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Maximum mass of magnetic white dwarfs

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Abstract We revisit the problem of the maximum masses of magnetized white dwarfs (WDs). The impact of a strong magnetic field on the structure equations is addressed. The pressures become anisotropic due to the presence of the magnetic field and split into parallel and perpendicular components. We first construct stable solutions of the Tolman-Oppenheimer-Volkoff equations for parallel pressures and find that physical solutions vanish for the perpendicular pressure when $B \gtrsim 10^{13}$ G. This fact establishes an upper bound for a magnetic field and the stability of the configurations in the (quasi) spherical approximation. Our findings also indicate that it is not possible to obtain stable magnetized WDs with super-Chandrasekhar masses because the values of the magnetic field needed for them are higher than this bound. To proceed into the anisotropic regime, we can apply results for structure equations appropriate for a cylindrical metric with anisotropic pressures that were derived in our previous work. From the solutions of the structure equations in cylindrical symmetry we have confirmed the same bound for $B \sim 10^{13}$ G, since beyond this value no physical solutions are possible. Our tentative conclusion is that massive WDs with masses well beyond the Chandrasekhar limit do not constitute stable solutions and should not exist.

Key words: magnetic fields — white dwarfs — equation of state

1 INTRODUCTION

Motivated by observations of thermonuclear supernovae that seem to require exploding white dwarf (WD) masses above the celebrated Chandrasekhar limit (Chandrasekhar 1931), a series of papers by Mukhopadhyay and collaborators (Das & Mukhopadhyay 2012, 2013) explored the magnetized version of the stellar structure and argued for a substantial increase in the maximum possible mass for large values of the magnetic field B, which quantizes the electronic energy levels. A great deal of interest has followed this suggestion and the problem has been addressed in a number of works. The main criticisms include an inconsistency with the virial theorem (Coelho et al. 2014) for large values of maximum mass and similar basic properties. The authors have responded to these criticisms, but the issue of the existence of these compact stars is still an open and important question.

From a theoretical point of view, the construction of fully consistent equilibrium solutions in the magnetized regime is still lacking, although hints for their existence and stability have been pointed out (Das & Mukhopadhyay 2013). The spatial distribution of the magnetic field seems to be an important ingredient for this issue, while the behavior of matter under extreme conditions leads to a consideration of the equation of state (EoS) in the Landau regime for the electron energy levels, which may change the effective description in terms of polytropic indexes and related quantities. Therefore, a step forward towards the solution of this problem would be an investigation into the stability of stellar models for highly magnetized matter and to identify the threshold values of B for the disappearance of stable solutions. In this paper we perform such an analysis within a definite relativistic framework and show that, at least within these simplified models, the magnetic field admitted on theoretical grounds cannot exceed 1.5×10^{13} G. Moreover, we confirm that the maximum masses do not grow beyond the Chandrasekhar value when the magnetic pressure is properly introduced via the stress-energy tensor. A brief discussion of the theoretical situation of putative high-mass WDs closes this work.

2 MODELS OF MAGNETIZED WHITE DWARFS

The scalar virial theorem (Lai & Shapiro 1991) has been generally employed to estimate the maximum magnetic field that a WD can sustain, with a mass $M = 1.4M_\odot$ and a radius $R = 0.005R_\odot$, which is around $B_{\max} \sim 10^{13}$ G. This value strongly suggests that a realistic model of a magnetized WD should feature quantized energy levels for the electrons, as has been done in many attempts to construct models that describe the microphysics of WDs as a magnetized fermion system (González Felipe et al. 2005).

In the approximation in which the magnetic field is constant and matter is allowed to settle in it, a breaking of the spherical symmetry of the star is apparent. This is not very relevant for low magnetic fields, but because we want to reach the extreme anisotropic regime, we have chosen to work in cylindrical coordinates in which the polar and equatorial radii differ and the deviation from spherical symmetry is naturally accounted for. An additional advantage of this procedure is that the construction of an anisotropic energy momentum tensor for the magnetized matter is very well-defined and straightforward.

In Manreza Paret et al. (2014), we first attempted to investigate this problem using a general cylindrically symmetric metric, with coordinates (t, r, ϕ, z). We followed the procedures of Trendafilova & Fulling (2011)) to solve Einstein equations for an axisymmetric model of a WD to take into account the anisotropy induced by the magnetic field. A constant magnetic field in (say) the z-direction defines two main directions in space, parallel and perpendicular to the magnetic field. The main approximation applied in that paper is to assume that all of the functions and variables only depend on the radial coordinates (r) and not on z and ϕ, so that we can solve for the dependence on the equatorial direction of the WD. However, this simple model could be useful for obtaining information about the effects of the magnetic field in terms of the shape (oblateness) of the WD and yield upper limits for the values of the magnetic field that this object can sustain.

The present paper builds on the results from Manreza Paret et al. (2014), and applies the same procedure to study the structure equation of a magnetized WD with the aim to confirm or refute the recent claims of super-Chandrasekhar masses for a magnetized WD (Das & Mukhopadhyay 2012).

3 MAGNETIZED WHITE DWARFS

The thermodynamical properties of matter in a magnetic field are obtained starting from the thermodynamical potential at zero temperature

$$\Omega_\mathrm{e} = -\frac{eB}{4\pi^2}\left[\sum_{l=0}^{l_{\max}} \alpha_l \left(\mu_\mathrm{e}\, p_F - \varepsilon_\mathrm{e}^2 \ln \frac{\mu_\mathrm{e} + p_F}{\varepsilon_\mathrm{e}}\right)\right], \qquad (1)$$

where $\mu_{\rm e}$ is the electron chemical potential, $l_{\rm max} = I[\frac{\mu_{\rm e}^2 - m_{\rm e}^2}{2eB}]$, $I[z]$ denotes the integer part of z, $\alpha_l = 2 - \delta_{l0}$ is the spin degeneracy of the l-Landau level, the Fermi momentum is $p_{\rm F} = \sqrt{{\mu_{\rm e}}^2 - \varepsilon_{\rm e}^2}$ and the rest energy is given by

$$\varepsilon_{\rm e} = \sqrt{m_{\rm e}^2 + 2|eB|l}\,. \tag{2}$$

The particle number density and magnetization are

$$N_{\rm e} \;=\; -(\partial\Omega_{\rm e}/\partial\mu_{\rm e}) = \frac{m_{\rm e}^2}{2\pi^2}\frac{B}{B_{\rm e}^c}\sum_{l=0}^{l_{\rm max}} \alpha_l p_{\rm F}\,, \tag{3}$$

$$\mathcal{M}_{\rm e} \;=\; -(\partial\Omega_{\rm e}/\partial B) = \frac{e}{4\pi^2}\left(\sum_{l=0}^{l_{\rm max}} \alpha_l \left[\mu_{\rm e} p_{\rm F} - \left[m_{\rm e}^2 + 4|eB|l\right]\ln\frac{\mu_{\rm e} + p_{\rm F}}{\varepsilon_{\rm e}}\right]\right)\,, \tag{4}$$

where $B_{\rm e}^{\rm c} = m_{\rm e}^2/|e| = 4 \times 10^{13}$ G is the critical magnetic field.

The energy density and the pressures parallel and perpendicular to the magnetic field can be written as

$$\epsilon \;=\; \Omega_{\rm e} + \mu_{\rm e} N_{\rm e} + N m_N \frac{A}{Z}\,, \tag{5a}$$

$$\mathcal{P}_{\|} \;=\; -\Omega_{\rm e}\,, \tag{5b}$$

$$\mathcal{P}_{\perp} \;=\; -\Omega_{\rm e} - B\mathcal{M}_{\rm e}\,, \tag{5c}$$

where $N m_N \frac{A}{Z}$ is the mass density term, N is the number of nucleons, m_N is the mass of nucleons, Z is the atomic number and A is baryon number. We assume that the white dwarfs are predominantly composed of ^{12}C and ^{16}O with $A/Z = 2$.

Components of the EoS including matter and field ($P_{\perp}^B = E^B = -P_{\|}^B = \frac{B^2}{8\pi}$) contributions have the following form

$$E \;=\; \varepsilon + \frac{B^2}{8\pi}\,, \tag{6}$$

$$P_{\|} \;=\; \mathcal{P}_{\|} - \frac{B^2}{8\pi}\,, \tag{7}$$

$$P_{\perp} \;=\; \mathcal{P}_{\perp} + \frac{B^2}{8\pi}\,. \tag{8}$$

In Figure 1 we show the EoS of the magnetized gas as derived from the above expressions. It can be noticed that the pressures do not differ much for low values of the magnetic field ($B \ll B_{\rm e}^c$), which is also true for the non-magnetic case $B = 0$. However, when $B \sim B_{\rm e}^c$, the difference in the pressures becomes quite large.

To quantify the anisotropy, we have defined the splitting coefficient as

$$\Delta = \frac{|P_{\perp} - P_{\|}|}{P(B \to 0)}\,. \tag{9}$$

We will use $\Delta \simeq \mathcal{O}(1)$ as a criterion to define the border separating the isotropic and the anisotropic regions, so that by applying the equation $\Delta(\mu_{\rm e}, B) = 1$, one can distinguish an anisotropic region from an isotropic one.

In Figure 2 we show the densities as a function of the magnetic field. The region above the dashed curve falls in the isotropic regime ($\Delta < 1$) and the region below the curve is in the anisotropic regime ($\Delta > 1$). The vertical lines are the solutions of the densities for a constant value of the magnetic field. We can see that for $B \lesssim 10^{12}$ G, all of the points lie in the isotropic region while for $B \gtrsim 10^{13}$ G there is a considerable number of points in the anisotropic region.

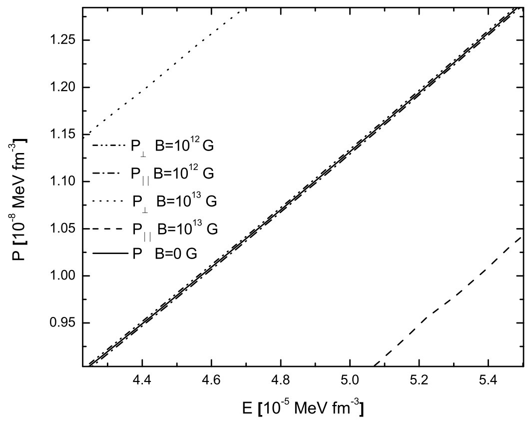

Fig. 1 EoS for magnetized electron gas. Notice the differences in the pressures when the magnetic field increases. The cases of $B = 0$ G and $B = 10^{12}$ G are almost indistinguishable from each other.

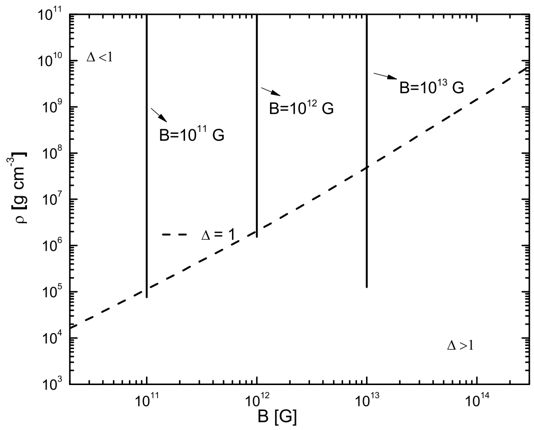

Fig. 2 Splitting of the pressures with respect to the magnetic field. The dashed line represents the solution of the equation $\Delta(\mu_e, B) = 1$ and the vertical lines are solutions of the densities for a constant magnetic field value.

In our numerical computations we will first use magnetic field values that are well within the isotropic region $B = 10^{12}$ G and also in the anisotropic region $B = 10^{13}$ G to compare the effects on the star structure. The next section will show the impact of this density-dependent field anisotropy on stellar structure.

4 TOV EQUATIONS FOR MAGNETIZED WHITE DWARFS

In order to set up the problem introduced by the magnetized matter EoS in the study of the structure of WDs, we will analyze the usual case first, assuming spherical symmetry and solving the resulting Tolman-Oppenheimer-Volkoff (TOV) equations (Misner et al. 1973). To find the static structure of a relativistic spherical star, we have to solve the well-known TOV equations.

$$\frac{dM}{dr} = 4\pi G\,E, \tag{10}$$

$$\frac{dP}{dr} = -G\frac{(E+P)(M+4\pi P r^3)}{r^2 - 2rM}, \tag{11}$$

with the boundary conditions $P(R) = 0$, $M(0) = 0$ and the EoS $E \to f(P)$.

The Mass-Radius curves obtained for magnetic fields $B = 10^{11}, 10^{12}$ G (that is, within the regime in which $\Delta < 1$) are in agreement with those obtained, for example, in Suh & Mathews (2000). Therefore, we confirm that quantization of the electronic levels for fields in this regime cannot increase the maximum mass of a WD sequence. However, if the terms $\propto B^2/8\pi$, representing pressure from the magnetic field, are omitted, higher values can be achieved. We believe that this unjustified omission is a significant part of the discussion on super-Chandrasekhar masses.

In Figure 3 we show the Mass-Radius diagram for different values of the magnetic field.

When we want to explore the anisotropic region, however, we find that anisotropy sets in for progressively larger regions of the WD when the value of the magnetic field is increased. Around $B = 10^{13}$ G, most of the star feels the anisotropy of the pressures and only the inner regions remain practically isotropic. This justifies the use of anisotropic solutions for cases with the highest fields.

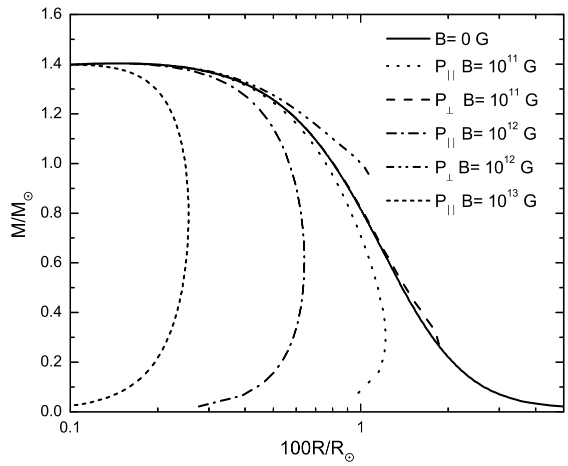

Fig. 3 Mass-Radius relation for the spherically symmetric case. Notice the differences that arise from using parallel or perpendicular pressures, and that we have not obtained masses beyond the Chandrasekhar limit.

5 ANISOTROPIC STRUCTURE EQUATIONS

To improve the structure equations in the presence of anisotropic pressures, in this section we consider an axisymmetric geometry which is more adequate to treat a magnetized fermion system. We follow the same procedure as used in Manreza Paret et al. (2014). The cylindrically symmetric metric reads

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dr^2 + r^2 d\phi^2 + e^{2\Psi}dz^2\,, \tag{12}$$

where Φ, Λ and Ψ are only functions of r which, as mentioned before, is the main approximation (Manreza Paret et al. 2014).

The energy momentum tensor for magnetized matter is given by (González Felipe et al. 2005)

$$\mathcal{T}^{\mu}{}_{\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & P_\perp & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\| \end{pmatrix}\,, \tag{13}$$

where E, $P_\|$ and $P_\perp$ are components of the EoS defined by (6), (7) and (8) respectively.

From the Einstein field equations in natural units and using conservation of energy and momentum ($\mathcal{T}^{\mu}{}_{\nu;\mu}$), we obtain the following four differential equations:

$$P'_\perp \;=\; -\Phi'(E+P_\perp) - \Psi'(P_\perp - P_\|), \tag{14a}$$

$$4\pi e^{2\Lambda}(E + P_\| + 2P_\perp) \;=\; \Phi'' + \Phi'(\Psi' + \Phi' - \Lambda') + \frac{\Phi'}{r}, \tag{14b}$$

$$4\pi e^{2\Lambda}(E + P_\| - 2P_\perp) \;=\; -\Psi'' - \Psi'(\Psi' + \Phi' - \Lambda') - \frac{\Psi'}{r}, \tag{14c}$$

$$4\pi e^{2\Lambda}(P_\| - E) \;=\; \frac{1}{r}(\Psi' + \Phi' - \Lambda')\,. \tag{14d}$$

This, together with the EoS having the properties $E \to f(P_\perp),\ P_\| \to f(E)$, is a system of differential equations in the variables

$$P_\perp,\ \ P_\|,\ \ E,\ \ \Phi,\ \ \Lambda,\ \ \Psi\,. \tag{15}$$

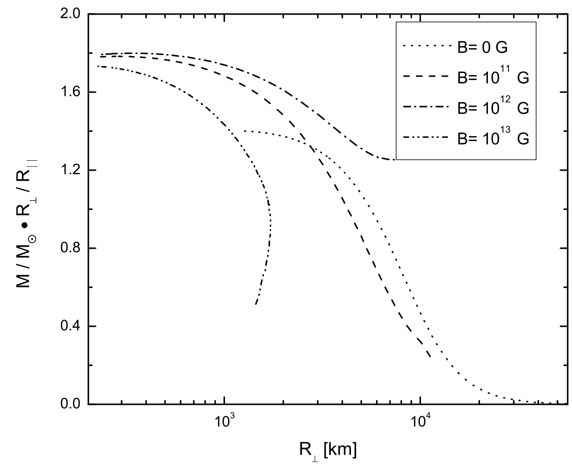

Fig. 4 Mass-Radius relation $(M\,R_\perp/R_\parallel)$ in solar masses for the cylindrically symmetric case. We have plotted curves for magnetic field values: $B = 0$ G (isotropic case $R_\perp = R_\parallel$), $B = 10^{11}$ G, 10^{12} G and $B = 10^{13}$ G. This last value of the magnetic field is the maximum value at which we have found stable configurations. Notice that in the $B \neq 0$ G cases the maximum value of the magnitude $(M/M_\odot\,R_\perp/R_\parallel)$ is always greater than in the isotropic case ($B = 0$ G), but this does not mean that the masses are greater than the Chandrasekhar limit because the magnitude of the parallel radius is undetermined in our model.

We consider $P_\perp(R_\perp) = 0$ which determines the radii of the star, in the equatorial (perpendicular) direction. Solutions for the system of Equations (14) are shown in Figure 4.

In Figure 4, we have plotted the magnitude $(M\,R_\perp/R_\parallel)$ in solar masses as a function of the perpendicular radius $R_\perp$. At first glance, Figure 4 shows maximum values for the quantity $MR_\perp/R_\parallel$. However, these values cannot be associated with maximum values of the WD masses because our model has an underdetermination of the parallel radius and the total mass of the star cannot be calculated. Our model allows us to determine a maximum field ($B \simeq 10^{13}$ G) beyond which the metric coefficients exhibit a divergent behavior. This value of the magnetic field coincides with the value at which the splitting of the pressures given by the parameter Δ is greater than 1, for $B = 10^{13}$ G. This result supports our interpretation that beyond this value of the magnetic field there are no stable solutions of the system, and points towards the end of the theoretical stellar sequences constructed from our assumptions.

6 CONCLUSIONS

We have revisited the role of anisotropic pressures in the description of the structure of a WD. Our findings show that when the splitting coefficient $\Delta > 1$, the differences in the pressures cannot be neglected and a different approach must be used to study the structure of the star. An axisymmetric geometry is more suitable than a spherical one for the solution of Einstein equations using a cylindrically symmetric metric. Our choice of the metric in the latter conditions is probably the simplest among all of the possible cylindrical metrics.

Taking into account the pressure anisotropy due to a magnetic field yields a critical field $B_\mathrm{c} \sim 10^{13}$ G for a magnetized WD, beyond which there are no stable equilibrium configurations. This bound for the value of the magnetic field is close to (but slightly lower than) what is obtained based on the scalar virial theorem.

Although in our model we cannot compute the total mass due to the assumption that all of the variables only depend on the perpendicular (equatorial) radius and not on the z-direction (Manreza Paret et al. 2014), this study is useful for confirming the existence of a maximum magnetic field for which the star may undergo an anisotropic collapse due to a magnetic instability. This point helps to clarify the claim of super-Chandrasekhar masses for a magnetized WD (Das & Mukhopadhyay 2012, 2013) and rules out the magnetic field being the reason for the existence of this kind of object. By the way, the recent paper Das & Mukhopadhyay (2014) makes use of extremely high magnetic fields, well above the Schwinger value, and clearly beyond the virial estimate; and also imposes a $\Gamma = 4/3$ polytrope as a model for the matter. This is at odds with previous claims (Das & Mukhopadhyay 2013) by the same authors that a $\Gamma = 2$ results from Landau quantization and hence it is clear that the latter does not stiffen the EoS needed to achieve the super-Chandrasekhar mass values.

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