

## A transitioning universe with time varying $G$ and decaying $\Lambda$

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**Abstract** We present a model of a universe that transitions from an early deceleration phase to the current acceleration phase under the framework of general relativity, in the presence of gravitational coupling  $G(t)$  and cosmological terms  $\Lambda(t)$ . Einstein's field equations have been solved by considering the time dependent deceleration parameter (DP) which renders the scale factor  $a = (t^n e^{kt})^{\frac{1}{m}}$  where  $m$ ,  $n$  and  $k$  are positive constants. The cosmological term ( $\Lambda(t)$ ) is found to be positive and a decreasing function of time, which supports the result obtained from observations of type Ia supernovae. The geometrical and kinematical features of the model are examined in detail.

**Key words:** early universe — large scale structure of universe

### 1 INTRODUCTION

The most striking discovery of modern physics is that the current universe is not only expanding but also accelerating. The late time accelerated expansion of the universe has been confirmed by observations of type Ia supernovae (SNe Ia) (Riess et al. 1998; Perlmutter et al. 1997). Observations also suggest that there has been a transition of the universe from an earlier deceleration phase to the current acceleration phase (Caldwell et al. 2006). The recent measurements of cosmic microwave background (CMB) anisotropy and the observations from SNe Ia demand a significant, positive cosmological constant (Perlmutter et al. 1997; Fujii 2000). In addition, observations of gravitational lensing indicate the presence of a non-zero  $\Lambda$ .

The cosmological term  $\Lambda$  and the gravitational coupling  $G$  are assumed to be constants in Einstein's theory of general relativity. However, alternative ideas about the variability of these parameters were started long ago. The idea of variable  $G$  was first introduced by Dirac (1937), though Dirac's arguments were based on cosmological considerations not directly related to Mach's principle. Later, Brans & Dicke (1961) formulated the scalar-tensor theory of gravitation which is based on the coupling between an adequate tensor field and a scalar field  $\phi$ , having the dimension of  $G^{-1}$ . Motivated by dimensional grounds with quantum cosmology, Chen & Wu (1990) considered the variation of the cosmological term to be  $\Lambda \propto R^{-2}$ . However, a number of authors argued in favor of the dependence  $\Lambda \propto t^{-2}$ . Subsequently, Arbab (2003) investigated cosmic acceleration with a positive cosmological constant. A positive cosmological constant helps overcome the age problem, connected, on one hand, with high estimates of the Hubble parameter and, on the other hand, with the age of globular clusters. Further, it seems that in order to retain the cold dark matter theory in a spatially flat universe, most of the critical density should be provided by a positive cosmological

constant (Efstathiou et al. 1990; Kofman et al. 1993). Observational data indicate that the cosmological constant, if nonzero, is smaller than  $10^{-55} \text{ cm}^{-2}$ . However, since everything that contributes to the vacuum energy acts as a cosmological constant, it cannot just be dropped without serious considerations. Moreover, expectations for  $\Lambda$  from particle physics exceed its present value by a factor of order  $10^{120}$ , in sharp contrast to observations. To explain this apparent discrepancy, the point of view has been adopted which allows the  $\Lambda$ -term to vary with time (Salim & Waga 1993; Matyjasek 1995). The idea is that during the evolution of the universe, the energy density of the vacuum decays into particles, thus leading to a decrease in the cosmological constant. As a result, particles are created, although the typical rate of creation is very small.

An anisotropic Bianchi type V cosmological model plays a significant role in understanding phenomena like the formation of galaxies during the early stage of evolution. The choice of anisotropic cosmological models permits one to obtain a more general cosmological model, in comparison to the Friedmann-Robertson-Walker (FRW) model. Theoretical arguments and recent observations of the CMB support the existence of an anisotropic phase that approaches an isotropic one. Therefore, it makes sense to consider models of the universe with an anisotropic background in the presence of gravitational coupling  $G$  and a cosmological term  $\Lambda$ . Among different anisotropic cosmological models, a Bianchi type V universe is a natural generalization of the open FRW model. Lorenz (1981) and Lorenz-Petzold (1985) investigated a tilted Bianchi type V cosmological model with matter and an electromagnetic field in higher dimensions. A large number of authors have studied the Bianchi type V cosmological model in different contexts (Beesham 1986; Banerjee & Sanyal 1988; Nayak & Sahoo 1989, 1996; Coley 1990; Singh & Singh 1991; Coley & Dunn 1992; Pradhan & Rai 2004). Singh & Chaubey (2006) initially considered a Bianchi type V universe as being a self consistent system with a gravitational field that is a binary mixture of perfect fluid and dark energy given by a cosmological constant. Furthermore, they have studied the evolution of a homogeneous anisotropic universe filled with viscous fluid, in the presence of the cosmological constant  $\Lambda$  (2007). Singh & Kale (2009) and recently Yadav et al. (2012) discussed anisotropic bulk viscous cosmological models with variable  $G$  and  $\Lambda$ .

In this paper, we present a model of the transitioning universe with  $G(t)$  and  $\Lambda(t)$ . To study the transition behavior of the universe, we assume the scale factor to be an increasing function of time which generates a time dependent deceleration parameter (DP). This paper is organized as follows. In Section 2, the model and field equations are presented. Section 3 deals with the scale factors and cosmological parameters. Finally the conclusions are presented in Section 4.

## 2 MODEL AND FIELD EQUATIONS

We consider space-time admitting a Bianchi type V group of motion to be in the form

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x} (B^2 dy^2 + C^2 dz^2) , \quad (1)$$

where  $A(t)$ ,  $B(t)$  and  $C(t)$  are the scale factors in the  $x$ ,  $y$  and  $z$  directions and  $\alpha$  is a constant.

The average scale factor ( $a$ ) and spatial volume of the Bianchi type V metric are given by

$$a = (ABC)^{\frac{1}{3}} , \quad (2)$$

$$V = a^3 = ABC . \quad (3)$$

The generalized mean Hubble's parameter ( $H$ ) is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3} (H_x + H_y + H_z) , \quad (4)$$

where  $H_x = \frac{\dot{A}}{A}$ ,  $H_y = \frac{\dot{B}}{B}$  and  $H_z = \frac{\dot{C}}{C}$  are the directional Hubble's parameters. An overdot denotes differentiation with respect to cosmic time  $t$ .

Since metric (1) is completely characterized by the average scale factor, let us consider that the average scale factor is increasing as a function of time by the following

$$a = (t^n e^{kt})^{\frac{1}{m}}, \tag{5}$$

where  $k \geq 0$ ,  $m > 0$  and  $n \geq 0$  are constant. It is important to note here that the ansatz for the scale factor generalized the one proposed by Yadav (2012a,b) and Pradhan & Amirhashchi (2011). Yadav (2012a,b) considered string and bulk viscous fluid to be the source of matter that describes the transition behavior of the universe, whereas Pradhan & Amirhashchi (2011) studied a dark energy model with a variable equation of state parameter. In this paper, we consider cosmic fluid filled with  $G(t)$  and  $\Lambda(t)$  to be the source of matter to describe the transition of the universe from the early decelerating phase to the current accelerating phase.

The value of DP ( $q$ ) for model (1) is found to be

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{mn}{(n + kt)^2}. \tag{6}$$

Equation (6) clearly indicates the time varying nature of DP ( $q$ ). Amendola (2003) and Riess et al. (2001) found that the expansion of the universe is accelerating at the present epoch, but it was decelerating in the past. It is however possible to have  $n = 0$  in Equation (5) for which we would have an inflationary universe. The sign of  $q$  indicates whether the model inflates or not. A positive sign of  $q$ , i.e.  $t \leq \frac{1}{k} [\sqrt{mn} - n]$ , corresponds to the standard decelerating model whereas the negative sign  $-1 \leq q < 0$  indicates inflation.

Einstein’s field equations read as

$$R_j^i - \frac{1}{2}g_j^i R - \Lambda g_j^i = -8\pi G T_j^i, \tag{7}$$

where  $T_j^i$  is the energy momentum tensor, which is given by

$$T_j^i = (\rho + p)v^i v_j - p g_j^i, \tag{8}$$

where  $\rho$  is the energy density,  $p$  is the isotropic pressure of the cosmic fluid, and  $v^i$  is the fluid four velocity vector. In the co-moving system of co-ordinates, we have  $v^i = (1, 0, 0, 0)$ .

Einstein’s field Equation (7) for the line element (1) leads to the following system of equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -8\pi G \gamma \rho + \Lambda, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -8\pi G \gamma \rho + \Lambda, \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -8\pi G \gamma \rho + \Lambda, \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = \rho + \Lambda, \tag{12}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{13}$$

Here, we have assumed, as usual, an equation of state  $p = \gamma\rho$ , where  $0 \leq \gamma \leq 1$  is constant.

The shear scalar ( $\sigma$ ) is obtained as

$$\sigma^2 = \frac{1}{3} \left[ \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \left( \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \right) \right]. \tag{14}$$

Equations (4), (12) and (14) allow us to write the analog of the Friedmann equation as

$$3H^2 = 8\pi G\rho + \sigma^2 + \Lambda + \frac{3\alpha^2}{A^2}. \quad (15)$$

Here, we obtain the same equations as in the case of constant  $G$  and  $\Lambda$ ; therefore the variability of  $G$  and  $\Lambda$  does not affect the equations. Finally, the generalized conservation equation can be obtained using Equations (9)–(11) in the differentiated form of Equation (12) and can be written as

$$8\pi G [\dot{\rho} + 3(1 + \gamma)\rho H] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0. \quad (16)$$

We assume that the conservation of energy momentum tensor of matter holds ( $T^i_j{}_{;j} = 0$ ) leading to

$$\dot{\rho} + 3(1 + \gamma)\rho H = 0, \quad (17)$$

leaving  $G$  and  $\Lambda$  as coupled fields

$$8\pi\rho\dot{G} + \Lambda = 0. \quad (18)$$

### 3 THE SCALE FACTORS AND COSMOLOGICAL PARAMETERS

Integrating Equation (13) and absorbing the constant of integration into  $B$  or  $C$ , we obtain

$$A^2 = BC. \quad (19)$$

From Equations (9)–(11) and taking the second integral of each, we obtain the following three relations, respectively,

$$\frac{A}{B} = b_1 \exp\left(x_1 \int a^{-3} dt\right), \quad (20)$$

$$\frac{A}{C} = b_2 \exp\left(x_2 \int a^{-3} dt\right), \quad (21)$$

$$\frac{B}{C} = b_3 \exp\left(x_3 \int a^{-3} dt\right), \quad (22)$$

where  $b_1, b_2, b_3, x_1, x_2$  and  $x_3$  are constants of integration.

From Equations (19)–(22) and (5), the metric functions can be explicitly written as

$$A(t) = (t^n e^{kt})^{\frac{1}{m}}, \quad (23)$$

$$B(t) = d (t^n e^{kt})^{\frac{1}{m}} \exp\left(\ell \int (t^n e^{kt})^{-\frac{3}{m}} dt\right), \quad (24)$$

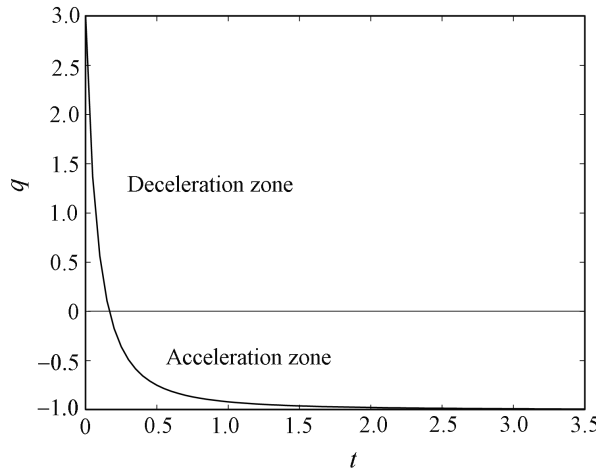
$$C(t) = d^{-1} (t^n e^{kt})^{\frac{1}{m}} \exp\left(-\ell \int (t^n e^{kt})^{-\frac{3}{m}} dt\right), \quad (25)$$

where  $d = (b_2 b_3)^{\frac{1}{3}}$  and  $\ell = \frac{x_2 + x_3}{3}$  with  $b_2 = b_1^{-1}$  and  $x_2 = -x_1$ .

Integrating Equation (17), we obtain

$$\rho = \rho_0 (t^n e^{kt})^{-\frac{3(1+\gamma)}{m}}, \quad (26)$$

where  $\rho_0$  is the positive constant of integration.



**Fig. 1** The plot of DP ( $q$ ) versus time ( $t$ ) with  $m = 1$ ,  $n = 0.25$  and  $k = 1.5$ .

The physical parameters such as the directional Hubble parameters ( $H_x, H_y, H_z$ ), average Hubble parameter ( $H$ ), shear scalar ( $\sigma$ ), expansion scalar ( $\theta$ ) and spatial volume ( $V$ ) are given by

$$H_x = \frac{1}{m} \left( \frac{n}{t} + k \right), \tag{27}$$

$$H_y = \frac{1}{m} \left( \frac{n}{t} + k \right) + \ell (t^n e^{kt})^{-\frac{3}{m}}, \tag{28}$$

$$H_z = \frac{1}{m} \left( \frac{n}{t} + k \right) - \ell (t^n e^{kt})^{-\frac{3}{m}}, \tag{29}$$

$$H = \frac{1}{m} \left( \frac{n}{t} + k \right), \tag{30}$$

$$\sigma^2 = \ell (t^n e^{kt})^{-\frac{6}{m}}, \tag{31}$$

$$\theta = \frac{3}{m} \left( \frac{n}{t} + k \right). \tag{32}$$

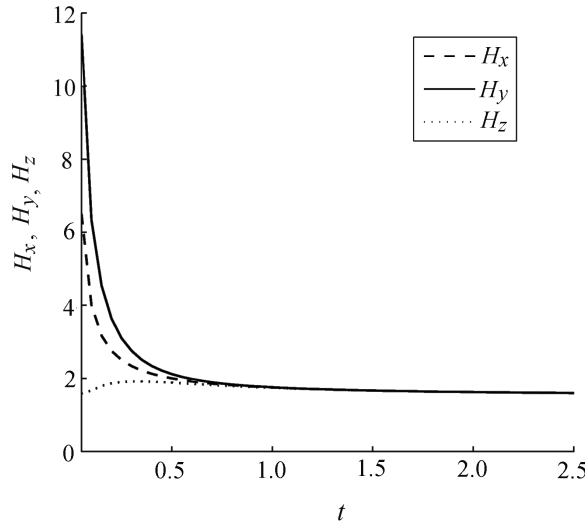
Equations (31) and (32) lead to

$$\frac{\sigma}{\theta} = \frac{\sqrt{\ell} m}{3} (t^n e^{kt})^{-\frac{3}{m}} \left( \frac{n}{t} + k \right)^{-1}. \tag{33}$$

From Equation (33), we observe that  $\lim_{t \rightarrow \infty} \left( \frac{\sigma}{\theta} \right) = 0$ . Thus the derived model approaches isotropy at the present epoch.

Figure 1 shows the dynamics of the DP from the early deceleration phase to the recent acceleration phase, whereas Figure 2 ensures that  $H_x, H_y$  and  $H_z$  evolve at an equal rate for late times, so the universe achieves isotropy at the present epoch.

It is observed that the scale factors  $A(t), B(t)$  and  $C(t)$ , along the spatial directions  $x, y$  and  $z$  respectively, vanish at  $t = 0$ . Thus the model has a point type singularity at  $t = 0$ . We obtain  $q = -1$  and  $\frac{dH}{dt} = 0$  as  $t \rightarrow \infty$ . The model under consideration has a time dependent DP and evolves to isotropy as  $t \rightarrow \infty$ , with  $\Lambda \rightarrow 0$ . Thus for large time, the model approaches the flat FRW model, which is very encouraging. It may be noted that, although current observations of SNe Ia and CMB



**Fig. 2** The plot of directional Hubble parameters versus time with  $m = 1, n = 0.25, k = 1.5$  and  $\ell = 0.65$ .

favor the accelerating models  $q < 0$ , they do not altogether rule out the decelerating ones which are also consistent with these observations. It is possible to fit the model with zero  $\Lambda$ , considering the extinction of light by metallic dust ejected from supernova explosions (Vishwakarma 2003).

The cosmological constant ( $\Lambda$ ) and gravitational constant ( $G$ ) are found to be

$$\Lambda = \frac{3}{m^2} \left( \frac{n}{t} + k \right)^2 - \ell^2 (t^n e^{kt})^{-\frac{6}{m}} - 3\alpha^2 (t^n e^{kt})^{-\frac{2}{m}} - \rho_0 (t^n e^{kt})^{-\frac{3(1+\gamma)}{m}}, \quad (34)$$

$$G = \frac{m}{24\pi(1+\gamma)} (t^n e^{kt})^{\frac{3(1+\gamma)}{m}} \left[ \frac{6n}{m^2 t^2} - \frac{6(\ell^2 + \alpha^2)}{m} t^n e^{kt} \right]. \quad (35)$$

From Equation (34), it is observed that the cosmological constant ( $\Lambda$ ) is a decreasing function of time. This behavior is clearly shown in Figure 3. Recent cosmological observations suggest the existence of a positive cosmological constant ( $\Lambda$ ) with magnitude  $\Lambda \left( \frac{G\hbar}{c^3} \right) \approx 10^{-123}$ . These observations on magnitude and redshift of SNe Ia suggest that our universe may be accelerating with induced cosmological density described by the cosmological  $\Lambda$ -term. Thus the model presented in this paper is consistent with the results of recent observations.

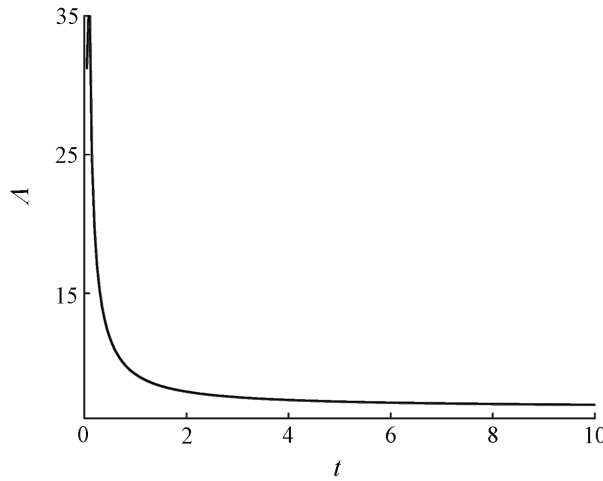
We can express Equations (9)–(12) in terms of  $H, q$  and  $\sigma$ , as

$$8\pi G\gamma\rho - \Lambda = (2q - 1)H^2 - \sigma^2 + \frac{\alpha^2}{a^2}, \quad (36)$$

$$8\pi G\gamma\rho - \Lambda = 3H^2 - \sigma^2 - \frac{3\alpha^2}{a^2}. \quad (37)$$

Equations (36) and (37) lead to

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{2\sigma^2}{3} - \frac{1}{6} 8\pi G(\rho + 3p). \quad (38)$$



**Fig. 3** Cosmological constant ( $\Lambda$ ) versus time ( $t$ ) with  $m = 1$ ,  $n = 0.25$ ,  $k = 1.5$ ,  $\gamma = 0.33$ ,  $\alpha = 0.15$  and  $\rho_0 = 1$ .

From Equation (38), it is shown that for  $\rho + 3p = 0$ , only the  $\Lambda$ -term contributes to the acceleration, which seems to show a relation between  $\Lambda$  and dark energy. The same is predicted by observations from the supernova legacy survey.

#### 4 RESULTS AND SUMMARY

In this paper, we have presented a model of the transitioning universe with gravitational coupling  $G(t)$  and cosmological term  $\Lambda(t)$  in the framework of general relativity. The spatial scale factors and volume scalar of the derived model vanish at  $t = 0$ . The energy density and pressure are infinite at this initial epoch. As  $t \rightarrow \infty$ , the scale factors diverge and  $\rho$  tends to zero. The shear scalar ( $\sigma$ ) is very large at the initial moment but decreases with cosmic time and vanishes as  $t \rightarrow \infty$ . The model leads to an isotropic state during the later time of its evolution. For  $n \neq 0$ , all matter and radiation are concentrated in the Big Bang and the model has a point type singularity at the initial moment. For  $n = 0$ , the universe has a non singular origin which seems reasonable for predicting the dynamics of the future universe. In the derived model,  $\lim_{t \rightarrow 0} \frac{\rho}{\theta^2}$  turns out to be constant. Thus matter is dynamically negligible near the origin and the model approaches homogeneity.

The cosmological constant ( $\Lambda$ ) is found to be a decreasing function of time and it approaches a small positive value at late time. A positive value of  $\Lambda$  corresponds to the negative effective mass density (repulsion). Hence we expect that in a universe with a positive value of  $\Lambda$ , the expansion tends to accelerate. Thus the derived model predicts an accelerating universe at the present epoch.

The age of the universe, in the derived model, is given by

$$T_0 = \frac{n}{m} H_0^{-1} - k,$$

which differs from the present estimate, i.e.  $T_0 = H_0^{-1} \approx 14$  Gyr. But if we take  $n = m$  and  $k = 0$ , the model is in good agreement with the present age of the universe.

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