quantum gravitational effects are expected to come into play which must be addressed by the theory of quantum gravity. However, as is well known, the full theory of quantum gravity is not available at the present stage, so we try to solve our problem within the framework of classical general relativity. Our aim in this paper is to find a non-singular bouncing solution (which is not new to cosmology) by constraining the Hubble parameter 'H' (which regulates the dynamics of the Universe). The bounce occurs at some finite (classical) value of the scale factor which may escape any quantum contributions. We can see that the system becomes overdetermined if any extra condition (which here we impose on the Hubble parameter) is assumed. This overdeterminacy can be compensated by introducing another entity into the field equations, the illustrious *dark energy* (DE). Nowadays, the theory of dark energy has become very popular and is a well established theory in modern cosmology, which is responsible for the current observed accelerating expansion of the Universe (Riess et al. 1998; Kowalski et al. 2008; Perlmutter et al. 1999; Amanullah et al. 2010; Rubin et al. 2013). In recent years, there has been a spurt of activity in ascertaining these accelerating models which are also supported by a number of observations such as Tegmark et al. (2004); Seljak et al. (2005); Wang & Mukherjee (2006); Bond et al. (1997); Eisenstein et al. (2005); Spergel et al. (2003, 2007); Komatsu et al. (2009, 2011); Hinshaw et al. (2009); Ade et al. (2014); Jain & Taylor (2003).

Though not much is known about this mysterious dark energy, it can be epitomized by a large-scale scalar field ϕ. For a scalar field with Lagrangian density $\mathcal{L} = \partial_\mu\phi\partial^\mu\phi - V(\phi)$, the stress energy tensor takes the form

$$T_{ij}^{\rm DE} = (\rho_\phi + p_\phi)U_iU_j + p_\phi \, g_{ij} \tag{7}$$

with its equation of state in the form $p_\phi = w_\phi\rho_\phi$, where w_ϕ is generally a function of time. Depending upon the dynamics of the field ϕ and its potential energy, this produces a number of candidates for dark energy. As is well known, the simplest and most preferred candidate of dark energy is Einstein's cosmological constant Λ (supported by a number of cosmological observations) for which w_ϕ is condensed to the value -1 (potential energy represented by a scalar field).

In Einstein's theory, dark energy can be introduced by supplanting $T_{ij}^{\rm M}$ by $T_{ij}^{\rm Total}$ in Equation (3), where

$$T_{ij}^{\rm Total} = T_{ij}^{\rm M} + T_{ij}^{\rm DE} = (\rho_t + p_t)U_iU_j + p_t \, g_{ij} \tag{8}$$

with $\rho_t = \rho + \rho_\phi$ and $p_t = p + p_\phi$. Now, the modified Einstein field equations are

$$\rho_t \; = \; 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} \, , \tag{9}$$

$$p_t \; = \; -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} \, . \tag{10}$$

The Bianchi identities entail that $T_{ij}^{\rm Total}$ has a vanishing divergence. We believe that the matter energy and dark energy are interacting naturally, which is a fundamental principle (Vishwakarma & Narlikar 2007). Although there are several candidates for dark energy (in the literature), we limit ourselves in the following to the case of only a cosmological constant. It is well known that cosmological constant Λ can also be represented as the inherent energy density of vacuum $\rho_{\rm v} = \Lambda$ (as we have taken $8\pi G = 1$) ascending from the zero point energy of quantum fluctuations which leads to the widely discussed cosmological constant problem. However, this problem is alleviated by considering a dynamically decaying vacuum energy (Abdussattar & Vishwakarma 1996 & the references therein).

Section 2 provides the dynamics of the Universe from the Hubble parameter where we have explained our main ansatz. In Sections 3, 4 and 5, we study some properties of our obtained model for different values of the curvature parameter of FLRW space-time. In Section 6, we discuss the parameters involved and put constraints on these parameters. At the end, we conclude our results in Section 7.

2 DYNAMICS OF THE UNIVERSE FROM THE HUBBLE PARAMETER

The observable parameters 'H' (Hubble parameter) and 'q' (deceleration parameter) are defined as

$$H = \dot{R}/R, \tag{11}$$

and

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{12}$$

In order to obtain the exact solution of the Einstein field equations, an extra condition is needed for which several authors have considered various forms such as $\Lambda \propto R^{-2}$ (Chen & Wu 1990), $\Lambda \propto H$, $\Lambda \propto H^2$ (Ali 2013; Arbab 1997), $\Lambda \propto t^{-2}$ (Vishwakarma 2001), $R \propto t^n$, $R \propto t^n \exp(t)$ (Banerjee & Das 2006), $q = \text{constant}$ (Berman 1991; Vishwakarma et al. 1999), $\rho = \rho_{\rm c}$ (Özer & Taha 1987), etc. A phenomenological approach to describe the cosmological evolution of decaying vacuum cosmology ($\Lambda(t)$CDM) has been studied by Wang and Meng (2005) based on a simple assumption about the form of the modified matter expansion rate in an attempt to unify almost all the current vacuum decaying models under one umbrella. Keeping in mind the current picture of an accelerating and flat Universe, Ray et al. (2007) have considered some specific dynamical models of the cosmological term for investigating the nature of dark energy and estimated the present values of some of the physical parameters which are in good agreement with the values suggested by Type Ia supernovae data and other experimental data. Recently, cosmological models based on the interaction between dark matter and dark energy appearing in several different cosmological scenarios have been pointed out by Barrow and Clifton (2006) which were further studied by Maia et al. (2015). By constraining the form of the deceleration parameter 'q' as $q = -\frac{\alpha}{t^2} + (\beta - 1)$, Abdussattar and Prajapati (2011) obtained a class of non-singular and bouncing cosmological models wherein the matter source is considered to be a perfect fluid and an interacting dark energy is represented by a dynamically decaying cosmological constant in a homogeneous and isotropic space-time. Berman (1983) has considered a special form of the Hubble parameter which leads to a constant deceleration parameter $q = m - 1$ and obtained a cosmological model with the variation of Λ (Berman 1991). In the quest for a negative value of the deceleration parameter consistent with observations, along the same line as that floated by Berman, here in this paper, we propose a specific form of the Hubble parameter given by

$$H = \frac{m}{\alpha t + \beta}\,, \tag{13}$$

which is the key ansatz of our paper. Here $m > 0, \alpha \neq 0$ and β are parameters. For $m = 1$ the model reduces to the model obtained by Berman. With the form of H given by Equation (13), Equation (11) on integration leads to the time variation of the scale factor as

$$R(t) = A(\alpha t + \beta)^{\frac{m}{\alpha}}\,, \tag{14}$$

where A is a constant of integration. Obviously, the diverse values of m and α will give rise to different cosmological models. Here, for simplicity we set the origin of the time coordinate at the bounce of these bouncing models.

It is easy to see from Equation (14) that at $t = 0$, $R = R_0 \neq 0$ (say, here and subsequently the suffix 'zero' signifies the value of the parameter at time $t = 0$). This implies

$$R = R_0 \beta^{-\frac{m}{\alpha}} (\alpha t + \beta)^{\frac{m}{\alpha}} . \tag{15}$$

The first and second order derivatives of the scale factor R are given by

$$\dot{R} = R_0 \beta^{-\frac{m}{\alpha}} m (\alpha t + \beta)^{\frac{m}{\alpha} - 1}, \tag{16}$$

and

$$\ddot{R} = R_0 \beta^{-\frac{m}{\alpha}} m (m - \alpha)(\alpha t + \beta)^{\frac{m}{\alpha} - 2}, \tag{17}$$

indicating that initially at time $t = 0$, we have $\dot{R} = R_0 \frac{m}{\beta}$ and $\ddot{R} = R_0 \frac{m(m-\alpha)}{\beta^2}$. This shows that the obtained model is free from initial singularity. Also, it starts with a finite acceleration and *finite velocity*. This is a significant deviation (the Universe starts with a finite acceleration and zero velocity) from the result obtained by Abdussattar and Prajapati (2011). The deceleration parameter is obtained using Equations (12) and (13) as

$$q = -1 + \frac{\alpha}{m} , \tag{18}$$

showing that the deceleration parameter is independent of time and the choice of α and m will suggest whether the expansion of the Universe is accelerated or decelerated.

With the help of Equations (15), (16) and (17), Equations (9) and (10) give

$$\rho_t \; = \; \frac{3m^2}{(\alpha t + \beta)^2} + \frac{3k}{R_0^2} \beta^{2\frac{m}{\alpha}} \frac{1}{(\alpha t + \beta)^{2\frac{m}{\alpha}}} , \tag{19}$$

$$p_t \; = \; \frac{m\,(2\alpha - 3m)}{(\alpha t + \beta)^2} - \frac{k}{R_0^2} \beta^{2\frac{m}{\alpha}} \frac{1}{(\alpha t + \beta)^{2\frac{m}{\alpha}}} , \tag{20}$$

yielding

$$\rho_t + p_t = \frac{2\alpha}{3m} \left(3H^2\right) + \frac{2k}{R_0^2} \beta^{2\frac{m}{\alpha}} \frac{1}{(\alpha t + \beta)^{2\frac{m}{\alpha}}} . \tag{21}$$

From Equation (21) we can see, for $\frac{\alpha}{m} = \frac{3}{2}$, the model would indicate $\rho = \rho_{\mathrm{c}}$ at adequately large times (for which the pressure of matter $p = 0$). The total active gravitational mass is obtained as

$$(\rho_t + 3p_t)\, R^3 = 6R_0^3 \beta^{-3\frac{m}{\alpha}} m\,(\alpha - m)\,(\alpha t + \beta)^{3\frac{m}{\alpha} - 2} \;, \tag{22}$$

which is negative, zero or positive in accordance with $\frac{\alpha}{m} \lesseqgtr 1$. Equation (19) suggests that at $t = 0$, $\rho_{t0} = \frac{3m^2}{\beta^2} + \frac{3k}{R_0^2}$ suggesting that $m > \frac{\beta}{R_0}$ for $k = -1$.

The age of the Universe is found to be $t_{\mathrm{p}} = \left(\frac{m}{\alpha}\right) H_{\mathrm{p}}^{-1} - \frac{\beta}{\alpha}$ and the radius of the Universe is given by $R_{\mathrm{p}} = \left(\frac{m}{\beta}\right)^{\frac{m}{\alpha}} \left(H_{\mathrm{p}}^{-1}\right)^{\frac{m}{\alpha}} R_0$, where the suffix 'p' represents the value at the present time.

In the following sections, we study some properties of the model in the early radiation dominated (RD) era and matter dominated (MD) era for different values of the curvature parameter.

3 $k = 0$ (SPATIALLY FLAT UNIVERSE)

3.1 RD Phase ($p = p_{\rm r} = \frac{1}{3}\rho_{\rm r}$)

In the early *pure radiation era*, the equation of state of matter is assumed to be $p = p_{\rm r} = \frac{1}{3}\rho_{\rm r}$. Equations (19) and (20) yield

$$\rho_{\rm r} = \frac{3}{2}\frac{\alpha m}{(\alpha t + \beta)^2}\,, \tag{23}$$

$$\rho_{\rm v} = \frac{3}{2}\frac{m(2m - \alpha)}{(\alpha t + \beta)^2}\,. \tag{24}$$

From Equations (23) and (24), we observe that at $t = 0$, we have $\rho_{\rm r0} = \frac{3\alpha m}{2\beta^2}$ and $\rho_{\rm v0} = \frac{3m(2m-\alpha)}{2\beta^2}$ suggesting that $\rho_{\rm r0} > 0$ in the beginning and $\rho_{\rm v0} > 0$ unless $\alpha > 2m$. The first order derivatives of $\rho_{\rm r}$ and $\rho_{\rm v}$ (Eqs. (23) and (24)) with respect to 't' yield

$$\dot{\rho}_{\rm r} = -\frac{3\alpha^2 m}{(\alpha t + \beta)^3}\,, \tag{25}$$

$$\dot{\rho}_{\rm v} = -\frac{3\alpha m(2m - \alpha)}{(\alpha t + \beta)^3}\,. \tag{26}$$

From Equations (25) and (26), it follows that $\dot{\rho}_{\rm r}$ and $\dot{\rho}_{\rm v}$ are negative showing that $\rho_{\rm r}$ and $\rho_{\rm v}$ are decreasing functions of time. Furthermore, the $\ddot{\rho}_{\rm r} > 0$ and $\ddot{\rho}_{\rm v} > 0$ at $t = 0$ imply that $\rho_{\rm r}$ and $\rho_{\rm v}$ are initially maximum but rapidly decrease by creating massive or massless particles.

We know the radiation energy density and temperature (T) are related by the relation

$$\rho_{\rm r} = \frac{\pi^2}{30}N(T)T^4\,, \tag{27}$$

in units with $k_B = c = \hbar = 1$. At temperature T, the effective number of spin degrees of freedom $N(T)$ is given by $N(T) = \frac{7}{8}N_{\rm f}(T) + N_{\rm b}(T)$, where $N_{\rm f}(T)$ and $N_{\rm b}(T)$ correspond to fermions and bosons respectively. We assume $N(T)$ to be constant throughout this era. From Equations (23) and (27) we obtain

$$T = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{(\alpha t + \beta)^2}\right]^{\frac{1}{4}}\,, \tag{28}$$

showing that at $t = 0$ we have $T_0 = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{\beta^2}\right]^{\frac{1}{4}}$ implying that the radiation temperature is also constant and is initially maximum.

3.2 MD Phase $p = p_{\rm m} \approx 0$, $\rho = \rho_{\rm m}$

In the present *matter dominated era*, the matter pressure is negligible, i.e. $p = p_{\rm m} \approx 0$ and $\rho = \rho_{\rm m}$. Equations (19) and (20) give

$$\rho_{\rm m} = \frac{2\alpha m}{(\alpha t + \beta)^2}\,, \tag{29}$$

$$\rho_{\rm v} = \frac{m(3m - 2\alpha)}{(\alpha t + \beta)^2}\,. \tag{30}$$

As $t \to \infty$, $\rho_{\rm m} \to 0$ and $\rho_{\rm v} \to 0$. Equations (29) and (30) can be written in terms of the Hubble parameter as

$$\rho_{\rm mp} = \left(\frac{2\alpha}{m}\right) H_{\rm p}^2 \,, \tag{31}$$

$$\rho_{\rm vp} = \left(3 - \frac{2\alpha}{m}\right) H_{\rm p}^2 \,. \tag{32}$$

4 $k = 1$ (NON-FLAT CLOSED UNIVERSE)

4.1 RD Phase ($p = p_{\rm r} = \frac{1}{3}\rho_{\rm r}$)

In this phase of evolution of the Universe, the radiation and vacuum energy densities are obtained from Equations (19) and (20) as

$$\rho_{\rm r} = \frac{3}{2}\left[\frac{\alpha m}{(\alpha t + \beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t + \beta)^{2\frac{m}{\alpha}}}\right], \tag{33}$$

$$\rho_{\rm v} = \frac{3}{2}\left[\frac{m(2m-\alpha)}{(\alpha t + \beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t + \beta)^{2\frac{m}{\alpha}}}\right]. \tag{34}$$

At $t = 0$, we have $\rho_{\rm r0} = \frac{3}{2}\left[\frac{\alpha m}{\beta^2} + \frac{1}{R_0^2}\right]$ and $\rho_{\rm v0} = \frac{3}{2}\left[\frac{m(2m-\alpha)}{\beta^2} + \frac{1}{R_0^2}\right]$. Differentiating Equations (33) and (34) with respect to t, we obtain

$$\dot{\rho}_{\rm r} = -\frac{3\alpha^2 m}{(\alpha t + \beta)^3} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t + \beta)^{2\frac{m}{\alpha}+1}}, \tag{35}$$

$$\dot{\rho}_{\rm v} = -\frac{3\alpha m(2m-\alpha)}{(\alpha t + \beta)^3} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t + \beta)^{2\frac{m}{\alpha}+1}}. \tag{36}$$

Equations (35) and (36) show that $\dot{\rho}_{\rm r}$ and $\dot{\rho}_{\rm v}$ are negative, implying that $\rho_{\rm r}$ and $\rho_{\rm v}$ are decreasing functions of time. Also $\ddot{\rho}_{\rm r} > 0$ and $\ddot{\rho}_{\rm v} > 0$ at $t = 0$, implying that $\rho_{\rm r}$ and $\rho_{\rm v}$ are initially maximum.

The radiation temperature (T) in this case is obtained from Equations (27) and (33) as

$$T = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{(\alpha t + \beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t + \beta)^{2\frac{m}{\alpha}}}\right]^{\frac{1}{4}}. \tag{37}$$

At $t = 0$, Equation (37) yields $T_0 = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{\beta^2}\right]^{\frac{1}{4}}$ which is the maximum value of T. We have here $k = 1$, i.e. the Universe is geometrically closed. So, it is possible to determine the time $t = t_{\rm cau}$ when the whole Universe becomes causally connected and $t_{\rm cau}$ is given by

$$\int_0^{t_{\rm cau}} \frac{dt}{R(t)} = \int_0^1 \frac{dr}{\sqrt{1-r^2}} = \frac{\pi}{2}\,. \tag{38}$$

Using Equation (15), we obtain

$$\int_0^{t_{\rm cau}} \frac{dt}{(\alpha t + \beta)^{\frac{m}{\alpha}}} = \frac{\pi}{2}R_0\beta^{-\frac{m}{\alpha}}\,, \tag{39}$$

which on integration yields

$$t_{\rm cau} = \frac{1}{\alpha}\left[\frac{\pi}{2}R_0\beta^{-\frac{m}{\alpha}}(\alpha - m) + \beta^{\frac{\alpha-m}{\alpha}}\right]^{\frac{\alpha}{\alpha-m}} - \frac{\beta}{\alpha}\,. \tag{40}$$

We find that the global causality is established at $t = t_{\rm cau}$ and supplying some particular values of m, α and β, we can determine $t_{\rm cau}$ from Equation (40).

4.2 MD Phase ($p = p_{\rm m} \approx 0$, $\rho = \rho_{\rm m}$)

In this phase of evolution of the Universe, we have

$$\rho_{\rm m} \;=\; \left[\frac{2\alpha m}{(\alpha t+\beta)^2} + \frac{2}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right], \tag{41}$$

$$\rho_{\rm v} \;=\; \left[\frac{m(3m-2\alpha)}{(\alpha t+\beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]. \tag{42}$$

As $t \to \infty$, $\rho_{\rm m} \to 0$ and $\rho_{\rm v} \to 0$. Equations (41) and (42) can be written in terms of the Hubble parameter as

$$\rho_{\rm mp} \;=\; \left(\frac{2\alpha}{m}\right) H_{\rm p}^2 + \frac{2}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_{\rm p}^{2\frac{m}{\alpha}}, \tag{43}$$

$$\rho_{\rm vp} \;=\; \left(3-\frac{2\alpha}{m}\right) H_{\rm p}^2 + \frac{1}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_{\rm p}^{2\frac{m}{\alpha}}. \tag{44}$$

5 $k = -1$ (NON-FLAT OPEN UNIVERSE)

5.1 RD Phase ($p = p_{\rm r} = \frac{1}{3}\rho_{\rm r}$)

Here, the radiation and vacuum energy densities are obtained as

$$\rho_{\rm r} \;=\; \frac{3}{2}\left[\frac{\alpha m}{(\alpha t+\beta)^2} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right], \tag{45}$$

$$\rho_{\rm v} \;=\; \frac{3}{2}\left[\frac{m(2m-\alpha)}{(\alpha t+\beta)^2} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]. \tag{46}$$

At $t = 0$, Equations (45) and (46) yield

$$\rho_{\rm r0} \;=\; \frac{3}{2}\left[\frac{\alpha m}{\beta^2} - \frac{1}{R_0^2}\right], \tag{47}$$

$$\rho_{\rm v0} \;=\; \frac{3}{2}\left[\frac{m(2m-\alpha)}{\beta^2} - \frac{1}{R_0^2}\right]. \tag{48}$$

Equation (47) suggests that $\frac{\beta^2}{\alpha m} < R_0^2$. If $\frac{\beta^2}{\alpha m} = R_0^2$, we get $\rho_{\rm r0} = 0$. From Equations (45) and (46), we observe that

$$\rho_{\rm r} \gtreqless 0 \quad \text{for} \quad t \gtreqless \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\sqrt{\alpha m}}\frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}$$

and

$$\rho_{\rm v} \gtreqless 0 \quad \text{for} \quad t \gtreqless \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\sqrt{m(2m-\alpha)}}\frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}.$$

The differentiation of (45) and (46) with respect to cosmic time 't' yields

$$\dot{\rho}_{\rm r} = -\frac{3\alpha^2 m}{(\alpha t+\beta)^3} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t+\beta)^{2\frac{m}{\alpha}+1}}, \tag{49}$$

$$\dot{\rho}_{\rm v} = -\frac{3\alpha m(2m-\alpha)}{(\alpha t+\beta)^3} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t+\beta)^{2\frac{m}{\alpha}+1}} \,. \tag{50}$$

$\dot{\rho}_{\rm r}$ becomes zero at $t = \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\alpha}\frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}$. Also $\dot{\rho}_{\rm v}$ becomes zero at $t = \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\sqrt{\alpha(2m-\alpha)}}\frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}$. At these points $\rho_{\rm r}$ and $\rho_{\rm v}$ are maximum.

In this case, the radiation temperature (T) is obtained from Equations (27) and (45) as

$$T = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{(\alpha t+\beta)^2} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]^{\frac{1}{4}} \,. \tag{51}$$

From Equation (51), at $t=0$, we have $T_0 = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{\beta^2} - \frac{1}{R_0^2}\right]^{\frac{1}{4}}$.

5.2 MD Phase ($p = p_{\rm m} \approx 0$, $\rho = \rho_{\rm m}$)

In this phase of evolution of the Universe, we have

$$\rho_{\rm m} = \left[\frac{2\alpha m}{(\alpha t+\beta)^2} - \frac{2}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right] , \tag{52}$$

$$\rho_{\rm v} = \left[\frac{m(3m-2\alpha)}{(\alpha t+\beta)^2} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right] . \tag{53}$$

As $t\to\infty$, $\rho_{\rm m}\to 0$ and $\rho_{\rm v}\to 0$. Equations (52) and (53) can be written in terms of the Hubble parameter as

$$\rho_{\rm mp} = \left(\frac{2\alpha}{m}\right)H_{\rm p}^2 - \frac{2}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_{\rm p}^{2\frac{m}{\alpha}} , \tag{54}$$

$$\rho_{\rm vp} = \left(3-\frac{2\alpha}{m}\right)H_{\rm p}^2 - \frac{1}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_{\rm p}^{2\frac{m}{\alpha}} . \tag{55}$$

The evolution of the Universe in our obtained model comprehensively depends on the choice of the parameters α, m and β. In the following section we discuss the consequences of the choice of these parameters α, m and β.

6 THE PARAMETERS AND THE MODEL

From Equation (12), we observe that for

$\alpha = m$, we have $q = 0$ (Expanding Universe without acceleration).
$\alpha < m$, we have $q < 0$ (Accelerated expansion of the Universe).
$\alpha > m$, we have $q > 0$ (Decelerated expansion of the Universe).

A statistical observation is given in the following Table 1 for different values of α and m, giving rise to different models.

For the best fit value of the deceleration parameter as suggested by the observations, $-\frac{1}{3} \leqslant q < 0$, we must have $\frac{2}{3} \leqslant \frac{\alpha}{m} < 0$. From Table 1, we see that for a model consistent with the observations, we should have $1.5 \le \alpha \le 3$ and $2.5 \le m \le 4$. The value of β should be constrained according to the curvature parameter. These values of α and m produce some interesting models with $0 < \beta < 2$, if the curvature parameter is $k=0$ or $k=1$, but are incompatible with $k=-1$ for higher values

Table 1 Comparison of Different Models for Different Values of 'α' and 'm'

Parameter	Exemplification	q	H	R
$\alpha = m$	$\alpha = 1, m = 1$	0	$\frac{1}{t+\beta}$	$R_0\beta^{-1}(t+\beta)$
	$\alpha = 2, m = 2$		$\frac{2}{2t+\beta}$	$R_0\beta^{-1}(2t+\beta)$
	$\alpha = 3, m = 3$		$\frac{1}{3t+\beta}$	$R_0\beta^{-1}(3t+\beta)$
$\alpha < m$	$\alpha = 1, m = 2$	$-\frac{1}{2}$	$\frac{2}{t+\beta}$	$R_0\beta^{-2}(t+\beta)^2$
	$\alpha = 2, m = 3$	$-\frac{1}{3}$	$\frac{3}{2t+\beta}$	$R_0\beta^{-\frac{3}{2}}(2t+\beta)^{\frac{3}{2}}$
	$\alpha = 3, m = 4$	$-\frac{1}{4}$	$\frac{4}{3t+\beta}$	$R_0\beta^{-\frac{4}{3}}(3t+\beta)^{\frac{4}{3}}$
$\alpha > m$	$\alpha = 2, m = 1$	1	$\frac{1}{2t+\beta}$	$R_0\beta^{-\frac{1}{2}}(2t+\beta)^{\frac{1}{2}}$
	$\alpha = 3, m = 2$	$\frac{1}{2}$	$\frac{2}{3t+\beta}$	$R_0\beta^{-\frac{2}{3}}(3t+\beta)^{\frac{2}{3}}$
	$\alpha = 4, m = 3$	$\frac{1}{3}$	$\frac{3}{4t+\beta}$	$R_0\beta^{-\frac{3}{4}}(4t+\beta)^{\frac{3}{4}}$

of β within this range as is clear from Equations (45) and (46). If the present value of the Hubble parameter is considered to be $H_{\rm p} = 72$ km s^{-1} Mpc^{-1}, then Equations (54) and (55) suggest that the value of β should be in the range $0 < \beta < 0.624$.

With a suitable choice of β in this range, $\beta = 0.2$, and taking $\alpha = 2$ and $m = 3$, we may obtain the following (with $H_{\rm p} = 72\ \rm km\ s^{-1}\ Mpc^{-1}$):

Age of the Universe $(t_{\rm p}) \approx 6 \times 10^{17}$ s; Radius of the Universe $(R_{\rm p}) \approx 1.7 \times 10^{28}\ R_0$; the energy densities obtained are given in Table 2.

Table 2 Present Values of Matter and Vacuum Energy Densities $(8\pi G = 1)$

Curvature Parameter	Matter energy density	Vacuum energy density
	$(10^{-30}$ s$^{-2})$	
$k = 0$	≈ 7.2594317	≈ 9.0742895
$k = 1$	≈ 7.2600242	≈ 9.074882
$k = -1$	≈ 7.2588392	≈ 9.0736970

7 CONCLUSIONS

In this paper we have obtained a class of non-singular and bouncing FLRW cosmological models wherein the matter source is supplied by a perfect fluid and an interacting dark energy is represented by a dynamically decaying cosmological constant by constraining the form of the Hubble parameter. Here, we have freedom associated with the parameters involved to obtain a suitable model of the Universe consistent with observations. For some specific values of these parameters we have obtained the age and radius of the Universe which are slightly greater than the age and radius obtained in the standard model. In all the three cases of the curvature parameters, the present values of the matter and vacuum energy densities are almost the same. The model is a simple generalization of the model obtained by Berman (1983).

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