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$f(T)$ modified teleparallel gravity as an alternative for holographic and new agegraphic dark energy models

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Abstract In the present work, we reconstruct different $f(T)$-gravity models corresponding to the original and entropy-corrected versions of the holographic and new agegraphic dark energy models. We also obtain the equation of state parameters of the corresponding $f(T)$-gravity models. We conclude that the original holographic and new agegraphic $f(T)$-gravity models behave like the phantom or quintessence model, whereas in the entropy-corrected models, the equation of state parameter can justify the transition from the quintessence state to the phantom regime as indicated by the recent observations.

Key words: modified theories of gravity — dark energy

1 INTRODUCTION

Recent observational data coming from type Ia supernovae surveys, large scale structure, and the cosmic microwave background anisotropy spectrum point toward the picture of a spatially-flat universe undergoing an accelerated expansion driven by a dominant negative pressure fluid, typically referred to as dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999). From cosmic observations, it has been shown that DE takes up about two-thirds of the total energy density. Although the nature and cosmological origin of DE are at present still enigmatic, a great variety of models have been proposed to describe the DE (for a review see Padmanabhan 2003; Peebles & Ratra 2003; Copeland et al. 2006).

One of the interesting alternative proposals for DE is modified gravity. It can naturally explain the unification of earlier and later cosmological epochs (Capozziello & Fang 2002). Moreover, modified gravity may serve in the role of dark matter (Sobouti 2007). There are some classes of modified gravities containing $f(R)$, $f(\mathcal{G})$ and $f(R,\mathcal{G})$ which are considered to be gravitational alternatives for DE (Starobinsky 1980; Capozziello et al. 2006; Sadjadi 2006; Hu & Sawicki 2007; Nojiri & Odintsov 2007; Nojiri & Odintsov 2011; Nozari & Azizi 2009; Karami & Khaledian 2011). The Lagrangian density of modified gravity theories f is an arbitrary function of R, $\mathcal{G}$ or both R and $\mathcal{G}$. Here, R and $\mathcal{G}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$ are the Ricci scalar and Gauss-Bonnet invariant term, respectively. Also $R_{\mu\nu\rho\sigma}$ and $R_{\mu\nu}$ are the Riemann and Ricci tensors, respectively. The field equations of these modified gravity theories are fourth order, which makes it difficult to obtain both exact and numerical solutions. Recently, a new modified gravity model was proposed by Bengochea

& Ferraro (2009) to describe the present accelerating expansion of the universe without resorting to DE. Instead of using the curvature defined via the Levi-Civita connection in general relativity (GR), the Weitzenböck connection is used in teleparallel gravity (TG) (Einstein 1930; Hayashi & Shirafuji 1981). As a result, the spacetime has no curvature but contains torsion. Similar to GR where the action is a curvature scalar R, the action of TG is a torsion scalar T. Following this line and in analogy with the $f(R)$ theory, Bengochea & Ferraro (2009) suggested a new model, named $f(T)$ theory, by generalizing the action of TG as a function of the torsion scalar T, and found that it can explain the observed acceleration of the universe. Indeed, there are some terms in the modified Friedmann equation in $f(T)$-gravity that can be identified as the effective DE that produces the accelerated expansion of the late-time universe (Bamba et al. 2011; Myrzakulov 2011; Zheng & Huang 2011; Karami & Abdolmaleki 2012). Models based on modified TG may also provide an alternative to inflation (Ferraro & Fiorini 2007). Another advantage of $f(T)$ theory is that its field equations are second order which are remarkably simpler than the fourth order equations of $f(R)$ theory (Wu & Yu 2010). Recently, $f(T)$-gravity has been extensively studied in the literature (Linder 2010; Wu & Yu 2010; Bamba et al. 2011; Chen et al. 2011; Myrzakulov 2011; Zheng & Huang 2011; Karami & Abdolmaleki 2012).

Viewing the $f(T)$-gravity model as an effective description of the underlying theory of DE motivates us to establish different models of $f(T)$-gravity according to some viable DE scenarios such as holographic DE (HDE), new agegraphic DE (NADE), entropy-corrected HDE (ECHDE) and entropy-corrected NADE (ECNADE). To do so, in Section 2 we review the theory of $f(T)$-gravity. In Sections 3, 4, 5 and 6 we reconstruct different $f(T)$-gravity models corresponding to the HDE, ECHDE, NADE and ECNADE models, respectively. Section 7 is devoted to conclusions.

2 $f(T)$-GRAVITY

In the framework of $f(T)$ theory, the action of modified TG is given by Bengochea & Ferraro (2009)

$$I = \frac{1}{2k^2} \int \mathrm{d}^4x \, e \left[f(T) + L_m \right] , \tag{1}$$

where $k^2 = M_P^{-2} = 8\pi G$ and $e = \det(e^i_\mu) = \sqrt{-g}$. Also T and L_m are the torsion scalar and the Lagrangian density of the matter inside the universe, respectively. Note that e^i_μ is the vierbein field which is used as a dynamical object in TG and has the following orthonormal property

$$\boldsymbol{e}_i \cdot \boldsymbol{e}_j = \eta_{ij} , \tag{2}$$

where $\eta_{ij} = \mathrm{diag}(-1, 1, 1, 1)$. Each vector $\boldsymbol{e}_i$ can be described by its components e^μ_i, where $i = 0, 1, 2, 3$ refers to the tangent space of the manifold and $\mu = 0, 1, 2, 3$ labels coordinates on the manifold. The metric tensor is obtained from the dual vierbein as

$$g_{\mu\nu}(x) = \eta_{ij} e^i_\mu(x) e^j_\nu(x) . \tag{3}$$

The torsion scalar T is defined as

$$T = S_\rho^{\ \mu\nu} T^\rho_{\ \mu\nu} , \tag{4}$$

where the non-null torsion tensor $T^\rho_{\ \mu\nu}$ is

$$T^\rho_{\ \mu\nu} = e^\rho_i (\partial_\mu e^i_\nu - \partial_\nu e^i_\mu) , \tag{5}$$

and

$$S_\rho^{\ \mu\nu} = \frac{1}{2} (K^{\mu\nu}_{\ \ \ \rho} + \delta^\mu_\rho T^{\alpha\nu}_{\ \ \ \alpha} - \delta^\nu_\rho T^{\alpha\mu}_{\ \ \ \alpha}) . \tag{6}$$

Also $K^{\mu\nu}{}_{\rho}$ is the contorsion tensor defined as

$$K^{\mu\nu}{}_{\rho} = -\frac{1}{2}(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}) \,. \tag{7}$$

Taking the variation of the action (1) with respect to the vierbein e^i_μ, one can obtain the field equations in $f(T)$-gravity as (Bengochea & Ferraro 2009)

$$S_i{}^{\mu\nu}\partial_\mu(T)f_{TT}(T) + \frac{1}{4}e_i^\nu f(T) + \Big[e^{-1}\partial_\mu(eS_i{}^{\mu\nu}) - e_i^\lambda T^\rho_{\mu\lambda}S_\rho{}^{\nu\mu}\Big]f_T(T) = \frac{k^2}{2}e_i{}^\rho T_\rho{}^\nu \,, \tag{8}$$

where subscript T denotes a derivative with respect to T, $S_i{}^{\mu\nu} = e_i{}^\rho S_\rho{}^{\mu\nu}$ and $T_{\mu\nu}$ is the energy-momentum tensor of matter. The field Equation (8) are second order which makes them simpler than the corresponding field equations in the other modified gravity theories like $f(R)$, $f(\mathcal{G})$ and $f(R,\mathcal{G})$ (Myrzakulov 2011).

Now if we consider the spatially-flat Friedmann-Robertson-Walker (FRW) metric for the universe as

$$g_{\mu\nu} = \mathrm{diag}\Big(-1, a^2(t), a^2(t), a^2(t)\Big) \,, \tag{9}$$

where a is the scale factor, then from Equation (3) one can obtain

$$e^i_\mu = \mathrm{diag}\Big(1, a(t), a(t), a(t)\Big) \,. \tag{10}$$

Substituting the vierbein (10) into (4) yields

$$T = -6H^2 \,, \tag{11}$$

where $H = \dot{a}/a$ is the Hubble parameter.

Taking $T^\mu_\nu = \mathrm{diag}(-\rho_m, p_m, p_m, p_m)$ for the matter energy-momentum tensor in the prefect fluid form and using the vierbein (10), the set of field equations (8) for $i = 0 = \nu$ reduces to (Bengochea & Ferraro 2009)

$$12H^2 f_T(T) + f(T) = 2k^2\rho_m \,, \tag{12}$$

and for $i = 1 = \nu$ yields

$$48H^2\dot{H}f_{TT}(T) - (12H^2 + 4\dot{H})f_T(T) - f(T) = 2k^2 p_m \,. \tag{13}$$

Here ρ_m and p_m are the energy density and pressure of the matter inside the universe, respectively, and they satisfy the conservation equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \,. \tag{14}$$

Note that Equations (12) and (13) are the modified Friedmann equations in the framework of $f(T)$-gravity in the spatially-flat FRW universe. One can rewrite Equations (12) and (13) as (Myrzakulov 2011)

$$\frac{3}{k^2}H^2 = \rho_m + \rho_T \,, \tag{15}$$

$$\frac{1}{k^2}(2\dot{H} + 3H^2) = -(p_m + p_T) \,, \tag{16}$$

where

$$\rho_T = \frac{1}{2k^2}(2Tf_T - f - T) \,, \tag{17}$$

$$p_T = -\frac{1}{2k^2}[-8\dot{H}Tf_{TT} + (2T - 4\dot{H})f_T - f + 4\dot{H} - T]\,, \qquad (18)$$

are the torsion contribution to the energy density and pressure which satisfy the energy conservation law

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0\,. \qquad (19)$$

In the case of $f(T) = T$, from Equations (17) and (18) we have $\rho_T = 0$ and $p_T = 0$. Therefore, Equations (15) and (16) are transformed to the usual Friedmann equations in GR.

The equation of state (EoS) parameter due to the torsion contribution is defined as

$$\omega_T = \frac{p_T}{\rho_T} = -1 + \frac{4\dot{H}(2Tf_{TT} + f_T - 1)}{2Tf_T - f - T}\,. \qquad (20)$$

Note that for the de Sitter universe, i.e. $\dot{H} = 0$, we have $\omega_T = -1$ which behaves like the cosmological constant.

In the subsequent sections, we reconstruct different $f(T)$-gravities according to the HDE, ECHDE, NADE and ECNADE models.

3 HOLOGRAPHIC $f(T)$-GRAVITY MODEL

Here we reconstruct the $f(T)$-gravity from the HDE model. The HDE proposal is motivated from the holographic principle, according to which, the number of degrees of freedom of a physical system should scale with the corresponding bounding area rather than with the volume ('t Hooft 1993; Susskind 1995; Cohen et al. 1999). By applying the holographic principle to cosmology, one can obtain the upper bound of the entropy contained in the universe (Fischler & Susskind 1998). Following this strategy, Li (2004) proposed the HDE density as

$$\rho_\Lambda = \frac{3c^2}{k^2R_h^2}\,, \qquad (21)$$

where c is a numerical constant and the future event horizon R_h is defined as

$$R_h = a\int_t^\infty \frac{\mathrm{d}t}{a} = a\int_a^\infty \frac{\mathrm{d}a}{Ha^2}\,. \qquad (22)$$

Li (2004) showed that the HDE model can drive the universe to accelerated expansion. Also the cosmic coincidence problem can be resolved by inflation in the HDE model, providing the minimal number of e-foldings.

Here, we assume two ansatzs for the scale factor which is usually considered for describing the accelerating universe in different modified gravities like $f(R)$, $f(\mathcal{G})$ and $f(R,\mathcal{G})$ (Starobinsky 1980; Capozziello et al. 2006; Sadjadi 2006; Hu & Sawicki 2007; Nojiri & Odintsov 2007; Nojiri & Odintsov 2011; Nozari & Azizi 2009; Karami & Khaledian 2011). The first ansatz is given by

$$a(t) = a_0(t_s - t)^{-h}, \qquad t \le t_s, \quad h > 0\,. \qquad (23)$$

Using Equations (11) and (23) one can obtain

$$H = \frac{h}{t_s - t}, \qquad T = -\frac{6h^2}{(t_s - t)^2}, \qquad \dot{H} = -\frac{T}{6h}\,. \qquad (24)$$

For the second ansatz of

$$a(t) = a_0t^h, \qquad h > 0\,, \qquad (25)$$

one can get

$$H = \frac{h}{t}, \qquad T = -\frac{6h^2}{t^2}, \qquad \dot{H} = \frac{T}{6h}\,. \tag{26}$$

For the first class of scale factors (23) and using Equation (24), the future event horizon R_h yields

$$R_h = a\int_t^{t_s} \frac{\mathrm{d}t}{a} = \frac{t_s - t}{h+1} = \frac{h}{h+1}\left(\frac{-6}{T}\right)^{1/2}. \tag{27}$$

Inserting Equation (27) into (21) one can obtain

$$\rho_\Lambda = -\frac{\gamma}{2k^2}T\,, \tag{28}$$

where

$$\gamma = c^2\left(\frac{h+1}{h}\right)^2. \tag{29}$$

Equating (17) with (28), i.e. $\rho_T = \rho_\Lambda$, we obtain the following differential equation

$$2Tf_T - f + (\gamma - 1)T = 0\,. \tag{30}$$

Solving Equation (30) yields the holographic $f(T)$-gravity model as

$$f(T) = \epsilon\sqrt{-T} + (1-\gamma)T\,, \tag{31}$$

where ϵ is an integration constant. Note that $T = -6H^2 < 0$.

Substituting Equation (31) into (20) one can obtain the EoS parameter of the torsion contribution as

$$\omega_T = -1 - \frac{2}{3h}, \qquad h > 0\,, \tag{32}$$

which is always smaller than -1 and corresponds to a phantom accelerating universe. Recent observational data indicate that the EoS parameter ω_T at present lies in a narrow strip around $\omega_T = -1$ and is quite consistent with being below this value (Copeland et al. 2006).

For the second class of scale factors (25) and using Equation (26), the future event horizon R_h reduces to

$$R_h = a\int_t^{\infty} \frac{\mathrm{d}t}{a} = \frac{h}{h-1}\left(\frac{-6}{T}\right)^{1/2}, \qquad h > 1\,, \tag{33}$$

where the condition $h > 1$ is obtained due to having a finite positive future event horizon. If we repeat the above calculations then we can obtain both the $f(T)$ and ω_T corresponding to the HDE for the second class of scale factors (25). The result for $f(T)$ is the same as Equation (31) where

$$\gamma = c^2\left(\frac{h-1}{h}\right)^2. \tag{34}$$

Also the EoS parameter is obtained as

$$\omega_T = -1 + \frac{2}{3h}\,, \qquad h > 1\,, \tag{35}$$

which describes an accelerating universe with the quintessence EoS parameter, i.e. $\omega_T > -1$. It should be mentioned that for $h > 1$, the EoS parameter (35) also takes place in the range of $-1 < \omega_T < -1/3$.

4 ENTROPY-CORRECTED HOLOGRAPHIC $f(T)$-GRAVITY MODEL

The ECHDE is the entropy-corrected version of the HDE model. The corrections arise in the black hole entropy in the loop quantum gravity (LQG) due to thermal equilibrium fluctuations and quantum fluctuations (Rovelli 1996; Ashtekar et al. 1998; Banerjee & Modak 2009). On this basis, Wei (2009) proposed the ECHDE density in the form

$$\rho_\Lambda = \frac{3c^2}{k^2R_h^2} + \frac{\alpha}{R_h^4}\ln\left(\frac{R_h^2}{k^2}\right) + \frac{\beta}{R_h^4}\,, \qquad (36)$$

where α and β are dimensionless constants. In the special case $\alpha = \beta = 0$, the above equation yields the well-known HDE density (21).

For the first class of scale factors (23), substituting Equation (27) into (36) one can get

$$\rho_\Lambda = -\frac{\gamma}{2k^2}T + \frac{1}{2k^2}\left[\sigma + \delta\ln\left(-\frac{\lambda}{T}\right)\right]T^2\,, \qquad (37)$$

where

$$\gamma = c^2\left(\frac{h+1}{h}\right)^2,\ \ \delta = \frac{k^2\alpha}{18}\left(\frac{h+1}{h}\right)^4,\ \ \lambda = \frac{6}{k^2}\left(\frac{h}{h+1}\right)^2,\ \ \sigma = \frac{k^2\beta}{18}\left(\frac{h+1}{h}\right)^4. \qquad (38)$$

Equating (17) with (37) one can get

$$2Tf_T - f + (\gamma-1)T - \left[\sigma + \delta\ln\left(-\frac{\lambda}{T}\right)\right]T^2 = 0\,. \qquad (39)$$

Solving the differential Equation (39) yields the entropy-corrected holographic $f(T)$-gravity model as

$$f(T) = \epsilon\sqrt{-T} + (1-\gamma)T + \frac{1}{3}\left\{\sigma + \delta\left[\frac{2}{3} + \ln\left(-\frac{\lambda}{T}\right)\right]\right\}T^2\,, \qquad (40)$$

where ϵ is an integration constant.

Substituting Equation (40) into (20) one can get

$$\omega_T = -1 - \frac{2}{3h}\times\left[1 + \left(\frac{\delta - [\sigma + \delta\ln(-\frac{\lambda}{T})]}{\gamma - [\sigma + \delta\ln(-\frac{\lambda}{T})]T}\right)T\right], \qquad h > 0\,. \qquad (41)$$

If we set $\delta = 0 = \alpha$ and $\sigma = 0 = \beta$ then Equations (40) and (41) reduce to (31) and (32), respectively.

Note that the time-dependent EoS parameter (41) in contrast with constant EoS parameter (32) can justify the transition from the quintessence state, $\omega_T > -1$, to the phantom regime, $\omega_T < -1$, as indicated by recent observations (Larson et al. 2011; Komatsu et al. 2011). To illustrate this transition in ample detail, the EoS parameter of the entropy-corrected holographic $f(T)$-gravity model, Equation (41), versus redshift $z = \frac{a_0}{a} - 1$ for the first class of scale factors, Equation (23), is plotted in Figure 1. Note that the torsion scalar T can be expressed in terms of redshift z. For the first class of scale factors (28) one can obtain

$$T = -\frac{6h^2}{(t_s-t)^2} = -\frac{6h^2}{(1+z)^{2/h}}.$$

Figure 1 demonstrates that for a set of free parameters $c = 0.818$ (Li et al. 2009), $\alpha = -5$, $\beta = 0.1$ and $h = 0.55$, ω_T crosses the -1 line twice. At the transition redshift $z_T \simeq 0.75$, we have

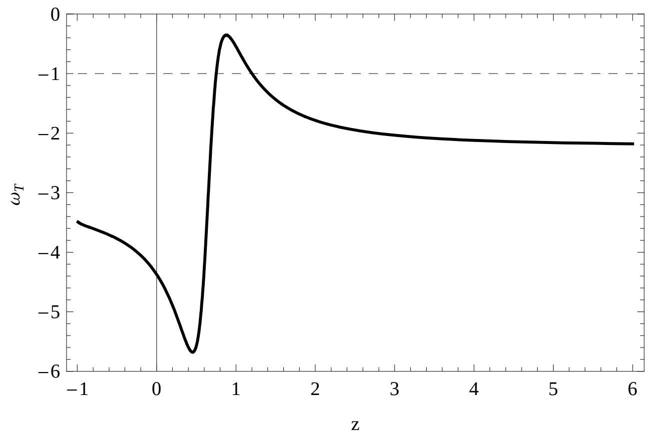

Fig. 1 The EoS parameter of the entropy-corrected holographic $f(T)$-gravity model, Eq. (41), versus redshift for the first class of scale factors, Eq. (23). Auxiliary parameters are: $c = 0.818$ (Li et al. 2009), $\alpha = -5$, $\beta = 0.1$ and $h = 0.55$.

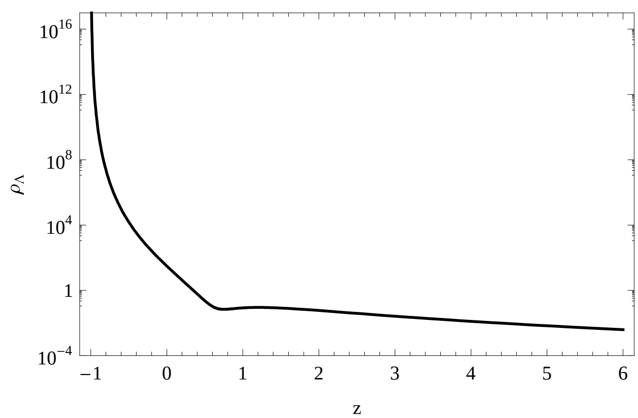

Fig. 2 The ECHDE density, Eq. (36), versus redshift for the first class of scale factors, Eq. (23). Auxiliary parameters are the same as in Fig. 1.

a direct transition from $\omega_T > -1$ (quintessence phase) to $\omega_T < -1$ (phantom phase). Whereas at $z_T \simeq 1.20$, the crossing direction is opposite, i.e. $\omega_T < -1 \to \omega_T > -1$. Crossing the -1 line twice in the direct and opposite transitions is in agreement with what was obtained recently for some $f(T)$-gravity models (see Wu & Yu 2011).

Considering Equations (41) and (37) it seems that at $T = \frac{\gamma}{\sigma+\delta\ln(-\lambda/T)}$, a singularity in ω_T and a change of sign in ρ_Λ appear. Regarding ω_T, Figure 1 shows that the EoS parameter of the entropy-corrected holographic $f(T)$-gravity model, Equation (41), does not show any singularity.

To check the change of sign in ρ_Λ given by Equation (37), we plot it in Figure 2. Figure 2 illustrates that for the first class of scale factors, although a future Big Rip singularity in the ECHDE density ($\rho_\Lambda \to \infty$) occurs at $z \to -1$ (or $t \to t_s$), the sign of ρ_Λ does not change. Also the EoS parameter remains finite at the future Big Rip singularity when $z \to -1$ (see again Fig. 1). It is also

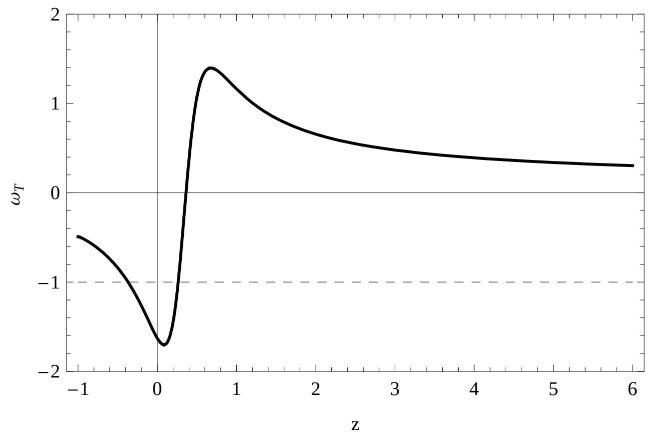

Fig. 3 The EoS parameter of the entropy-corrected holographic $f(T)$-gravity model, Eq. (43), versus redshift for the second class of scale factors, Eq. (25). Auxiliary parameters are: $c = 0.818$ (Li et al. 2009), $\alpha = -13$, $\beta = 12$ and $h = 1.31$.

interesting to note that Figure 2 demonstrates that the local minimum and maximum points of ρ_Λ occur at the transition redshifts when $\omega_T = -1$ (see Fig. 1). This can also be shown analytically. From Equation (17), $\mathrm{d}\rho_T/\mathrm{d}T = 0$ yields

$$2Tf_{TT} + f_T - 1 = 0.$$

Inserting the above relation into Equation (20) gives $\omega_T = -1$.

For the second class of scale factors (25), the resulting $f(T)$ is the same as Equation (40) where

$$\gamma = c^2\left(\frac{h-1}{h}\right)^2, \ \ \delta = \frac{k^2\alpha}{18}\left(\frac{h-1}{h}\right)^4, \ \ \lambda = \frac{6}{k^2}\left(\frac{h}{h-1}\right)^2, \ \ \sigma = \frac{k^2\beta}{18}\left(\frac{h-1}{h}\right)^4. \qquad (42)$$

Also the EoS parameter is obtained as

$$\omega_T = -1 + \frac{2}{3h} \times \left[1 + \left(\frac{\delta - [\sigma + \delta\ln(-\frac{\lambda}{T})]}{\gamma - [\sigma + \delta\ln(-\frac{\lambda}{T})]T}\right)T\right], \qquad h > 1\,. \qquad (43)$$

Here also in order to make R_h be finite positive, the parameter h should be in the range of $h > 1$. One notes that the dynamical EoS parameter (43) in contrast with the constant EoS parameter (35) can accommodate the transition from $\omega_T > -1$ to $\omega_T < -1$ at the recent stage.

Figure 3 displays the evolution of the EoS parameter of the entropy-corrected holographic $f(T)$-gravity model, Equation (43), versus redshift z for the second class of scale factors, Equation (25). In this case, the torsion scalar T can be expressed in terms of redshift z as

$$T = -\frac{6h^2}{t^2} = -6h^2(1+z)^{2/h}.$$

Figure 3 like Figure 1 shows that the -1 line is crossed twice for another set of free parameters $c = 0.818$ (Li et al. 2009), $\alpha = -13$, $\beta = 12$ and $h = 1.31$. At $z_T \simeq 0.26$ we have a direct transition (i.e. $\omega_T > -1 \rightarrow \omega_T < -1$). Also an opposite transition occurs in the future at $z_T \simeq -0.37$. Furthermore, Figure 3 indicates that there is no singularity in the dynamical EoS parameter (43). Note that also the sign of the ECHDE density (36) for the second class of scale factors (25) remains unchanged (see Fig. 4).

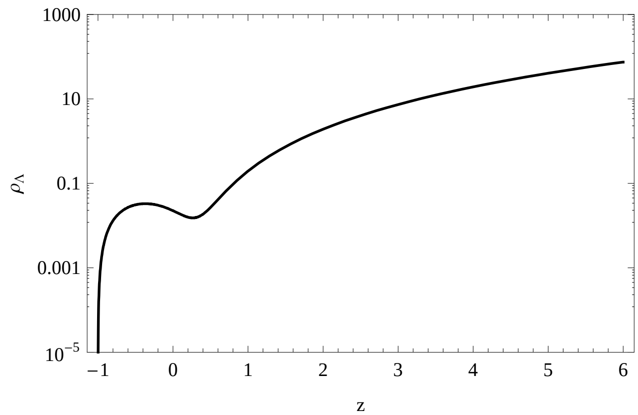

Fig. 4 The ECHDE density, Eq. (36), versus redshift for the second class of scale factors, Eq. (25). Auxiliary parameters are the same as in Fig. 3.

5 NEW AGEGRAPHIC $f(T)$-GRAVITY MODEL

The NADE model is another approach for explaining DE. This model assumes that the DE density arises from the spacetime and matter field fluctuations in the universe (Karolyhazy 1966; Maziashvili 2007). Cai (2007) defined the ADE density as $\rho_\Lambda = 3n^2k^{-2}T^{-2}$, where n is a numerical constant and T is the age of the universe. However, the original ADE model had some difficulties. For example it suffers from difficulty in describing the matter-dominated epoch. Therefore, the NADE density was proposed by Wei & Cai (2008a) as

$$\rho_\Lambda = \frac{3n^2}{k^2\eta^2}\,, \tag{44}$$

in which the old cut-off T was replaced with conformal time η defined as

$$\eta = \int \frac{\mathrm{d}t}{a} = \int \frac{\mathrm{d}a}{Ha^2}\,. \tag{45}$$

It was found that the coincidence problem could be solved naturally in the NADE model (Wei & Cai 2008b). Note that although evolution behavior of the NADE is similar to that of the HDE, some essential differences exist between them. In particular, the NADE model is free from the drawback concerning the causality problem which exists in the HDE model (Wei & Cai 2008a).

For the first class of scale factors (23), the conformal time η by the help of Equation (24) yields

$$\eta = \int_t^{t_s} \frac{\mathrm{d}t}{a} = \frac{h^{h+1}}{a_0(h+1)}\left(\frac{-6}{T}\right)^{\frac{h+1}{2}}\,. \tag{46}$$

Substituting Equation (46) into (44) gives

$$\rho_\Lambda = \frac{\gamma}{2k^2}T^{h+1}\,, \tag{47}$$

where

$$\gamma = \frac{6n^2a_0^2(h+1)^2}{(-6h^2)^{h+1}}\,. \tag{48}$$

Equating (17) with (47) yields

$$2Tf_T - f - T - \gamma T^{h+1} = 0\,. \tag{49}$$

Solving Equation (49) gives the new agegraphic $f(T)$-gravity model as

$$f(T) = \epsilon\sqrt{-T} + T + \frac{\gamma}{1+2h}T^{h+1}\,, \tag{50}$$

where ϵ is an integration constant. Inserting Equation (50) into (20) gives

$$\omega_T = -1 - \frac{2(h+1)}{3h}, \qquad h > 0\,, \tag{51}$$

which is always smaller than -1 like the EoS parameter of the holographic $f(T)$-gravity model (32), and it behaves as a phantom type DE.

For the second class of scale factors (25) and using (26), the conformal time η is obtained as

$$\eta = \int_0^t \frac{\mathrm{d}t}{a} = \frac{h^{1-h}}{a_0(1-h)}\left(\frac{-6}{T}\right)^{\frac{1-h}{2}}, \quad 0 < h < 1\,, \tag{52}$$

where the condition $h < 1$ is necessary due to having a finite positive conformal time. The resulting $f(T)$ is

$$f(T) = \epsilon\sqrt{-T} + T + \frac{\gamma}{1-2h}T^{1-h}\,, \tag{53}$$

where

$$\gamma = \frac{6n^2a_0^2(1-h)^2}{\left(-6h^2\right)^{1-h}}\,. \tag{54}$$

Also the EoS parameter of the new agegraphic $f(T)$-gravity model is obtained as

$$\omega_T = -1 + \frac{2(1-h)}{3h}, \quad 0 < h < 1\,, \tag{55}$$

which shows a quintessence-like EoS parameter $\omega_T > -1$. Here in order to have $-1 < \omega_T < -1/3$, the parameter h should be in the range of $1/2 < h < 1$.

6 ENTROPY-CORRECTED NEW AGEGRAPHIC $f(T)$-GRAVITY MODEL

More recently, very similar to the ECHDE model, the ECNADE density was proposed by Wei (2009) as

$$\rho_\Lambda = \frac{3n^2}{k^2\eta^2} + \frac{\alpha}{\eta^4}\ln\left(\frac{\eta^2}{k^2}\right) + \frac{\beta}{\eta^4}\,, \tag{56}$$

which closely mimics that of the ECHDE density (36) and R_h is replaced with the conformal time η. In the special case $\alpha = \beta = 0$, Equation (56) yields the NADE density (44). The motivation for taking the energy density of the modified NADE in the form (56) comes from the fact that both the NADE and HDE models have the same origin. Indeed, it was argued that the NADE models are the HDE model with different infrared length scales (Myung & Seo 2009).

For the first class of scale factors (23), substituting Equation (46) into (56) yields

$$\rho_\Lambda = \frac{\gamma}{2k^2}T^{h+1} + \frac{1}{2k^2}\left[\sigma + \delta\ln\left(\frac{\lambda}{T^{h+1}}\right)\right]T^{2(h+1)}\,, \tag{57}$$

where

$$\gamma = \frac{6n^2{a_0}^2(h+1)^2}{(-6h^2)^{h+1}}, \qquad \delta = \frac{2k^2\alpha {a_0}^4(h+1)^4}{(-6h^2)^{2(h+1)}},$$
$$\lambda = \frac{(-6h^2)^{h+1}}{k^2{a_0}^2(h+1)^2}, \qquad \sigma = \frac{2k^2\beta {a_0}^4(h+1)^4}{(-6h^2)^{2(h+1)}}\,. \tag{58}$$

Equating (17) with (57) gives

$$2Tf_T - f - T - \gamma T^{h+1} - \left[\sigma + \delta \ln\left(\frac{\lambda}{T^{h+1}}\right)\right] T^{2(h+1)} = 0\,. \tag{59}$$

Solving the differential Equation (59) one can obtain the entropy-corrected new agegraphic $f(T)$-gravity model as

$$f(T) = \epsilon\sqrt{-T} + T + \frac{\gamma}{1+2h}T^{h+1} + \frac{1}{3+4h} \times \left\{\sigma + \delta\left[\frac{2(1+h)}{3+4h} + \ln\left(\frac{\lambda}{T^{h+1}}\right)\right]\right\} T^{2(h+1)}\,, \tag{60}$$

where ϵ is an integration constant. Inserting Equation (60) into (20) gives

$$\omega_T = -1 - \frac{2}{3}\left(\frac{h+1}{h}\right) \times \left[1 + \left(\frac{-\delta + [\sigma + \delta\ln(\frac{\lambda}{T^{h+1}})]}{\gamma + [\sigma + \delta\ln(\frac{\lambda}{T^{h+1}})]T^{h+1}}\right)T^{h+1}\right], \qquad h > 0\,. \tag{61}$$

If we set $\delta = 0 = \alpha$ and $\sigma = 0 = \beta$ then Equations (60) and (61) reduce to (50) and (51), respectively. Note that the time-dependent EoS parameter (61) in contrast with constant EoS parameter (51) can justify the transition from $\omega_T > -1$ to $\omega_T < -1$.

Figure 5 illustrates the EoS parameter of the entropy-corrected new agegraphic $f(T)$-gravity model, Equation (61), for the first class of scale factors, Equation (23). Here for a set of free parameters $n = 2.716$ (Wei & Cai 2008b), $\alpha = -7.5$, $\beta = -14.8$ and $h = 2.5$, the direct and opposite transitions occur at $z_T \simeq 0.82$ and 1.44, respectively. Besides, Figure 5 reveals that there is no any singularity in the dynamical EoS parameter (61). Note that here also the sign of the ECNADE density (57) for the first class of scale factors (23) does not change (see Fig. 6).

For the second class of scale factors (25), the resulting $f(T)$ is

$$f(T) = \epsilon\sqrt{-T} + T + \frac{\gamma}{1-2h}T^{1-h} + \frac{1}{3-4h} \times \left\{\sigma + \delta\left[\frac{2(1-h)}{3-4h} + \ln\left(\frac{\lambda}{T^{1-h}}\right)\right]\right\} T^{2(1-h)}\,, \tag{62}$$

where

$$\gamma = \frac{6n^2{a_0}^2(1-h)^2}{(-6h^2)^{1-h}}, \qquad \delta = \frac{2k^2\alpha {a_0}^4(1-h)^4}{(-6h^2)^{2(1-h)}},$$
$$\lambda = \frac{(-6h^2)^{1-h}}{k^2{a_0}^2(1-h)^2}, \qquad \sigma = \frac{2k^2\beta {a_0}^4(1-h)^4}{(-6h^2)^{2(1-h)}}\,. \tag{63}$$

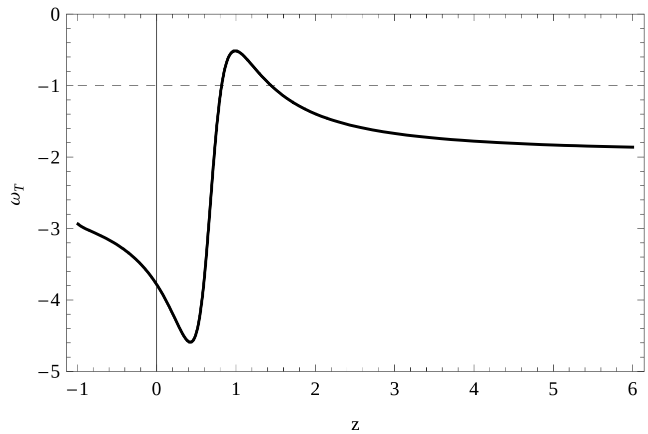

Fig. 5 The EoS parameter of the entropy-corrected new agegraphic $f(T)$-gravity model, Eq. (61), versus redshift for the first class of scale factors, Eq. (23). Auxiliary parameters are: $n = 2.716$ (Wei & Cai 2008b), $\alpha = -7.5$, $\beta = -14.8$ and $h = 2.5$.

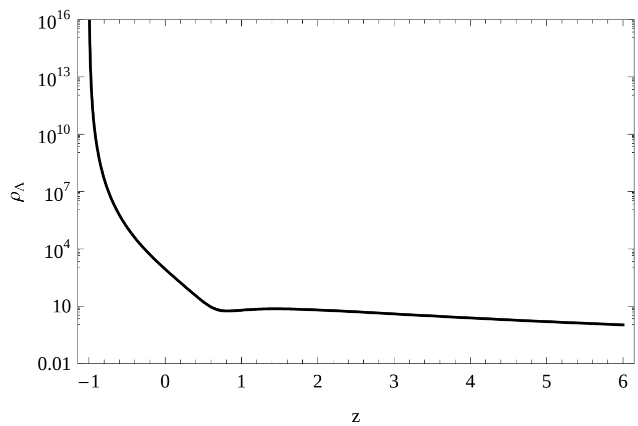

Fig. 6 The ECNADE density, Eq. (56), versus redshift for the first class of scale factors, Eq. (23). Auxiliary parameters are the same as in Fig. 5.

Also the EoS parameter can be obtained as

$$\omega_T = -1 + \frac{2}{3}\left(\frac{1-h}{h}\right) \times \left[1 + \left(\frac{-\delta + [\sigma + \delta \ln{(\frac{\lambda}{T^{1-h}})}]}{\gamma + [\sigma + \delta \ln{(\frac{\lambda}{T^{1-h}})}]T^{1-h}}\right) T^{1-h}\right], \qquad 0 < h < 1\,. \quad (64)$$

Here also in order to have a finite positive conformal time η, the parameter h should be in the range of $0 < h < 1$. Contrary to the constant EoS parameter (55), the dynamical EoS parameter (64) can accommodate the transition from $\omega_T > -1$ to $\omega_T < -1$ at the recent stage.

Figure 7 presents the evolution of the EoS parameter of the entropy-corrected new agegraphic $f(T)$-gravity model, Equation (64), for the second class of scale factors, Equation (25). Here also like Figure 5, for another set of free parameters $n = 2.716$ (Wei & Cai 2008b), $\alpha = -44$, $\beta = -10$

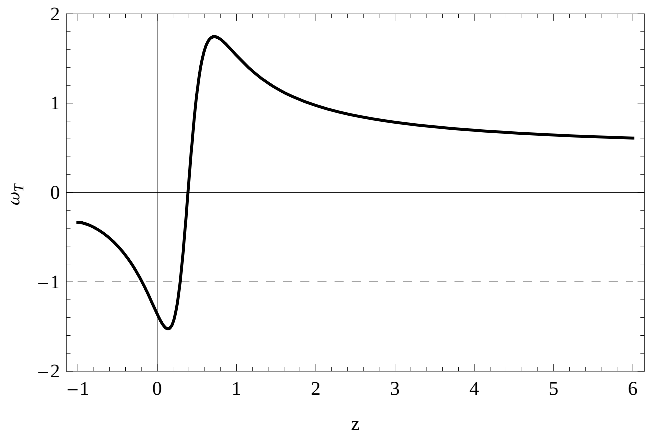

Fig. 7 The EoS parameter of the entropy-corrected new agegraphic $f(T)$-gravity model, Eq. (64), versus redshift for the second class of scale factors, Eq. (25). Auxiliary parameters are: $n = 2.716$ (Wei & Cai 2008b), $\alpha = -44$, $\beta = -10$ and $h = 0.5$.

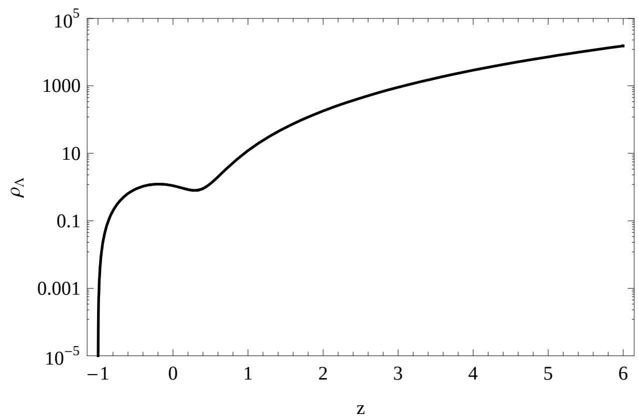

Fig. 8 The ECNADE density, Eq. (56), versus redshift for the second class of scale factors, Eq. (25). Auxiliary parameters are the same as in Fig. 7.

and $h = 0.5$, ω_T crosses the -1 line twice at $z_T \simeq 0.29$ and -0.19 corresponding to the direct and opposite transitions, respectively. Besides, Figure 7 demonstrates that there is no singularity in the dynamical EoS parameter (64). Note that here also the sign of the ECNADE density (56) for the second class of scale factors (25) does not change (see Fig. 8).

7 CONCLUSIONS

Here, we considered the original and entropy-corrected versions of the HDE and NADE models. Among various candidates explaining cosmic accelerated expansion, only the HDE and NADE models are based on the entropy-area relation. However, this definition can be modified by the inclusion of quantum effects, motivated from the LQG. Hence the ECHDE and ECNADE were introduced by addition of correction terms to the energy densities of the HDE and NADE, respectively.

We investigated the HDE, ECHDE, NADE and ECNADE in the framework of $f(T)$-gravity. Among other approaches related with a variety of DE models, a very promising approach to DE is related with the modified TG known as $f(T)$-gravity, in which DE emerges from the modification of torsion. The class of $f(T)$-gravity theories is an intriguing generalization of Einstein's new GR, taking a curvature-free approach and using a connection with torsion. It is analogous to the $f(R)$ extension of the Einstein-Hilbert action of standard GR, but has the advantage of the second order field equations. We reconstructed different theories of modified gravity based on the $f(T)$ action in the spatially-flat FRW universe for two classes of scale factors containing i) $a = a_0(t_s - t)^{-h}$ and ii) $a = a_0 t^h$ which were consistent with the original and entropy-corrected versions of the HDE and NADE scenarios. Furthermore, we obtained the EoS parameter of the corresponding $f(T)$-gravity models. Our calculations show that for the first class of scale factors, the EoS parameter, of both the holographic and new agegraphic $f(T)$-gravity models, always behaves like that of phantom DE, whereas for the second class, the EoS parameter behaves like quintessence DE. Interestingly, the EoS parameter of both the entropy-corrected holographic and new agegraphic $f(T)$-gravity models can cross the phantom-divide line twice. For the first class of scale factors $a = a_0(t_s - t)^{-h}$, the EoS parameter of both the entropy-corrected holographic and new agegraphic $f(T)$-gravity models has an opposite transition ($\omega_T < -1 \rightarrow \omega_T > -1$) in the far past and a direct transition ($\omega_T > -1 \rightarrow \omega_T < -1$) in the near past. For the second class $a = a_0 t^h$, the EoS parameter of the aforementioned models has a direct transition in the near past and an opposite transition in the future. It is interesting to note that the direct transition from the non-phantom (quintessence) phase to the phantom one in the near past is consistent with the recent cosmological observational data.

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