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Sensitivity study of high eccentricity orbits for Mars gravity recovery *

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Abstract By linear perturbation theory, a sensitivity study is presented to calculate the contribution of the Mars gravity field to the orbital perturbations in velocity for spacecrafts in both low eccentricity Mars orbits and high eccentricity orbits (HEOs). In order to improve the solution of some low degree/order gravity coefficients, a method of choosing an appropriate semimajor axis is often used to calculate an expected orbital resonance, which will significantly amplify the magnitude of the position and velocity perturbations produced by certain gravity coefficients. We can then assess to what degree/order gravity coefficients can be recovered from the tracking data of the spacecraft. However, this existing method can only be applied to a low eccentricity orbit, and is not valid for an HEO. A new approach to choosing an appropriate semimajor axis is proposed here to analyze an orbital resonance. This approach can be applied to both low eccentricity orbits and HEOs. This small adjustment in the semimajor axis can improve the precision of gravity field coefficients and does not affect other scientific objectives.

Key words: planets and satellites — Mars methods — analytical variables: gravity, resonance

1 INTRODUCTION

Mars is the second closest planet to the Earth in the solar system and it has many similarities to Earth; therefore, there have been many missions to explore Mars since the 1960s. The gravity field of Mars plays an important role in ensuring successful fulfilment of Mars missions and achieving the desired orbit configuration for specific scientific objectives. Mars gravity field models are mainly derived from the tracking data of available spacecrafts orbiting Mars, and orbital geometry is a key issue in the recovery of the Mars gravity field.

The evolution of a spacecraft moving around Mars is dominated by perturbing forces due to the non-spherical part of gravity from Mars, whose effects can be classified as secular or periodic perturbations. The former corresponds to the precession of the orbit in which the argument of periapsis ω, the right ascension of ascending node Ω and the mean anomaly M undergo a secular change proportional to all even zonal harmonics, while the semimajor axis a, the eccentricity e and

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the inclination i do not undergo a secular change. On the other hand, all gravitational coefficients contribute, at different levels, to periodic variations, which can be calculated by linear analytical methods (Rosenblatt & Dehant 2010). Using the relations between Keplerian orbital elements and Cartesian coordinates in radial, transverse and normal components, the contribution of each gravity coefficient to the orbital perturbation in position and velocity can be obtained and then compared with the precision of the tracking system. A theoretical upper limit for the sensitivity of an orbit to the gravity field can finally be determined (Lemoine 1992).

In practice, the formal error or/and the signal-to-noise ratio (SNR) are usually used to judge whether a certain coefficient of gravity can be reliably estimated or not. On the other hand, the sensitivity of a specific orbit to the gravity field of Mars can also be evaluated and quantified in terms of the magnitude of perturbations in both orbital positions and velocities. This information, associated with the coverage and accuracy of the tracking data, provides insight into the strength or weakness of each spacecraft when treated as a sensor for the Martian gravity field. Indeed, this method makes use of analytical perturbations on the orbit caused by a particular gravity coefficient, which is identical to the SNR method, since the perturbation is the 'signal' while the precision of tracking data is the 'noise.'

The perturbations of a spacecraft in position and velocity are not very sensitive to the high degree/order coefficients, therefore, these coefficients are usually not well determined (i.e. have a low accuracy) from the tracking data of spacecrafts. One possible way to improve this situation is to put the spacecraft on a resonant orbit with the rotation of Mars, by choosing an appropriate semimajor axis. This resonance will increase the perturbations on the orbital velocity caused by these degree/order coefficients, thereby providing a much better determination with respect to the case without resonance. This small adjustment in the semimajor axis can improve the gravity field solution and does not significantly change the original orbit. For example, Klokočník et al. (2003), using this resonance effect on CHAMP's recovery of 46 order coefficients, indicate that this method can provide useful checks or improvements. This resonance analysis can also be applied to spacecrafts orbiting Mars. An analysis of the Mars Global Surveyor (MGS) mission shows that a small adjustment in the semimajor axis can induce various higher-order resonances and help in the recovery of the Mars gravity field (Klokocnik et al. 2010). However, the existing method is only valid for low eccentricity orbits, and problems still persist for high eccentricity orbits (HEOs). The new approach proposed in this paper to analyze the orbital resonance can be applied to both low eccentricity orbits and HEOs.

Linear perturbation theory is often applied for sensitivity studies using orbital motion. The basics of this theory are briefly described in Section 2. This sensitivity study is applied to the MGS in Section 3 to evaluate which coefficients can be better determined from the associated orbit tracking data. Considering the fact that the existing method can only find the exact resonance of low eccentricity orbits, the new approach, valid for both low eccentricity orbits and HEOs, is proposed in Section 4 as the main content of this paper, to analyze the exact orbital resonances. The exact resonances for near circular orbits (such as MGS) and assumed HEOs are searched for and also plotted in Section 4. As a validation of this new approach for HEO resonance, in Section 5 we use the sensitivity study to calculate the velocity perturbations on the orbit induced by gravity coefficients for the two cases 8:3 and 29:11. The results confirm that our method of searching the semimajor axis for exact resonances is valid for an HEO. Section 6 gives a summary and discussion. We hope that this information will be useful for the orbit design of future missions to Mars.

2 THEORY OF ORBITAL PERTURBATION

The external gravitational potential can be represented by an expansion of spherical harmonics as,

$$U(r,\phi,\lambda) = \frac{GM}{r} + \frac{GM}{r}\sum_{l=1}^{\infty}\sum_{m=0}^{l}\left(\frac{a_e}{r}\right)^l \bar{P}_{lm}(\sin\phi)[\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda]\,, \tag{1}$$

where $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are fully normalized harmonic coefficients of degree l and order m, and $\bar{P}_{lm}$ is the fully normalized associated Legendre function. a_e is the semimajor axis of the reference ellipsoid of Mars (3396.2 km). GM is the product of the universal constant of gravitation and the mass of Mars. r, ϕ and λ are respectively the distance, latitude and longitude of the spacecraft in the coordinate frame centered on Mars.

The summation term in Equation (1) is the non-spherical perturbing potential R, which can be expressed in terms of the six Keplerian orbital elements by means of a change of variables (Kaula 1966)

$$R = \frac{GM}{a}\sum_{l=2}^{K}\left(\frac{a_e}{a}\right)^l \sum_{m=0}^{l}\sum_{p=0}^{l}\bar{F}_{lmp}(i)\sum_{q=-Q}^{Q}G_{lpq}(e)S_{lmpq}(\omega, M, \Omega, \theta)\,, \tag{2}$$

where $\bar{F}_{lmp}(i)$ is a function of the orbital inclination i and $G_{lpq}(e)$ is the eccentricity function. p and q are summation indexes. Q is a number that depends on the eccentricity, which will be discussed later, and S_{lmpq} is a function defined by harmonic coefficients, ω, M, Ω and sidereal time θ.

The orbit and its variations are usually described by Lagrange planetary equations, whose solutions are given by linear perturbation theory. The solution requires a reference orbit in which the mean semimajor axis $\bar{a}$, the eccentricity $\bar{e}$ and the inclination $\bar{i}$ are constant, and the argument of periapsis ω, the right ascension of ascending node Ω and the mean anomaly M undergo a linear variation with time. Their linear change at rates $\dot{\omega}$, $\dot{\Omega}$ and $\dot{M}$ can be represented as (Kaula 1966)

$$\begin{aligned}
\dot{\omega} &= \frac{d\omega}{dt} = \frac{3nC_{20}a_e^2}{4(1-\bar{e}^2)^2\bar{a}^2}(1 - 5\cos^2\bar{i})\,, \\
\dot{\Omega} &= \frac{d\Omega}{dt} = \frac{3nC_{20}a_e^2}{2(1-\bar{e}^2)^2\bar{a}^2}\cos\bar{i}\,, \\
\dot{M} &= \frac{dM}{dt} = n - \frac{3nC_{20}a_e^2}{4(1-\bar{e}^2)^2\bar{a}^2}(3\cos^2\bar{i} - 1)\,,
\end{aligned} \tag{3}$$

where n is the mean motion, $n = \sqrt{\frac{GM}{\bar{a}^3}}$. At epoch t, the reference orbit is described by the mean elements $\bar{a}$, $\bar{e}$, $\bar{i}$, ω_0, Ω_0 and M_0 at t_0, with linear change of rates $\dot{\omega}$, $\dot{\Omega}$ and $\dot{M}$. Given the reference orbit, and integrating both sides of the Lagrange equations, the periodic perturbations can be expressed as (Kaula 1966)

$$\Delta\alpha = \sum_{l=2}^{K}\sum_{m=0}^{l}\sum_{p=0}^{l}\sum_{q=-Q}^{Q}\Delta\alpha_{lmpq}\,, \tag{4}$$

where α stands for any of the six Keplerian elements. More details about $\Delta\alpha_{lmpq}$ can be found in Kaula (1966). For a spacecraft in a low eccentricity orbit, Q is usually taken as 1 or 2, but for one in an HEO, this value has to be increased to make the linear analytical perturbations converge to a reasonable accuracy.

Linear perturbation theory is only valid within the so-called Laplace limit, i.e. $e \approx 0.662$. This condition is valid for most of the present spacecrafts orbiting Mars, so in these cases this approach is still valuable for sensitivity studies of the Mars gravity field. When e is close to the Laplace limit, Equation (4) converges slowly and one needs to increase the value of Q, although this method becomes inefficient. If a target orbit's eccentricity exceeds the Laplace limit, an alternative numerical method described in Lemoine (1992) and Rosborough & Lemoine (1991) can be used.

After calculating perturbations $\Delta\alpha_{lmpq}$, the next step is to transform the Keplerian perturbations into radial, transverse and normal components of orbital position and velocity perturbations. The magnitude of the position and velocity perturbations can then be directly compared with the precision of the tracking data (range or range-rate measurements) to ascertain which coefficients of the gravity field may be determined with sufficient accuracy. In general, two trajectories are then produced: the reference trajectory (a secularly precessing ellipse as already explained above) and the perturbed trajectory, with the latter being defined by adding the linear perturbations induced by each coefficient of the gravity field (i.e. C_{lm} or S_{lm} for a given pair of (l, m)) to the reference orbit.

These two trajectories are then converted to and subtracted in Cartesian space and the resulting differences are rotated into the radial, transverse and normal directions. Thus, a time series of the differences in position and velocity is directly obtained, from which the root mean square (RMS) of the differences can be computed for each component. Then, by comparing the RMS of the differences with the capabilities/precision of the tracking system, we can assess to what degree/order the gravity field can be recovered from the tracking data of the spacecraft (Rosborough & Lemoine 1991).

It is necessary to mention that contributions to the orbital perturbations by some high degree/order gravity terms are usually very small if there is no resonance, so that they cannot be determined very well from tracking data. Thus the main goal of this work is to search for one (or more) orbital resonances by choosing a reasonable semimajor axis, so that the orbital perturbations by certain high degree/order gravity coefficients are amplified with respect to the precision of the tracking data and thereby become determinable. This will be done by using the linear analytical method described in this section rather than a numerical integration method.

3 CASE STUDIES: A SPACECRAFT IN A NEARLY CIRCULAR ORBIT

The MGS spacecraft is in a nearly circular orbit. For gravity recovery, the best quality tracking data of MGS were acquired in the three weeks of the Gravity Calibration Orbit (GCO) phase and one month of the mapping transition phase using a stewed high gain antenna. In these phases, the MGS periapsis altitude reached 380 km. The MGS tracking data were collected by NASA's Deep Space Network (DSN), and the X band data were used for gravity recovery. The precision of the two-way and three-way Doppler data is better than 0.05 mm s^{-1} for an integration time of 10 s, while the precision of the one-way Doppler data is better than 1 mm s^{-1} for an integration time of 10 s (Yuan et al. 2001).

In order to analyze the velocity perturbation induced on MGS by the Mars gravity field, we used an analytical method to simulate the velocity perturbation during the GCO phase. In this simulation, we used the GMM-2B spherical harmonic expansion of the static field (Lemoine et al. 2001) as input, which is complete up to a degree and order of 80. The MGS ephemeris is given by the software SPICE (*www.naif.jpl.nasa.gov/naif/aboutspice.html*).

Figure 1 shows the time evolution of some orbital elements of MGS during the GCO phase. Regarding the initial parameters of the MGS reference orbit, $\bar{a}$, $\bar{e}$ and $\bar{i}$ can be obtained by averaging a, e and i, while ω_0, Ω_0 and M_0 can be obtained by least-squares fitting. The values obtained in this way are the initial parameters $\bar{a} = 3795.95$ km, $\bar{e} = 0.0063$, $\bar{i} = 92.9°$, $\omega_0 = 225.92°$, $\Omega_0 = 351.05°$ and $M_0 = 194.68°$, while $\dot{\omega}$, $\dot{\Omega}$ and $\dot{M}$ can be obtained from Equation (3).

Figure 2 summarizes the velocity perturbations of MGS induced by different coefficients; only the contributions of these gravity coefficients giving perturbations larger than 0.1 mm s^{-1} (at the level of DSN observation capabilities) are plotted. Figure 2 clearly shows that the largest perturbations are mainly due to zonal gravity coefficients, but other peaks, like those at orders 13, 25, 38 and 50, are induced by resonance effects (see Sect. 4 for details). Comparing the values of Figure 2 with the observation precision of DNS (0.05 mm s^{-1} for two-/three-way Doppler measurements over 10 s), one can deduce that MGS tracking data can only provide a full solution of the Mars gravity field up to degree and order 50–60. Although some higher degree and order terms can also induce significant perturbations by resonance effects (Marty et al. 2009), *a priori* constraints on these coefficients above a certain higher degree is still needed to solve these coefficients. Usually, these constraints are defined by a power rule derived from Kaula's theory (Lemoine et al. 2001) that

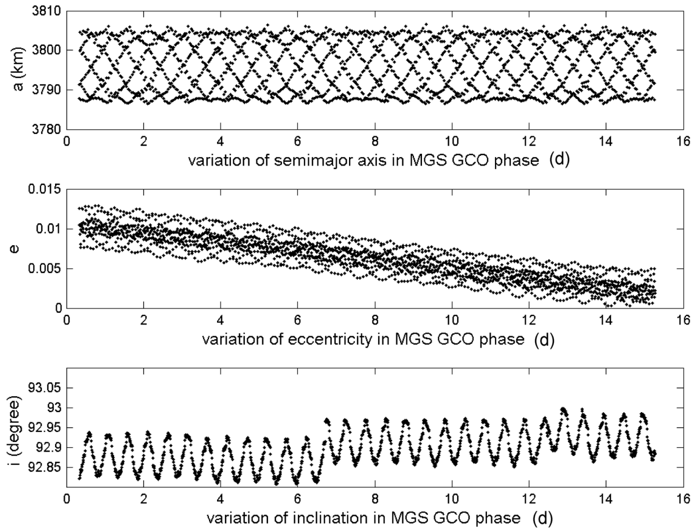

Fig. 1 Diagram of variation of orbital elements in MGS GCO phase. The GCO phase started on 1999 February 4 and lasted three weeks.

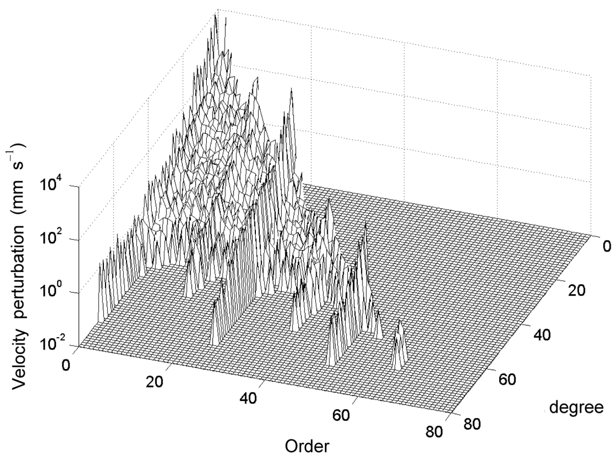

Fig. 2 The velocity perturbations on the orbit of MGS (GCO phase) by Mars gravity coefficient (C_{lm}, S_{lm}) pairs. Only perturbations larger than 0.1 mm s^{-1} are plotted.

is represented by the power of Mars gravity coefficients in the spectral domain (Yuan et al. 2001). For the above reason, the GMM-2B model is only well determined up to degree 60 (Lemoine et al. 2001).

4 RESONANCE EFFECTS

Resonance is a common effect experienced by almost all spacecraft orbiting around a rotating planet. This phenomenon occurs when the motion of the spacecraft and the rotation of the planet satisfies certain relationships, and it plays an important role in studying the sensitivity of the orbits to the associated gravity field. Indeed, the analysis of the velocity perturbation shown in Section 2 confirms the importance of the resonance effect on the orbit in determining the Mars gravity field.

In this and the next section, we will discuss under which conditions the resonance will occur for both MGS (being representative of a circular polar orbit) and HEO.

As shown by Reigber (1989), large perturbations will occur if the argument $\dot{\psi}_{lmpq}$ is very small or becomes singular, i.e. resonance exists if

$$\dot{\psi}_{lmpq} = (l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega} - \dot{\theta}) \approx 0\,, \tag{5}$$

where $\dot{\theta}$ is the angular rate of rotation for Mars. To simplify the analysis, we are mainly interested in the case $q = 0$. In this situation, Equation (5) can be written as

$$\alpha(\dot{\omega} + \dot{M}) \approx \beta(\dot{\theta} - \dot{\Omega})\,, \tag{6}$$

where α and β are some pairs of mutually prime integers. The exact resonance occurs when $\frac{\beta}{\alpha}$ is commensurable, i.e. the spacecraft completes β nodal periods when Mars rotates α times relative to the spacecraft's precessing orbital plane. α and β can be correspondingly represented as $\alpha\gamma = (l-2p); \beta\gamma = m; \gamma = 1, 2, 3, \cdots$.

If exact resonance occurs, $\dot{\psi}$ in Equation (5) is equal to zero (Klokočník et al. 2003). Writing $\dot{M}$ as $\dot{M} = \dot{\sigma} + n$, we obtain from Equation (3)

$$\dot{\sigma} = -\frac{3nC_{20}a_e^2}{4(1-\bar{e}^2)^{\frac{3}{2}}\bar{a}^2}(3\cos^2\bar{i} - 1). \tag{7}$$

Putting $\dot{\sigma}$ into Equation (5), and using α and β in place of $l - 2p$ and m respectively, then

$$\dot{\psi}_{\frac{\beta}{\alpha}} = \alpha(\dot{\omega} + n + \dot{\sigma}) + \beta(\dot{\Omega} - \dot{\theta}) \approx \alpha n - \beta\dot{\theta}. \tag{8}$$

The condition $\dot{\psi}_{\frac{\beta}{\alpha}} = 0$, required for exact resonance, implies

$$n = \frac{\beta}{\alpha}(\dot{\theta} - \dot{\Omega}) - \dot{\omega} - \dot{\sigma}. \tag{9}$$

For a spacecraft in a nearly circular orbit such as MGS, using Equations (3) and (7) in (9), neglecting $\bar{e}^2$ terms, and replacing n by $\frac{\beta}{\alpha}\dot{\theta}$ in the correction term, we obtain

$$n = \frac{\beta}{\alpha}\dot{\theta}\{1 - \frac{3}{2}J_2(\frac{R}{\bar{a}})^2[4\cos^2\bar{i} - \frac{\beta}{\alpha}\cos\bar{i} - 1]\}, \tag{10}$$

where $J_2 = -\sqrt{5}C_{20}$.

There are two ways to compute the mean semimajor axis $\bar{a}$ from Equation (10). The first one takes $\frac{\beta}{\alpha}\dot{\theta}$ as an initial estimate for n and by subsequent iterations on Equation (10), the mean element $\bar{a}$ is obtained. However, this method can only be applied to a low eccentricity orbit and is not valid for an HEO.

A new approach is proposed here. Starting with $n^2\bar{a}^3 = GM$ and letting periapsis altitude a_p change in a certain range, one can find the range of n. Then from the equation $\alpha n - \beta\dot{\theta} = 0$, one can obtain the range of $\frac{\beta}{\alpha}$.

Taking a spacecraft in a nearly circular orbit as an example, in this case $\bar{a}$ can be written as $\bar{a} \approx r + a_p$. If the periapsis altitude a_p changes from 200 to 400 km, the range of $\frac{\beta}{\alpha}$ becomes $12.4826 \sim 13.5382$. Letting $\alpha = 1, 2, 3, \cdots$, one can obtain the corresponding values of β with the conditions that β should be an integer and that $\frac{\beta}{\alpha}$ should be commensurable. The value of $\frac{\beta}{\alpha}$ is then put into Equation (9) to get the value of n. Finally, the value of $\bar{a}$ can be obtained from relation $\bar{a} = (\frac{GM}{n^2})^{\frac{1}{3}}$.

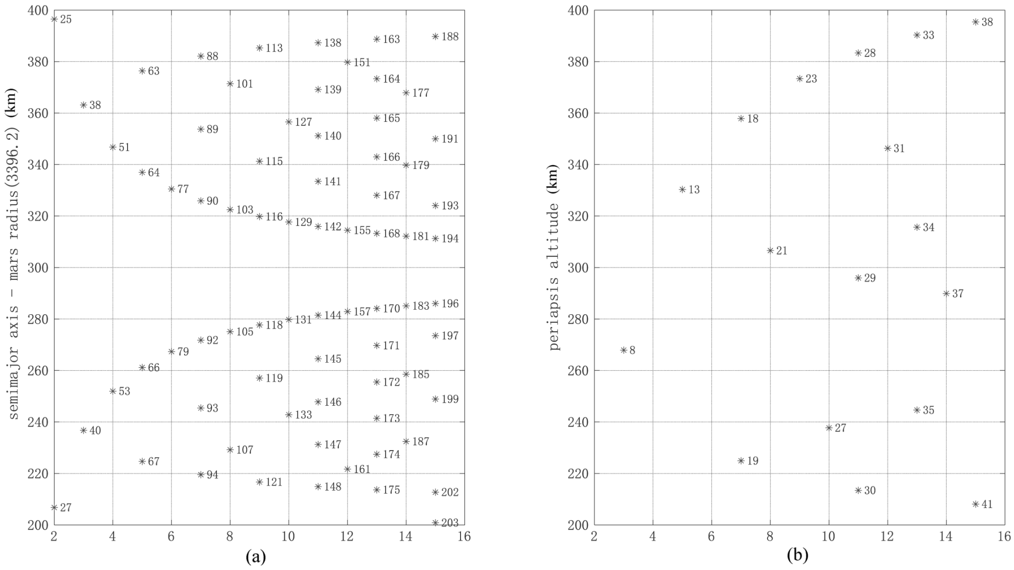

Fig. 3 The resonance diagram for a spacecraft orbiting Mars with periapsis altitude between 200 and 400 km. The X axis indicates the α and the Y axis indicates the value of the periapsis altitude when the exact resonance occurs. The dots in the figure mean the location of the β/α resonance and the numbers after the dots denote β. (a) A nearly circular orbit with inclination 92.9° like MGS, (b) An HEO with inclination 98° and eccentricity 0.65507.

In Equation (9), $\dot{\omega}$, $\dot{\sigma}$ and $\dot{\Omega}$ also affect the value of n, but these effects are much smaller than that of $\dot{\theta}$. From our simulation, the effects of $\dot{\omega}$, $\dot{\sigma}$ and $\dot{\Omega}$ on $\bar{a}$ are smaller than 1 km. Therefore, in the later computing process, we use the mean value of $\dot{\omega}$, $\dot{\sigma}$ and $\dot{\Omega}$ to simplify the procedure. For most spacecrafts in orbit, since the exact resonance condition is not satisfied, the result given by $\bar{a}$ has a reasonable accuracy.

From this approach, some possible resonances of a spacecraft moving around Mars in a nearly circular and a nearly polar orbit (such as MGS) are found. In this calculation, the inclination is fixed at $\bar{i} = 92.9°$, and the periapsis altitude a_p changes from 200 to 400 km. The results of a_p when the exact resonances occur are presented in Figure 3(a). X and Y axes indicate the α and the value of a_p respectively. Dots in the figure indicate the locations of $\frac{\beta}{\alpha}$ resonances and the numbers after the dots denote values of β. In the MGS GCO phase, the periapsis altitude a_p is about 399.75 km, and the corresponding resonance is 25:2. In the whole mission of MGS, its orbits, with large variation in altitude due to maneuvers, have passed through various higher order resonances (for example, 188:15), which was also discussed in Klokocnik et al. (2010). Figure 3(b) shows the result of a similar analysis for an HEO, which will be described in the next section.

5 CASE STUDIES: THE HEOS

The main tasks of a spacecraft in an HEO include exploring the space environment and surveying the geology and other surface features of Mars. It can also provide some valuable tracking data to improve the determination of the Mars gravity field (Wu et al. 2009), especially when a specific orbit can be designed with a reasonable $\bar{a}$ to have a resonance. This is one of the main purposes of this paper.

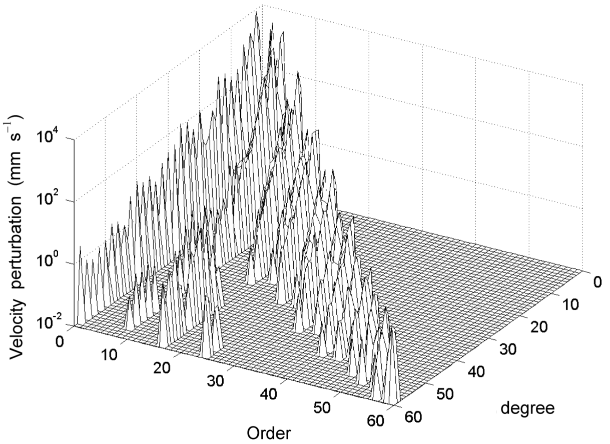

Fig. 4 The velocity perturbations on the HEO (resonance of 8:3) by Mars gravity coefficient (C_{lm}, S_{lm}) pairs. Only the perturbations larger than 0.1mm s^{-1} are plotted.

We put our method of finding the resonance condition to the test by considering, as a case study, a Mars mission to be launched in December 2018; reaching its destination after 10 mon, it maneuvers into an elliptical orbit around Mars. The spacecraft in orbit carries out its main scientific tasks from a high eccentricity elliptical orbital with the following parameters: the apoapsis altitude is 14 150 km; the periapsis altitude is 260 km. This gives the orbital parameters $\bar{a} = 10\,602$ km and $\bar{e} = 0.65507$, and we will also suppose $\bar{i} = 98°$.

Using the same method as for the nearly circular orbit in Section 4, one can find the exact resonance conditions for this spacecraft in an HEO, and the results are shown in Figure 3(b). It is worth noting that the eccentricity $\bar{e}$ is very close to, but smaller than, the Laplace limit, so the analytical method presented in Sections 2 and 4 can still be adopted by only using a larger value of Q (as the convergence becomes slow), although the computational cost will increase. In this case, we took the value of $Q = 50$, so the result of the perturbing potential calculated from Equation (2) differs from that calculated from Equation (1) by less than 5%. Such an HEO satisfies the conditions for an 8:3 orbital resonance, and the velocity perturbations induced on the spacecraft by the Mars gravity field are shown in Figure 4 where, as before, only terms greater than 0.1 mm s^{-1} are plotted.

The Chinese VLBI Network (CVN) organized three experiments that tracked the Mars Express spacecraft and performed orbit determination in 2008 and 2009. It is shown that the 5 s integrated three-way Doppler measurement noise is 0.3 mm s^{-1}, at roughly the same noise level as the European Space Agency. These experiments demonstrate that China has the capability to track and determine the trajectory of a spacecraft in orbit around Mars (Cao et al. 2010). Considering the results of Figure 4 and those of the CVN tracking precision, it can be stated that the orbital data can provide the solution of most of the gravity coefficients up to degree and order $50 \sim 60$. There are some large perturbations dominated by zonal spherical harmonics and those of orders 8, 16 and 24, which are induced by the resonance effects. Comparing Figure 4 with Figure 2, it is shown that these coefficients induce larger velocity perturbations on this spacecraft in an HEO, at least by 1 mm s^{-1}, with respect to those on MGS.

In conclusion, this HEO can also contribute to or complement spacecrafts in circular orbits (such as MGS) in order to recover the Mars gravity field; i.e. incorporating these HEO tracking data with MGS data may improve some low and mid degree/order terms in Mars gravity field models.

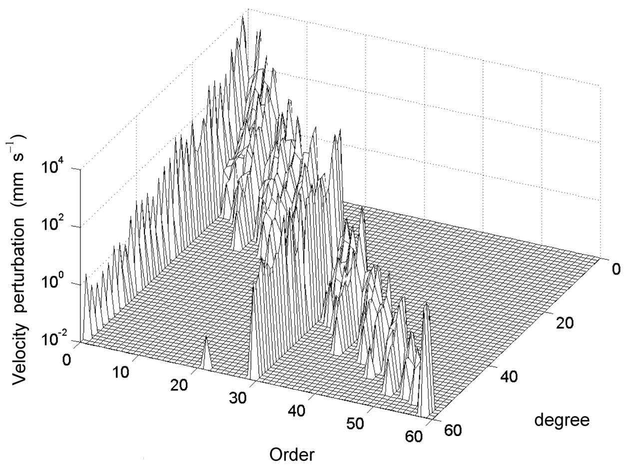

Fig. 5 The velocity perturbations on the HEO (resonance of 29:11) by Mars gravity coefficient (C_{lm}, S_{lm}) pairs. Only the perturbations larger than 0.1 mm s^{-1} are plotted.

In order to obtain more information on the perturbation of an HEO by higher degree/order gravity coefficients, one can choose another appropriate parameter a_p in orbit design so that a higher order resonance exists, such as 21:8, 29:11, etc. Taking the 29:11 resonance as another example, it can be identified from Figure 3(b) that the periapsis altitude is about 294 km and the mean semimajor axis $\bar{a}$ is 10 700 km. Figure 5 shows the corresponding velocity perturbations in this case. Comparing Figure 5 with Figure 4, it is clearly shown that the coefficients of order 29 give much larger perturbations in a 29:11 resonance orbit than in an 8:3 orbital resonance.

From the results of the two simulated HEOs and their comparisons with MGS, one can find that, by choosing a set of appropriate orbital parameters a_p to make a favorable resonance, the perturbation on the orbital velocity by Mars gravity coefficients of a certain degree/order can be significantly enlarged, so that these gravity coefficients can be solved with better precision from the tracking data. At the same time, the adjustment of a_p in the orbital design is small and does not affect other assigned scientific objectives.

6 SUMMARY AND DISCUSSION

The aim of this paper is to simulate and investigate the value of the HEO's tracking data in the recovery of the Mars gravity field. A new approach is proposed to search the orbital resonance via choosing an appropriate semimajor axis $\bar{a}$. The expected resonance will significantly enlarge the magnitude of perturbation in position and velocity produced by specific harmonic coefficients of the Mars gravity field, eventually leading to a better determination of their values. This can be done with a small adjustment of $\bar{a}$ that does not significantly alter the original orbit, and can help the recovery of some higher degree and order gravity coefficients.

One problem of the linear analytical method in Section 2 is the rate of convergence. Actually, for an orbit with high eccentricity, the convergence is much slower than for the orbits with $e < 0.1$. This is because for an orbit with $e < 0.1$, the value of Q in Equation (2) in the linear analytical method can be generally taken as 1 or 2. However, for an orbit with high eccentricity, the value of Q has to be increased, thus decreasing the computational efficiency. As in Rosborough & Lemoine (1991), Q is taken as 40 for $M9$ ($e = 0.62$), and their RMS values for position and velocity perturbation agree very well with the results from the numerical software (Geodyn) within a reasonable precision of 5%. Therefore, the linear analytical method for some high eccentricity (below the Laplace limit) orbit simulation and analysis is still applicable, with an appropriate value of Q to ensure a sufficient precision. Following this principle, Q is determined to be 50 by comparing the difference between Equations (1) and (2). This issue and the details of this method are beyond the scope of this paper and will be discussed in the next paper.

The resonance effect may also increase the observation noise, especially for a constellation of two satellites such as GRACE. However, some existing studies that recover Earth's gravity field have demonstrated that the resonance effect on a single spacecraft can be employed to improve gravity field coefficients. For example, in Klokočník et al. (2003), the authors make use of the resonance effect to improve CHAMP's 46th order coefficient recovery with good results. Moreover, there is no "GRACE" plan for Mars at present. Considering that the tracking data from spacecrafts in orbit around Mars are limited, using the resonance effect for a given HEO to improve some specific coefficients of the Mars gravity field may become a useful choice.

The White Paper on Chinese Space and National 11th Five-Year Plan declared deep space exploration to be the main arena of development for the Chinese space industry, and Mars exploration to be a focus of the Chinese space program after lunar exploration (Wu et al. 2009). The first Chinese spacecraft that planned to travel Mars, Yinhuo-1 (YH-1), had four scientific objectives, one of which, by utilizing the unique nature of the YH-1 orbit (such as low inclination), was to improve the existing Martian gravity field model from orbit tracking information. The probe unfortunately failed to enter the designed orbit in November 2011 due to the failure of a Russian launcher. However, with continuous development, Mars is becoming a major destination of future Chinese deep space exploration. The technical heritage of the YH-1 mission and the improvement of the Chinese deep space network has laid a solid foundation for a future Mars program. Future Chinese missions to Mars may focus on the space environment and geology of this planet, and a spacecraft in an HEO may be adopted. As in the case of YH-1, tracking data are usually only valuable for the determination of low degree and order gravity coefficients. This paper however has shown that a small change of the orbit design, via resonance, can upgrade the value of tracking data of such an HEO in Mars gravity recovery. We hope this optimum orbit design method can serve as a reference for future Mars programs.

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