RAA 2015 Vol. **15** No. **10**, 1695–1700 doi: 10.1088/1674–4527/15/10/006
http://www.raa-journal.org *http://www.iop.org/journals/raa*

***R**esearch in*
***A**stronomy and*
***A**strophysics*

Orbit and spin evolution of synchronous binary stars on the main sequence (a theoretical improvement to the analytical method)

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Received 2014 August 28; accepted 2014 November 27

Abstract This paper provides a method to study the solution of equations for synchronous binary stars with large eccentricity on the main sequence. The theoretical results show that the evolution of the eccentricity is linear with time or follows an exponential form, and the semi-major axis and spin vary with time in an exponential form that are different from the results given in a previous paper. The improved method is applicable in both cases of large eccentricity and small eccentricity. In addition, the number of terms in the expansion of a series with small eccentricity is very long due to the series converging slowly. The advantage of this method is that it is applicable to cases with large eccentricity due to the series converging quickly. This paper chooses the synchronous binary star V1143 Cyg that is on the main sequence and has a large eccentricity ($e = 0.54$) as an example calculation and gives the numerical results. Lastly, the evolutionary tendency including the evolution of orbit and spin, the time for the speed up of spin, the circularization time, the orbital collapse time and the life time are given in the discussion and conclusion. The results shown in this paper are an improvement on those from the previous paper.

Key words: binaries: close — rotation — evolution

1 INTRODUCTION

Tidal friction can synchronize the orbit and spin of two components in a binary system. In general, synchronization is stronger in an old binary system due to tidal friction so that synchronization of the orbit and rotation in a binary system is the result of the interaction of mutual tidal friction. The set of equations describing non-synchronization of the orbit and rotation are given by Zahn (1989). Solving the set of non-synchronous equations given by Zahn must use numerical integration. However, if we solve the equation for the synchronous case, we may reduce the non-synchronous equation to the synchronous case, for which we may utilize the solution derived by an analytical method or numerical method. Zahn & Bouchet (1989) used these methods to study the orbital circularization of late-type binaries. Li (2012) used the analytical method to solve the evolution of orbit and spin of a synchronous binary star with small eccentricity, β Per (also called Algol). Li (2013) also used a numerical method to study the evolution of a synchronous binary star that has a large eccentricity named EK Cep. However, the analytical method used by the author is only applicable to a binary star with small eccentricity like β Per ($e = 0.015$) and it is not suitable for cases with large eccentricity. In this paper the author provides a method which is appropriate for binary stars with large eccentricity, but the eccentricity varies with time in a linear form, which is different from the result of Li (2012), hereafter called the previous paper. Therefore, this paper represents an improvement on the previous paper.

2 THE MAIN RESULTS IN THE PREVIOUS PAPER

The equations describing the secular evolution of the orbital elements (a, e) and rotational angular velocity Ω due to the tidal friction in non-synchronous binary stars are given by Zahn (1989). However, the non-synchronous equations were reduced to the following synchronous equations in the previous paper by using the coefficients of tidal friction $\lambda^{12} = \lambda^{21} = \lambda^{10} = \lambda^{32} = \lambda$, but $\lambda^{22} \neq \lambda$. (Zahn 1989)

$$\frac{1}{a}\frac{da}{dt} = -114q(1+q)\frac{\lambda}{t_f}e^2\left(\frac{R}{a}\right)^8, \tag{1}$$

$$\frac{1}{e}\frac{de}{dt} = -21q(1+q)\frac{\lambda}{t_f}\left(\frac{R}{a}\right)^8, \tag{2}$$

$$\frac{1}{\Omega}\frac{d\Omega}{dt} = 36q^2\left(\frac{M}{I}\right)\frac{\lambda}{t_f}e^2a^2\left(\frac{R}{a}\right)^8. \tag{3}$$

All symbols are the same as in the previous paper. M and R denote the mass and radius of the primary star. $q = M'/M$, M' denotes the mass of the secondary star, and $\lambda = k_2$, which represents the constant of apsidal motion. I is the moment of inertia of the primary star. a and e denote the semi-major axis and eccentricity respectively. Ω is the angular velocity of the primary star. The convective friction time t_f is given by Zahn & Bouchet (1989)

$$t_f = \left(\frac{MR^2}{L}\right)^{1/3} = t_{f_\odot}(M/M_\odot)^{1/3}(T_e/T_{e_\odot})^{-4/3}. \tag{4}$$

The previous paper that combines Equation (1) with Equation (2) yielded the first solution for the semi-major axis

$$a = a_0 \exp\left[\frac{19}{7}(e^2 - e_0^2)\right]. \tag{5}$$

Combining Equation (1) with Equation (3) provides the solution for the angular velocity of the primary star

$$\Omega = \Omega_0 \exp\left[-\frac{9}{57}\left(\frac{q}{1+q}\right)\frac{M}{l}(a^2 - a_0^2)\right]. \tag{6}$$

Substitution of the solution (5) into Equation (2) produces the solution for eccentricity described in the previous paper

$$t - t_0 = -\frac{\ln(\frac{e}{e_0}) + \frac{1}{2}c(e^2 - e_0^2)}{21q(1+q)Q}, \tag{7}$$

where

$$Q = \frac{\lambda}{t_f}\left(\frac{R}{a_0}\right)^8 \exp\left(\frac{152}{7}e_0^2\right). \qquad c = \frac{152}{7}. \tag{8}$$

In the previous paper when we studied a star with small eccentricity, such as β Per with $e = 0.015$, the second term $\frac{1}{2}c(e^2 - e_0^2)$ could be neglected due to it being small enough, and the solution was given by

$$e = e_0 \exp[-21q(1+q)Q(t - t_0)]. \tag{9}$$

3 THEORETICAL IMPROVEMENT TO THE ANALYTICAL METHOD

The results of the previous paper are not applicable to a binary system with large eccentricity because in the case of a large eccentricity, the second term $\frac{1}{2}c(e^2 - e_0^2)$ cannot be neglected. Because the solution (9) is not applicable to a star with large eccentricity, we must find an improved method for these cases, which is the aim of this paper.

It is well known that an equation for a series expansion is

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots (-1)^{n-1}\frac{x^n}{n}, \quad (-1 < x \leq 1)$$

by letting $1 + x = e^2$, $\therefore x = e^2 - 1$ (e: eccentricity).

The above formula can be transformed as

$$\ln e^2 = (e^2-1) - \frac{(e^2-1)^2}{2} + \frac{(e^2-1)^3}{3} - \cdots (-1)^{n-1}\frac{(e^2-1)^n}{n}, \quad (-1 < x < 1). \tag{10}$$

In the series expansion we can see that if we retain the term to order e^2 and neglect higher order terms e^4, e^6, e^8, $e^{10}\ldots$, the results that remain are only ne^2 and $-\sum_{n=1}^{m}\frac{1}{n}$. The expression expressed in terms of n^{th} is

$$\ln e^2 = ne^2 - \sum_{n=1}^{m}\frac{1}{n}, \quad (n = 1, 2, 3, 4, \cdots, m). \tag{11}$$

The first term of the numerator on the right side of Equation (7) can be written as

$$\ln\left(\frac{e}{e_0}\right) = \frac{1}{2}\ln\left(\frac{e^2}{e_0^2}\right) = \frac{1}{2}\Big(\ln e^2 - \ln e_0^2\Big) = \frac{1}{2}\Big(ne^2 - \sum_{n=1}^{m}\frac{1}{n} - ne_0^2 + \sum_{n=1}^{m}\frac{1}{n}\Big) = \frac{1}{2}n\Big(e^2 - e_0^2\Big). \tag{12}$$

Here n is an undetermined constant, i.e., n denotes a number such that we may retain the first term to the n^{th} term and neglect all lower order terms. This assumes a different value for different binary star systems. It is determined mainly by the eccentricity.

Substituting ($\ln\frac{e}{e_0}$) for Equation (12) into Equation (7), the solution for the eccentricity is obtained by the formula

$$e^2 - e_0^2 = -\frac{42q(1+q)Q(t-t_0)}{n+c}, \qquad c = 152/7\,, \tag{13}$$

$$\therefore e = e_0\Big[1 - \frac{42q(1+q)Q(t-t_0)}{e_0^2(n+c)}\Big]^{1/2}. \tag{14}$$

$$\delta e = e - e_0 = e_0\Big[\Big(1 - \frac{42q(1+q)Q(t-t_0)}{e_0^2(n+c)}\Big)^{1/2} - 1\Big]. \tag{15}$$

Also, substituting Equation (13) into Equation (5), we get

$$a = a_0 \exp\Big[-\frac{114q(1+q)Q(t-t_0)}{n+c}\Big]. \tag{16}$$

$$\therefore \delta a = a_0\Big[\exp\Big(-\frac{114q(1+q)Q(t-t_0)}{n+c}\Big) - 1\Big]. \tag{17}$$

$$\because a^2 - a_0^2 = a_0^2\Big[\exp\Big(-\frac{228q(1+q)Q(t-t_0)}{n+c}\Big) - 1\Big]. \tag{18}$$

Substituting Equation (18) into Equation (6), we obtain

$$\Omega = \Omega_0 \exp\Big\{-\frac{9}{57}\Big(\frac{q}{1+q}\Big)\frac{M}{I}a_0^2\Big\{\exp\Big[\frac{-228q(1+q)Q(t-t_0)}{n+c}\Big]-1\Big\}\Big\}, \tag{19}$$

$$\delta\Omega = \Omega_0\Big\{\exp\Big\{-\frac{9}{57}\Big(\frac{q}{1+q}\Big)\frac{M}{I}a_0^2\Big\{\exp\Big[\frac{-228q(1+q)Q(t-t_0)}{n+c}\Big]-1\Big\}\Big\}-1\Big\}. \tag{20}$$

4 NUMERICAL RESULTS FOR THE SYNCHRONOUS BINARY STAR V1143 CYG THAT HAS A LARGE ECCENTRICITY

We choose the eclipsing binary V1143 Cyg, which has a large eccentricity, as a case for an example calculation. Both components of the system V1143 Cyg (HD 185912) are similar main sequence stars with spectral type F. Synchronization of the system was analyzed by the following authors.

The analysis by Tan et al. (1995): The synchronization of the orbit with respect to the spin of the primary star: $(V\sin i)_{\rm syn} = 8.9$ (km s^{-1}): $(V\sin i)_M = 9$ (km s^{-1}), $F-1 = 0.011$. This is a synchronous star. The synchronization of orbit relative to the spin of the secondary star: $(V\sin i)_{\rm syn} = 8.5$ (km s^{-1}): $(V\sin i)_M = 20$ (km s^{-1}), $F-1 = 1.35$. This is a non-synchronous star. The analysis by Li (2004) showed that synchronization of the orbit relative to the spin of the primary star $Q_1 = 0.98$ (type A: synchronization). Synchronization of the orbit relative to the spin of the secondary star $Q = 0.34$ (type D: non-synchronization). According to the conclusions above, this paper studies the synchronization of the orbit relative to the spin of the primary star and does not consider the synchronization of the orbit relative to the spin of the secondary star.

The eccentricity in the orbit of V1134 Cyg is given by Hegedüs (1988), Batten et al. (1989) and Tan et al. (1995): $e_0 = 0.54$.

The physical and orbital parameters are provided by Brancewicz & Dworak (1980) and Tan et al. (1995). This paper cites data from the latter. The parameters we use are $P = 7.6408$ (d), $M = 1.29(M_\odot)$, $M' = 1.28(M_\odot)$, $R = 1.35(R_\odot)$, $R' = 1.28(R_\odot)$, $q = 0.99$, $a = 22.24(R_\odot)$, and $i = 78°$.

The luminosity L and effective temperature T_e are from data given by Popper (1980): $\log L = 0.41 (L = 2.5704L_\odot)$, $\log T_e = 3.806$ ($T_e = 6397$ K). Zahn & Bouchet (1989) gave $T_{e\odot} = 5770$ K and $t_{f\odot} = 0.433$ yr.

The period of the apsidal motion U is cited from the data published by Hegedüs (1988): $U = 10\,725$ yr.

The velocity of rotation of the primary star may be calculated from $\Omega_0 = \frac{V}{R} = 9.7978 \times 10^{-7}$ rad s^{-1}, $V = \frac{9}{\sin i}$ km s^{-1}, and $i = 78°$.

At first, it is necessary to determine the value of n (number). Substitution of $e = 0.54$ into the expanded expression (10) yields

$$\ln e^2 = -0.7084 - 0.2509 - 0.1185 - 0.0630 - 0.0357 - 0.0211 - 0.0128 - 0.0079 - \cdots. \tag{21}$$

We can see that all terms smaller than the fourth term -0.0630 may be neglected, thus, we can retain the first to the fourth terms. Hence we may take n (number) as four ($n = 4$).

By substituting the above data into the following formulae, we can obtain

$$t_f = t_{f\odot}(M/M_\odot)^{1/3}(T_e/T_{e\odot})^{-4/3} = 0.4107\ \text{yr}. \tag{22}$$

$$\lambda = k_2 = \frac{P/U}{(\frac{R}{a})^5(1+16\frac{M'}{M}) + (\frac{R'}{a})^5(1+16\frac{M}{M'})} = 0.0788 \tag{23}$$

from Cowling (1938).

Substituting $t_{f\odot}$, $\lambda = k_2$, $n = 4$, $c = 152/7$ and R_0, a_0, e_0 into Equation (8), we get

$$Q = 1.9883 \times 10^{-8}\ \mathrm{yr}^{-1}\,. \tag{24}$$

$$\frac{M}{I} = \frac{1}{KR^2} = 9.3466 \times 10^{-12}\ \mathrm{km}^{-2}, \quad (K = 0.1212). \tag{25}$$

We take $t - t_0 = 100$ yr = cy (the evolutional time) and substitute values of a_0, e_0, Ω_0 and q, n, c into Equations (15), (17) and (20), then we find the solution for the orbital and spin evolution of V1143 Cyg per century

$$\delta a = -0.000385 R_\odot\ \mathrm{cy}^{-1}\,, \tag{26}$$

$$\delta e = -0.000059 e_0\ \mathrm{cy}^{-1}\,, \tag{27}$$

$$\delta\Omega = 0.00623\Omega_0\ \mathrm{cy}^{-1}\,. \tag{28}$$

The timescale of circularization is given by Equation (2)

$$t_{\rm cir} = \frac{e}{\dot{e}} = \left(\frac{t_f}{k_2}\right)\frac{1}{21q(1+q)}\left(\frac{a}{R}\right)^8 = 6.83 \times 10^8\ \mathrm{yr}. \tag{29}$$

The time for speed up of spin is

$$t_\Omega = \frac{\Omega}{\dot{\Omega}} = \frac{t_f}{36q^2\left(\frac{M}{I}\right)ke^2a^2\left(\frac{R}{a}\right)^8} = 1.35 \times 10^6\ \mathrm{yr}. \tag{30}$$

The orbital decay (the collapse time of the system) is

$$t_a = \frac{a}{\dot{a}} = \frac{t_f}{114q(1+q)ke^2\left(\frac{R}{a}\right)^8} = 4.32 \times 10^9\ \mathrm{yr}. \tag{31}$$

The lifetime is

$$t_{\rm life} = M/\dot{M} = 2.10 \times 10^{14}\ \mathrm{yr}. \tag{32}$$

By using the expressions from Nieuwenhuijzen & de Jager (1990)

$$\log\dot{M} = -14.02 + 1.24\log(L/L_\odot) + 0.81\log(R/R_\odot) + 0.16(M/M_\odot)\,,$$

$$\dot{M} = -6.14 \times 10^{-13} M_\odot\ \mathrm{yr}^{-1}\,.$$

5 DISCUSSION AND CONCLUSIONS

(1) The improved method in this paper is not only applicable to a synchronous binary star with large eccentricity, but also to a synchronous binary star with small eccentricity.

(2) For choosing a value for the number n, the larger the eccentricity is, the smaller the value of n that can be used, which is due to the fact that the series (10) converges quickly, such as $n = 4$ for V1143 Cyg ($e = 0.54$) in this paper. On the other hand, the smaller the eccentricity is, the larger the value of n can be, which is due to the series (10) converging slowly, such as for β Per ($e = 0.015$) in the previous paper. Substituting $e = 0.015$ into the series (10), we get $\ln e^2 = -0.9997 - 0.4997 - 0.3330 - 0.2497 - 0.1997 - 0.1663 - 0.1425 - 0.1247 - 0.1108 - 0.0997 - 0.0996\cdots$. All the terms after the $10^{\rm th}$ term 0.0997 can be neglected. Thus, we may retain the first term to the $10^{\rm th}$ term and hence $n = 10$.

(3) The improved method is different from the method described in the previous paper. In terms of theoretical results, the eccentricity is linear with time in this paper. However, the eccentricity varies with time in an exponential form in the previous paper. Although the semi-major axis and the rotational angular velocity of the primary star also vary with time in an exponential form, the forms of both formulae are different, which is due to the fact that the eccentricity is linear with time. This aspect represents an improvement on the previous paper.

(4) On the other hand, if we change Equation (10) to be $e^2 - e_0^2 = \frac{2}{n}\ln(\frac{e}{e_0})$, and then substitute it into Equation (7), we get the result that the eccentricity still varies with time in an exponential form, i.e.

$$\ln\left(\frac{e}{e_0}\right) = \frac{21q(1+q)Q(t-t_0)}{(1+\frac{c}{n})}, \tag{33}$$

$$e = e_0 \exp\left[\frac{21q(1+q)Q(t-t_0)}{(1+\frac{c}{n})}\right], \tag{34}$$

$$\delta e = e_0 \left\{\exp\left[\frac{21q(1+q)Q(t-t_0)}{(1+\frac{c}{n})}\right] - 1\right\}. \tag{35}$$

We can see that although the forms of solutions (33), (34) and (35) have exponential forms like in the previous paper, both solutions are different because the solution of the previous paper is only applicable to cases with small eccentricity but the solution of this paper suits cases with higher eccentricity.

(5) The conclusion of this paper is that the evolutionary tendency of system V1143 Cyg is such that the semi-major axis and eccentricity decrease with time, especially the eccentricity which decreases to nearly zero until the circularization time. This paper also infers that in this system, at first the speed up of spin of the primary star occurs at time 1.35×10^6 yr, and then the orbital circularization occurs at time 6.83×10^8 yr, and lastly the collapse of the system occurs at 4.92×10^9 yr before the life time of this system is at an end in 2.10×10^{14} yr.

References

Batten, A. H., Fletcher, J. M., & MacCarthy, D. G. 1989, Publications of the Dominion Astrophysical Observatory Victoria, 17, 1

Brancewicz, H. K., & Dworak, T. Z. 1980, Acta Astronomica, 30, 501

Cowling, T. G. 1938, MNRAS, 98, 734

Hegeüs, T. 1988, Bulletin d'Information du Centre de Donnees Stellaires, 35, 15

Li, L.-S. 2004, Journal of Astrophysics and Astronomy, 25, 203

Li, L.-S. 2012, RAA (Research in Astronomy and Astrophysics), 12, 1673

Li, L.-S. 2013, Astronomical Research & Technology (Publ. Nat Astron. Obs. China), 10, 249 (in Chinese, Abstract in English)

Nieuwenhuijzen, H., & de Jager, C. 1990, A&A, 231, 134

Popper, D. M. 1980, ARA&A, 18, 115

Tan, H., Pan, K., & Wang, X. 1995, Acta Astrophysica Sinica, 15, 57

Zahn, J.-P. 1989, A&A, 220, 112

Zahn, J.-P., & Bouchet, L. 1989, A&A, 223, 112