# Anisotropic massive strings in the scalar-tensor theory of gravitation

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**Abstract** We present the model of an anisotropic universe with string fluid as the source of matter within the framework of the scalar-tensor theory of gravitation. An exact solution of field equations is obtained by applying Berman's law of variation to Hubble's parameter which yields a constant value of the deceleration parameter. The nature of classical potential is examined for the model under consideration. It has also been found that the massive strings dominate in the early universe and finally disappear from the universe. This is in agreement with current astronomical observations. The physical and dynamical properties of the model are also discussed.

**Key words:** early universe — massive strings — scalar-tensor theory — classical potential

## 1 INTRODUCTION

On the basis of coupling between an adequate tensor field and scalar field $\phi$, Brans & Dicke (1961) formulated the scalar-tensor theories of gravitation. The scalar field $\phi$ has the dimension of $G^{-1}$, and therefore $\phi^{-1}$ plays the role of a time varying gravitational constant $G$. This theory is more consistent with Mach's principle and is less reliant on the absolute properties of space. A detailed survey of Brans-Dicke (BD) theory has been done by Singh & Rai (1983). In fact, the notion of time-dependent $G$ was first conceived by Dirac (1938), though Dirac's arguments were based on cosmological considerations not directly concerned with Mach's principle. Recently Jamil et al. (2009), Jamil & Debnath (2011) and Chakraborty et al. (2012a,b) have investigated the cosmological model of the universe with variable $G$ in a different physical context.

In BD theory, which is a generalization of general relativity, a scalar field $\phi$, in addition to the metric tensor $g_{ij}$ and a dimensionless coupling constant $\omega$, was introduced. For a large value of coupling constant $\omega$ (i.e. $\omega > 500$), BD theory follows the result of general relativity. The holographic dark energy in the framework of BD theory with a chameleon scalar field has been investigated by Setare & Jamil (2010). In this connection, Sheykhi et al. (2012a,b), Sheykhi & Jamil (2011), Karami et al. (2011) and Jamil et al. (2011) explored BD theory to study power law entropy corrected dark energy and chameleon cosmology. In 1985, Saez and Ballester developed a scalar-tensor theory in which a dimensionless scalar field is coupled with a metric. This coupling used to give a satisfactory description of the weak fields. This scalar-tensor theory plays an important role in solving the missing matter problem and removing the graceful exit problem in non flat Friedmann-Robertson-Walker cosmologies and the inflation era (1997) respectively. Singh & Agrawal (1991, 1992), Reddy

et al. (2006, 2008), Socorro et al. (2010) and recently Jamil et al. (2012) have studied the cosmological model within the framework of a Saez-Ballester scalar-tensor theory of gravitation in different physical contexts.

Among the different cosmological structures of the universe, the cosmic string models have enjoyed wide acceptance because they give rise to density perturbations which lead to the formation of galaxies (1985). Firstly, Letelier (1979) described the gravitational effect of massive strings which are formed by geometric strings with particles attached along their extension. The cosmic strings play a significant role in the early stage of evolution of the universe before the creation of particles because cosmic strings have one-dimensional topological defects associated with spontaneous symmetry breaking whose plausible production site is only a cosmological phase transition in the early universe. At the observational front, Pogosian et al. (2003) have shown that the cosmic strings are not responsible for either the cosmic microwave background (CMB) fluctuations or the observed clustering of galaxies. This means that strings disappear from the present universe, leaving only particles, but they were driving the early universe.

In recent years, Bianchi universes have been gaining an increasing interest and tremendous impetus in observational cosmology. In connection with data from the Wilkinson Microwave Anisotropy Probe (Hinshaw et al. 2009; Jaffe et al. 2005) it has been discovered that the standard cosmological model requires positive and dynamic cosmological parameters, a case which resembles Bianchi morphology (2006, 2007). According to this result, the universe should achieve the following features: i) a slightly anisotropic spatial geometry in spite of inflation, and ii) a non trivial isotropization history of the universe due to the presence of an anisotropic energy source. The anomalies found in the CMB and large scale structure observations stimulated a growing interest in the anisotropic cosmological model of the universe. Here we confine ourselves to the case of a Bianchi V model whose spatial section is flat but the expansion rate is direction dependent. Recently Yadav et al. (2011) and Yadav (2012) studied Bianchi V string cosmological models in general relativity. In this paper, we discuss Einstein's field equations in the scalar-tensor theory of gravitation for Bianchi V space-time, filled with string fluid as the source of matter. An exact solution of the field equations is obtained by applying the law of variation of Hubble's parameter, firstly proposed by Berman (1983). This law yields the constant value of the deceleration parameter (DP).

The paper is organized as follows: In Section 2, we have provided the basic equations in connection to the proposed model. Sections 3 and 4 deal with, respectively, the field equations and the solution of field equations. Finally the conclusion is presented in Section 5.

## 2  METRIC AND BASIC EQUATIONS

A spatially homogeneous and anisotropic Bianchi V space-time is described by the line element

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x} \left( B^2 dy^2 + C^2 dz^2 \right) , \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the scale factors in different spatial directions and $\alpha$ is a constant.

We define the average scale factor $(a)$ of a Bianchi-type V model as

$$a = (ABC)^{\frac{1}{3}} . \tag{2}$$

The spatial volume is given by

$$V = a^3 = ABC . \tag{3}$$

Therefore, the mean Hubble's parameter $(H)$ reads as

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \left( H_1 + H_2 + H_3 \right) , \tag{4}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters toward $x$, $y$ and $z$ respectively. An overdot denotes differentiation with respect to cosmic time $t$.

We define the kinematical quantities such as expansion scalar $(\theta)$, shear scalar $(\sigma)$ and anisotropy parameter $(A_m)$ as follows:

$$\theta = u^i_{;i} \,, \tag{5}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \,, \tag{6}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \,, \tag{7}$$

where $u^i = (1,0,0,0)$ is a matter four velocity vector and

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P^\alpha + u_{j;\alpha}P^\alpha_i\right) - \frac{1}{3}\theta P_{ij} \,. \tag{8}$$

Here, the projection vector $P_{ij}$ has the form

$$P_{ij} = g_{ij} - u_i u_j \,. \tag{9}$$

The expansion scalar $(\theta)$ and shear scalar $(\sigma)$, in Bianchi V space-time, have the form

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \,, \tag{10}$$

$$2\sigma^2 = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3} \,. \tag{11}$$

Here, $(;)$ stands for a covariant derivative with respect to cosmic time $t$.

## 3 FIELD EQUATIONS

We consider a homogeneous and anisotropic Bianchi V metric coupled with scalar field $\phi$. Our model is based on the Saez-Ballester theory of gravitation which implements a coupling of a dimensionless scalar field with the metric.

We assume the Lagrangian

$$L = R - \omega\phi^r\phi_{,i}\phi^{,i} \,, \tag{12}$$

where $R$, $\omega$ and $r$ represent the scalar curvature, coupling constant and dimensionless arbitrary constant respectively.

For the scalar field having the dimensions of $G^{-1}$, the Lagrangian (12) is not physically admissible because two terms on the right hand side of Equation (12) have different dimensions. However, it is a suitable Lagrangian in the case of the dimensionless scalar field.

From the above Lagrangian, we can establish the action

$$I = \int_\Sigma (L + 8\pi L_m)(-g)^{\frac{1}{2}}dx^1 dx^2 dx^3 dx^4 \,, \tag{13}$$

where $L_m$ is the matter Lagrangian, $g$ is the determinant of the matrix $g_{ij}$, $x^k$ are the coordinates and $\Sigma$ is an arbitrary region of integration.

The variational principle $\delta I = 0$ leads to the field equations

$$G_{ij} - \omega\phi^r\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,\ell}\phi^{,\phi}\right) = -8\pi T_{ij},$$

$$2\phi^r \phi^{;i}_{;i} + r\phi^{r-1}\phi_{,\ell}\phi^{,\ell} = 0\,. \tag{14}$$

Equation (14) is obtained by considering arbitrary independent variations of the metric and scalar field vanishing at the boundary of $\Sigma$.

Since the action $I$ is a scalar, it can be easily proved that the equation of motion

$$T^{ij}_{;i} = 0 \tag{15}$$

is a consequence of the field equations.

The energy momentum tensor for a cloud of massive strings and perfect fluid distribution is taken as

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij} - \lambda x_i x_j\,, \tag{16}$$

where $p$ is isotropic pressure; $\rho$ is the proper energy density for the cloud of strings with particles attached to them; $\lambda$ is the string tension density; $x^i$ is a unit space-like vector representing the direction of the string. The vectors $u^i$ and $x^i$ satisfy the conditions $u_i u^i = -x_i x^i = -1$ and $v^i x_i = 0$.

Choosing $x^i$ parallel to $\frac{\partial}{\partial x}$, we have

$$x^i = (A^{-1}, 0, 0, 0)\,. \tag{17}$$

Here, the cosmic string has been directed along the $x$-axis.

It is important to note that the string is one-dimensional and infinitely long, hence it has only one non zero pressure component. Thus it brings anisotropy into the system, which is not favored by observational results.

If the particle density of the configuration is denoted by $\rho_p$, then

$$\rho = \rho_p + \lambda\,. \tag{18}$$

Einstein's field equations (in gravitational units $c = 1$, $8\pi G = 1$) are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}\,. \tag{19}$$

Einstein's field Equations (19) for the line-element (1) lead to the following system of equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2 + \lambda\,, \tag{20}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2\,, \tag{21}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2\,, \tag{22}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = \rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2\,, \tag{23}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0\,, \tag{24}$$

$$\ddot{\phi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{r}{2\phi}\dot{\phi}^2 = 0\,. \tag{25}$$

The energy conservation equation $T^{ij}_{;j} = 0$ yields

$$\dot{\rho} + 3(\rho + p)H - \lambda\frac{\dot{A}}{A} = 0\,. \tag{26}$$

## 4  SOLUTION OF THE FIELD EQUATIONS

We have a system of six Equations (20)−(25) involving seven unknown variables, namely $A$, $B$, $C$, $p$, $\rho$, $\lambda$ and $\phi$. Therefore, in order to completely solve the field equations, we need at least one suitable physical assumption among the unknown variables. So, we constrain the system of equations with the law of variation for Hubble's parameter proposed by Berman (1983), which yields a constant value of DP. This law reads

$$H = Da^{-n}, \tag{27}$$

where $D$ and $n$ are positive constants. In this paper, we show how the constant DP models with metric (1) behave in the presence of a string fluid and dimensionless scalar field $\phi$. DP $(q)$, an important observational quantity, is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{28}$$

From Equations (4) and (27), we get

$$\dot{a} = Da^{-n+1}. \tag{29}$$

Integration of Equation (29) leads to

$$a = (nDt + c_1)^{\frac{1}{n}}, \quad (n \neq 0). \tag{30}$$

It is important to note here that for $n = 0$, the model has a non singular origin and it evolves with exponential expansion which seems reasonable to predict the dynamics of the future universe. Since we are looking for a model of the universe, which describes the dynamics of the universe from the Big Bang to the present epoch, in this paper the case $n = 0$ has been omitted.

Integrating Equation (24) and absorbing the constant of integration in $B$ or $C$, without loss of generality, we obtain

$$A^2 = BC. \tag{31}$$

Subtracting Equation (21) from Equation (22) and taking the second integral, we get the following relation

$$\frac{B}{C} = d_1 \exp\left[x_1 \int \frac{dt}{V}\right], \tag{32}$$

where $d_1$ and $x_1$ are constants of integration.

From Equations (3), (30), (31) and (32), the metric function can be explicitly written as

$$A = (nDt + c_1)^{\frac{1}{n}}, \tag{33}$$

$$B = \sqrt{d_1}(nDt + c_1)^{\frac{1}{n}} \exp\left[\frac{x_1}{2D(n-3)}(nDt + c_1)^{\frac{n-3}{3}}\right], \tag{34}$$

$$C = \frac{1}{\sqrt{d_1}}(nDt + c_1)^{\frac{1}{n}} \exp\left[-\frac{x_1}{2D(n-3)}(nDt + c_1)^{\frac{n-3}{3}}\right], \tag{35}$$

provided that $n \neq 3$.

Inserting Equation (4) into Equation (25) and then integrating, we obtain

$$\dot{\phi}^2 \phi^r = d_2 a^{-6}. \tag{36}$$

Here, $d_2$ is a constant of integration.

The average Hubble's parameter $(H)$, isotropic pressure $(p)$, proper energy density $(\rho)$, string tension density $(\lambda)$ and particle energy density $(\rho_p)$ are found to be

$$H = \frac{D}{nDt + c_1}, \tag{37}$$

$$p = \alpha^2 (nDt + c_1)^{-\frac{2}{n}} - \left(\frac{2\omega d_2 - x_1^2}{4}\right)(nDt + c_1)^{-\frac{6}{n}} - 3(1-n)D^2(nDt + c_1)^{-2}, \quad (38)$$

$$\rho = 3D^2(nDt + c_1)^{-2} - \frac{(x_1^2 + 2\omega d_2)}{4}(nDt + c_1)^{-\frac{6}{n}} - 3\alpha^2(nDt + c_1)^{-\frac{2}{n}}, \quad (39)$$

$$\lambda = \frac{(x_1^2 - \omega d_2)}{4}(nDt + c_1)^{-\frac{6}{n}}, \quad (40)$$

$$\rho_p = 3D^2(nDt + c_1)^{-2} - \frac{3x_1^2}{4}(nDt + c_1)^{-\frac{6}{n}} - 3\alpha^2(nDt + c_1)^{-\frac{2}{n}}. \quad (41)$$

The above solutions identically satisfy the energy conservation Equation (26), as expected.

The spatial volume $(V)$, expansion scalar $(\theta)$ and DP $(q)$ are given by

$$V = (nDt + c_1)^{\frac{3}{n}}, \quad (42)$$

$$\theta = 3D(nDt + c_1)^{-1}, \quad (43)$$

$$q = n - 1. \quad (44)$$

We observe that at $t = -\frac{c_1}{nD}$, the spatial volume vanishes while all other parameters diverge. Therefore, the model has a Big Bang singularity at $t = -\frac{c_1}{nD}$. This singularity is point type because the directional scale factors $A(t)$, $B(t)$ and $C(t)$ vanish at the initial moment. From Equation (43), it is clear that for $n = 1$, the universe expands with a constant rate. However, the recent observations of type Ia supernovae (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998, 2004 and Tonry et al. 2003) reveal that the present universe is accelerating and the value of DP lies somewhere in the range $-1 < q < 0$. It follows that one can choose the value of $n$ in the range $0 < n < 1$ to ensure that the derived model is consistent with observations.

In the derived model, the scale factors increase with time, but the contribution of exponential terms to the scale factors $B$ and $C$ becomes negligible for sufficiently large time, i.e. for sufficiently large time we have $A(t) \approx B(t) \approx C(t)$. This may be observed from Equations (33)−(35). Thus, initially the growth of scale factors takes place at different rates due to effective contribution of exponential terms in $B$ and $C$, but later the scale factors grow at the same rate. Therefore, in the derived model, the early anisotropic universe becomes isotropic at later times.

The scalar function $(\phi)$ may be obtained as

$$\phi = \left[\frac{\phi_0(r+2)}{2D(n-3)}(nDt + c_1)^{\frac{n-3}{n}}\right]^{\frac{2}{r+2}}, \quad (45)$$

where $\phi_0$ is the constant of integration.

The shear scalar $(\sigma)$ and anisotropy parameter $(A_m)$ are read as

$$\sigma = \frac{x_1}{2}(nDt + c_1)^{-\frac{3}{n}}, \quad (46)$$

$$A_m = \frac{x_1^2}{6D^2}(nDt + c_1)^{\frac{2n-6}{n}}. \quad (47)$$

The behavior of scalar function $(\phi)$ is depicted in Figure 1. From Equation (47), it is clear that for $n < 1$, the anisotropy parameter $(A_m)$ vanishes at late time. The behavior of $A_m$ versus time is shown in Figure 2.

The behaviors of $\rho$, $\rho_p$ and $\lambda$ are depicted in Figure 3. From Equations (40) and (41), it is clear that for $n < 1$ and for a large value of time, $\frac{\rho_p}{\lambda} > 1$. This means that the particles dominate the strings at later times which confirms the disappearance of strings in the present day observations.
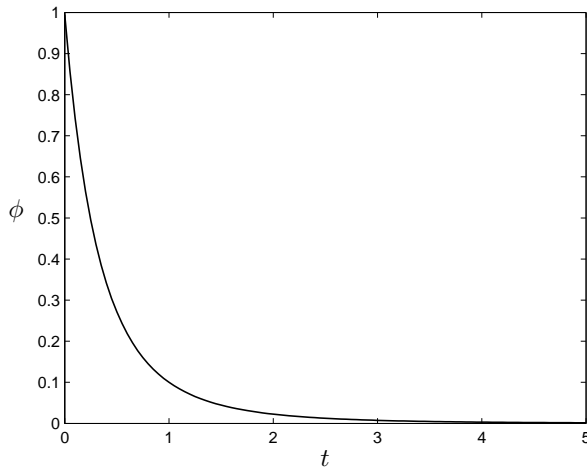
**Fig. 1** Plot of scalar function ($\phi$) vs. time.
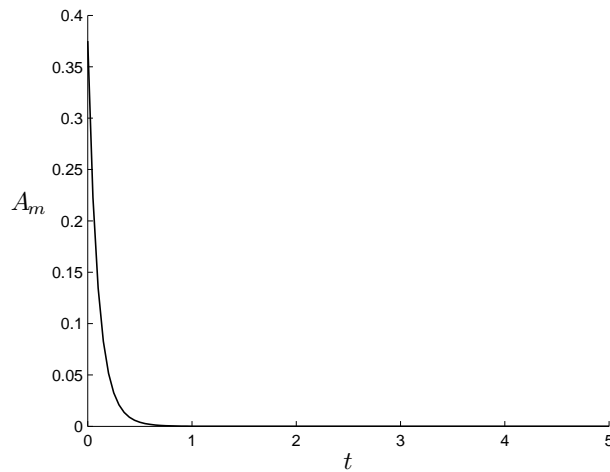


**Fig. 2** Plot of anisotropy parameter ($A_m$) vs. time.

From Equation (42), we obtain
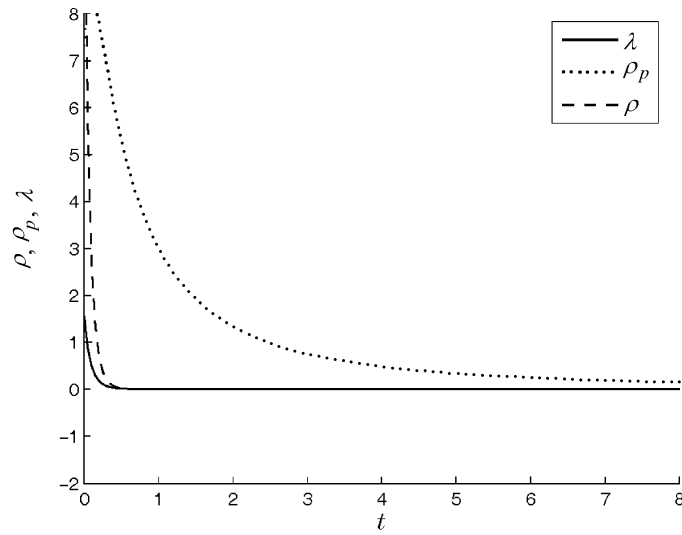
$$\dot{V} = 3D(nDt + c_1)^{\frac{3-n}{n}} . \tag{48}$$

According to Saha & Boyadjiev (2004), the equation of motion of a single particle with unit mass under force $\boldsymbol{F}(V)$ can be described as
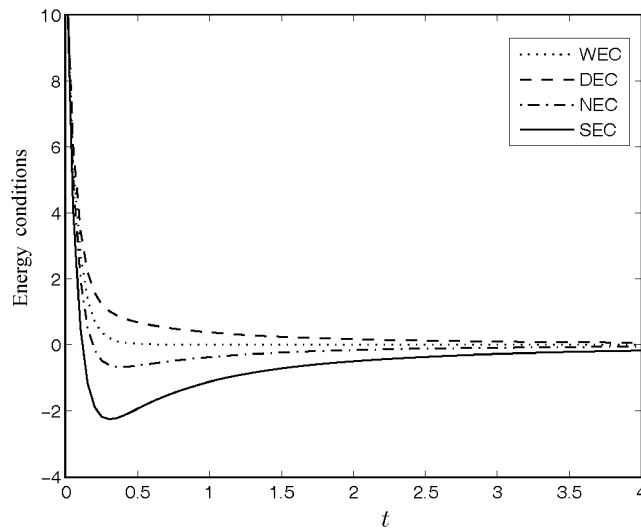
$$\dot{V} = \sqrt{2[\in -U(V)]} , \tag{49}$$

where $U(V)$ and $\in$ are the classical potential of force $\boldsymbol{F}$ and amount of energy respectively.

From Equations (48) and (49), we obtain

$$U(V) = 2 \in -9D^2(nDt + c_1)^{\frac{6-2n}{n}} . \tag{50}$$

**Fig. 3** Plot of proper energy density ($\rho$), string tension density ($\lambda$) and particle energy density ($\rho_p$) vs. time.



**Fig. 4** The left hand side of energy conditions vs. time.

In connection with Hubble's parameter, the classical potential ($U$) is given by

$$U(V) = 2 \in -9D^{\frac{6}{n}} H^{\frac{2n-6}{n}} \,. \tag{51}$$

Figure 4 plots the left hand side of the energy conditions versus time. We observe that the weak energy condition and dominant energy condition are satisfied in the derived model. The null energy condition is violated in the early universe but it is eventually satisfied in the present universe. It
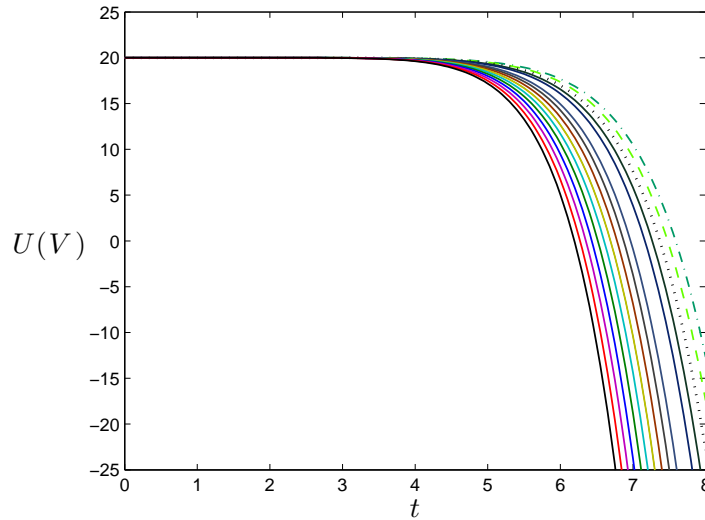
**Fig. 5** The classical potential vs. time for different values of $n$.

can also be observed that the strong energy condition (SEC) is violated in the derived model. The violation of SEC gives a reversal in the gravitational effect which may be a possible cause for late time accelerated expansion of the universe. Thus due to all the energy conditions, one finds $\rho \geq 0$ and $\rho_p \geq 0$. Together with the fact that the sign of $\lambda$ is unrestricted, it may also take values that are positive, negative or zero. In the derived model, $\lambda$ starts with a positive value and approaches zero with the passage of time (Fig. 3).

Figure 5 plots the classical potential with respect to time in the presence of a string fluid as the source of matter. We observe that $U(V)$ shows positive and negative behaviors with respect to time.

For a physically viable model, the speed of sound $(v_s)$ should be less than the speed of light $(c)$. In gravitational units, we have taken $c = 1$, therefore the condition $0 \leq v_s \leq 1$ must be satisfied for a physically acceptable model of the universe.

From Equations (38) and (39), the speed of sound is given by

$$v_s^2 = \frac{dp}{d\rho} = \frac{6(2\omega d_2 - x_1^2)(nDt + c_1)^{\frac{-3-n}{n}} + 24(1-n)nD^2 - 8\alpha^2(nDt + c_1)^{\frac{1-n}{n}}}{24nD^2 + 6(x_1^2 + 2\omega d_2)(nDt + c_1)^{\frac{-3-n}{n}} + 24\alpha^2(nDt + c_1)^{\frac{1-n}{n}}}. \tag{52}$$

Figure 6 shows that the velocity of sound $(v_s)$ lies between 0 and 1.

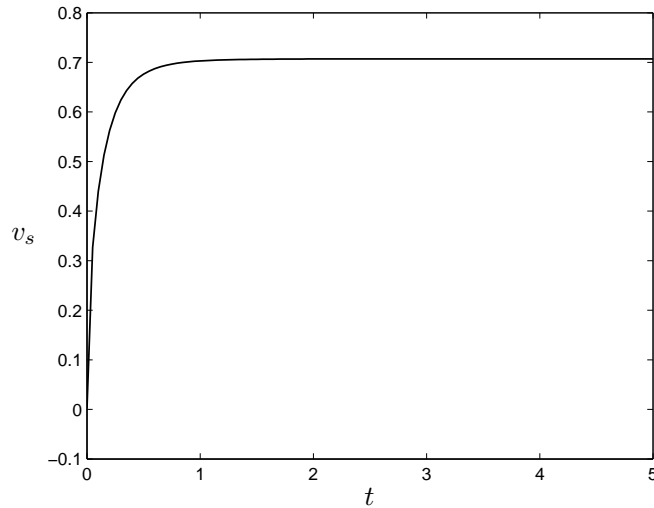We can express Equations (20)−(23) in terms of $H$, $q$ and $\sigma$ as

$$\rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2 = 3H^2 - \sigma^2 - \frac{3\alpha^2}{A^2}, \tag{53}$$

$$p - \frac{1}{2}\omega\phi^r\dot{\phi}^2 - \frac{\lambda}{3} = (2q - 1)H^2 - \sigma^2 + \frac{\alpha^2}{A^2}. \tag{54}$$

From Equations (53) and (54), we obtain

$$\frac{\ddot{a}}{a} = \frac{\lambda}{3} + \frac{1}{3}\omega\phi^r\dot{\phi}^2 - \frac{2}{3}\sigma^2 - \frac{1}{6}(\rho + 3p). \tag{55}$$

This is Raychaudhuri's equation for a given distribution and it is identically satisfied by the solution presented in this paper.

**Fig. 6** The speed of sound vs. time.

## 5  CONCLUSIONS

In this paper, we have studied a Bianchi V string cosmological model in the scalar-tensor theory of gravitation. The study reveals that the sting tension density $(\lambda)$ vanishes at the present epoch, which is why strings disappear from the present universe but they were playing a significant role in the expansion of the early universe. The derived model is singular in nature and it has a Big Bang singularity at $t = -\frac{c_1}{nD}$. Thus the universe starts evolving from the infinite Big Bang singularity at $t = -\frac{c_1}{nD}$ and expands with a power law expansion rate. The spatial volume is zero at initial moment $t = -\frac{c_1}{nD}$. At this instant, the physical parameters $\rho$, $p$, $\lambda$, $\rho_p$, $H$ and $\sigma$ all assume infinite values. These parameters are a decreasing function of time and ultimately tend to zero for a sufficiently large value of time. The spatial volume tends to zero as $t \to \infty$. Thus, the universe is essentially an empty space-time for large $t$.

The age of the universe is given by

$$T = \frac{1}{(q+1)} H^{-1} - \frac{c_1}{(q+1)D}.$$

Thus the age of the universe increases with $-1 < q < 0$ which shows the derived model is consistent with observations.

We have also discussed the classical potential with respect to time and have observed that the classical potential changes its nature with evolution of the universe. In the early universe, it is found to be positive and grows at a constant rate, but at late times, it is dominated by a negative value and decreases rapidly with time.

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