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On the Lorentz factor of superluminal sources

Chika Christian Onuchukwu^{1,2} and Augustine A. Ubachukwu²

¹ Department of Industrial Physics, Anambra State University Uli, Nigeria; onuchukwu71chika@yahoo.com

² Department of Physics and Astronomy, University of Nigeria Nsukka, Nigeria

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Abstract We investigate the properties of features seen within superluminal sources often referred to as components. Our result indicates a fairly strong correlation of $r \sim 0.5$ for quasars, $r \sim 0.4$ for galaxies and $r \sim 0.7$ for BL Lac objects in our sample between component sizes and distances from the stationary core. The assumption of free adiabatic expanding plasma enables us to constrain the Lorentz factor for superluminal sources. Our estimated Lorentz factor of $\gamma \sim 9-13$ for quasars, $\gamma \sim 7-11$ for galaxies and $\gamma \sim 4-9$ for BL Lac objects indicates that BL Lacs have the lowest range of Lorentz factors.

Key words: galaxies: general — galaxies: active — galaxies: jets — methods: analytical — methods: statistical — methods: data analysis

1 INTRODUCTION

A large number of superluminal motions have been observed in jets from numerous classes of astrophysical objects using Very Long Baseline Interferometry (VLBI). The most prominent sources of these jets are Active Galactic Nuclei (AGNs), where the jet emission contributes significantly to the spectrum of the source (Körding & Falcke 2004). These extragalactic radio jets have been intensively studied for a number of important reasons. These include their association with a supermassive accreting black hole, which plays a substantial role in the formation and evolution of galaxies – the building blocks of the universe. Studies and applications of the astrophysics of these radio jets can help reveal black holes' energy and interactions with their environments. Understanding the formation, collimation, acceleration, and propagation of these radio jets will aid our understanding of the laws of physics under extreme conditions, such as relativistic speed and high magnetic field strength.

In recent years, VLBI observations of AGN jets (e.g. see Kellermann et al. 2004; Britzen et al. 2007, 2008; the MOJAVE program; the TANAMI project) have enabled morphological studies and motions of features (often referred to as components) seen within these jets. These observations provide us with many values of the apparent luminosity, size, radial distance away from the core and proper motion/apparent transverse speed of components moving relativistically along the jets. Though these quantities are of considerable interest, the intrinsic properties (Lorentz factor – bulk speed/pattern speed, angle with respect to the line of sight) are more fundamental and necessary for kinematic characterization of these sources.

The jets can, at the simplest level, be modeled by two intrinsic properties, Lorentz factor (γ) and angle with respect to the line of sight (θ). Homan (2012) noted that by studying the apparent

speed distribution of a large, flux-density limited sample of AGN jets, we probe the underlying Lorentz factor distribution of the parent population, assuming that the apparent speeds of moving jet components are good tracers of the underlying flow velocity of the beam (Vermeulen & Cohen 1994; Lister & Marscher 1999).

2 THEORY DESCRIBING THE RELATIONSHIP

In the relativistic beam model of extragalactic radio sources (EGRS), the cores of these sources are believed to be responsible for the ejection of the observed VLBI jet components, and thus observed properties are expected to be correlated. As these components are ejected, they move away from the core. Assuming ballistic motion, the observed radial distance (D) from the core should be fore-shortened due to the combined relativistic motion and orientation effect, thus we can write (see Ubachukwu 1998)

$$D = D_0 \sin \theta, \tag{1}$$

where D_0 is the intrinsic length of the jet component from the core in its rest frame, and θ is the viewing angle. From the orientation based argument, the observed apparent speed β_a with which these components move out from the core can be defined as

$$\beta_{\rm a} = \gamma \beta \delta \sin \theta, \tag{2}$$

where δ is the Doppler factor defined by $\delta = \gamma^{-1}(1 - \beta \cos \theta)^{-1}$ and γ is the Lorentz factor, which is related to β , the bulk speed defined in units of c (the speed of light) by $\gamma = (\sqrt{1 - \beta^2})^{-1}$. The maximum apparent speed $(\beta_a)_{max}$ for a given Lorentz factor γ occurs at a critical angle θ_c given by $\beta = \cos \theta_c$ which implies that $\sin \theta_c = \gamma^{-1}$. The Doppler factor for a given Lorentz factor reaches its maximum for $\beta = \cos \theta_c$ and is given by $\delta = \gamma$. Equation (2) can thus be written approximately as

$$(\beta_{\rm a})_{\rm max} \approx \beta \gamma.$$
 (3)

Thus the time (t_D) taken for the component to move away from the core to an observed maximum radial distance D_{max} is given by

$$t_{\rm D} = \frac{D_{\rm max}}{\beta\gamma}.\tag{4}$$

Kovalev et al. (2005) noted that most new jet features typically increase in size and/or decrease in flux density as a result of adiabatic expansion and/or synchrotron losses. Thus, as these components move away from the stationary core, they are expected to expand sideways. Assuming free adiabatic expansion, we follow previous studies (e.g. de Young 2002; Körding & Falcke 2004) to define the expansion speed (which is expected to equal the sound speed) as

$$\beta_{\rm s} = \frac{1}{\sqrt{\frac{1}{\Gamma - 1} + \frac{m_{\rm p} n_{\rm tot} c^2}{\Gamma P}}},\tag{5}$$

where $m_{\rm p}$ is the proton mass, $n_{\rm tot}$ is the total number density of particles, Γ is the adiabatic index, and P is the thermodynamic equilibrium pressure. The adiabatic index is usually taken to be ~ 4/3 or ~ 5/3 for relativistic and non-relativistic expansion respectively. Körding & Falcke (2004) showed that for photon emitting plasma (which is what we expect from a radio component) the maximal expansion speed will be

$$\beta_{\rm s,max} = \sqrt{\Gamma - 1}.\tag{6}$$

Thus the time (t_R) taken for the source to reach an observed maximum size R_{max} is given by

$$t_{\rm R} = \frac{R_{\rm max}}{\sqrt{\Gamma - 1}}.$$
(7)

For a free ballistic adiabatic expanding gas in an isotropic environment, with constant expansion speed and linear speed away from the core, the time to expand to the maximum size $t_{\rm R}$ should correlate with the time to reach the maximum radial distance $t_{\rm D}$. Thus, from Equations (4) and (7) we have

$$\frac{R_{\max}}{\sqrt{\Gamma - 1}} = \frac{D_{\max}}{\sqrt{\gamma^2 - 1}}.$$
(8)

By implication, a linear regression fit to Equation (8) in the form

$$\log D_{\max} = \log R_{\max} + \log \sqrt{\left(\frac{\gamma^2 - 1}{\Gamma - 1}\right)} \tag{9}$$

will enable us estimate the average Lorentz factor and thus place some useful constraint on the angle with respect to the line of sight to the observer. Also, the $\log D_{\max} - \log R_{\max}$ plot is expected to yield a slope of unity, which can be tested using a well-defined source sample. We choose a logarithmic form of the relationship due to the wide spread in R_{\max} and D_{\max} .

3 DATA ANALYSIS/RESULT

Our analyses were based on the Caltech-Jodrell Bank Flat-Spectrum Sources (CJF) defined by Taylor et al. (1996) which is a complete flux-limited VLBI sample of 293 flat spectrum radio sources drawn from the 6 cm and 20 cm Green Bank Surveys. Of the 293 sources in the original sample, we obtained information on 799 jet components for 237 sources from Britzen et al. (2007, 2008), with at least one observed component's apparent velocity for each source, and other parameters like proper motion, distance of each component with respect to a stationary core and sizes (major axis) of each component. This sample is large, and the observational strategy and data reduction are homogeneous. Thus, given its size, completeness, and range of observed source properties, it will allow a meaningful statistical investigation of apparent motions with many correlations between other parameters.

In our analysis, we assumed that a source contains an ideal beam – one that is straight and narrow, with the pattern speed β_p equal to the bulk speed β_b . Many sources, however, are seen to have more than one moving component, with different values of the apparent transverse speed β_a . We selected the component with the fastest speed for sources with more than one component, believing that it is the component most likely to have the same speed as the beam (see Cohen et al. 2007) and would have reached a maximum distance for a given time. Moreover, Kellermann et al. (2004) suggested that there is a characteristic speed for each jet which is probably reflected by the fastest speed observed for the jet component. The final sample consists of 177 quasars, 41 galaxies and 19 BL Lac objects. However, Britzen et al. (2008) noted that selecting the components based on the brightest components is a good representative of the sample and adequate for population studies. Thus, in this paper, we will compare our results obtained from selection based on the fastest components, with those based on the brightest components. We point out that sources with only one observed component are represented in both selections by that component.

Assuming the radius of the semi-major axis of each source represents the size, and transforming the angular distance of each component from the stationary core to a linear distance, for the selection based on fastest/brightest components, we estimated the Lorentz factor of each source from Equation (8) assuming $4/3 \le \Gamma \le 5/3$. The histogram plots of the estimated Lorentz factor for each source in each class of objects are shown in Figures 1–6. Generally, the distributions are similar for $\Gamma = 4/3$ and $\Gamma = 5/3$ for each class of objects, though they are shifted to higher values for $\Gamma = 5/3$ and are generally positively skewed. The median value of the Lorentz factor is estimated for each class of object for $4/3 \le \Gamma \le 5/3$. For selection based on fastest components, the values are $\sim 9 \pm 2 - 11 \pm 2$; $\sim 6 \pm 1 - 10 \pm 2$; and $\sim 4 \pm 1 - 6 \pm 1$ respectively for quasars, galaxies and BL



Fig. 1 Histogram plot of estimated Lorentz factor for the selection based on fastest components for quasars (*solid line* for $\Gamma = 5/3$; *dashed line* for $\Gamma = 4/3$.)



Fig. 3 Histogram plot of estimated Lorentz factor for the selection based on fastest components for galaxies (*solid line* for $\Gamma = 5/3$; *dashed line* for $\Gamma = 4/3$).



Fig. 2 Histogram plot of estimated Lorentz factor for the selection based on brightest components for quasars (*solid line* for $\Gamma = 5/3$; *dashed line* for $\Gamma = 4/3$).



Fig. 4 Histogram plot of estimated Lorentz factor for the selection based on brightest components for galaxies (*solid line* for $\Gamma = 5/3$; *dashed line* for $\Gamma = 4/3$).

Lac objects. Similarly, for the selection based on brightest components we have $\sim 6 \pm 1 - 10 \pm 2$; $\sim 5 \pm 1 - 8 \pm 1$ and $\sim 5 \pm 1 - 8 \pm 1$ for quasars, galaxies and BL Lac objects respectively for $4/3 \leq \Gamma \leq 5/3$. The quoted errors were estimated from the observed error associated with the parameters obtained from Britzen et al. (2007, 2008).

We also fitted the observed D - R data to Equation (9). The result gives: $\log D = (0.7 \pm 0.3) \log R + 1.2 \pm 0.1$ for quasars; $\log D = (0.3 \pm 0.5) \log R + 1.1 \pm 0.1$ for galaxies; and $\log D = (0.6 \pm 0.4) \log R + 0.9 \pm 0.1$ for BL Lac objects. This corresponds to a distribution in Lorentz factor of: $\gamma \sim 7 - 17$ for quasars; $\gamma \sim 6 - 13$ for galaxies; and $\gamma \sim 4 - 9$ for BL Lac objects for $\frac{4}{3} \le \Gamma \le \frac{5}{3}$, with correlation coefficients between D - R of $r \sim 0.6, 0.4$, and 0.8 respectively; there



Fig. 5 Histogram plot of estimated Lorentz factor for the selection based on fastest components for BL Lacs (*solid line* for $\Gamma = 5/3$; *dashed line* for $\Gamma = 4/3$).



Fig.7 Plot of $\log D_{\max}$ against $\log R_{\max}$ for the fastest components with error bars representing standard deviation (legend: quasars – *red diamonds*; galaxies – *blue circles*; BL Lacs – *green squares*).



Fig. 6 Histogram plot of estimated Lorentz factor for the selection based on brightest components for BL Lacs (*solid line* for $\Gamma = 5/3$; *dashed line* for $\Gamma = 4/3$).



Fig. 8 Plot of $\log D_{\max}$ against $\log R_{\max}$ for the brightest components with error bars representing standard deviation (legend: quasars – *red diamonds*; galaxies – *blue circles*; BL Lacs – *green squares*).

is a fairly strong correlation, especially for quasars and BL Lac objects. The spread in the Lorentz factor results also implies a distribution in the critical angle with respect to the line of sight θ_c of $\sim 3^\circ - 8^\circ, 4^\circ - 10^\circ$, and $6^\circ - 16^\circ$ for quasars, radio galaxies and BL Lacs respectively.

Similar analyses using selection based on brightest components yield: $\log D = (0.7 \pm 0.3) \log R + 1.1 \pm 0.1$ for quasars; $\log D = (0.9 \pm 0.4) \log R + 1.0 \pm 0.1$ for galaxies; and $\log D = (0.1 \pm 0.4) \log R + 0.5 \pm 0.2$ for BL Lac objects, with correlation coefficients $r \sim 0.6, 0.5$ and 0.3 respectively. The corresponding Lorentz factors for different classes of sources are: $\gamma \sim 5 - 12$ for

quasars; $\gamma \sim 5-9$ for galaxies; and $\gamma \sim 2-5$ for BL Lac objects for $\frac{4}{3} \leq \Gamma \leq \frac{5}{3}$, corresponding to a distribution in θ_c of $\sim 5^\circ - 10^\circ$; $\sim 6^\circ - 12^\circ$; and $\sim 12^\circ - 36^\circ$ for quasars, radio galaxies and BL Lac objects respectively. The plot of $\log D_{\max}$ against $\log R_{\max}$ for the fastest components is shown in Figure 7, while the plot of $\log D_{\max}$ against $\log R_{\max}$ for the brightest components is shown in Figure 8. In general, the Lorentz factor obtained using the fastest components is greater than that obtained using the brightest components. We emphasize that values estimated using the selection based on brightest components may represent the lower limits of the Lorentz factors for each class of objects, and consequently the upper limit of the viewing angle.

4 DISCUSSION AND CONCLUSIONS

We have used the relationship between the observed component's radial distance D and component size R to estimate the Lorentz factor γ and the corresponding angle with respect to the line of sight θ for a selection based on the fastest/brightest components. Our results show a stronger D – R correlation for selections based on the fastest components than for that based on the brightest components for quasars, radio galaxies and BL Lac sub-samples. Our results also indicate a lack of apparent correlation between D and R in the BL Lac data for selection based on the brightest component. This suggests that selection based on the brightest components may not be appropriate for the determination of the Lorentz factor or for characterizing superluminal motion for this class of objects. In general, the result on the analysis of the CJF sample obtained here showed that galaxies have lower intrinsic Lorentz factors (lower bulk speed) than quasars. There are few BL Lacs in the sample, but the result showed that BL Lacs have the lowest range of Lorentz factors, in agreement with those in previous studies (e.g. Gabuzda et al. 1994; Morganti et al. 1995; Kellermann et al. 2007). This is an indication that BL Lac objects are low power sources (Bicknell 1994). We note that Equation (9) yields a slope of unity for the plot of the R - D data. The results from our regression analysis approximate the theoretical value, especially for quasars, and in general for selection based on the fastest components. This is an indication that the underlying assumption is plausible. These results do show that selections based on the fastest component are a good indicator of the bulk speed.

We note that our estimated values for angle with respect to the line of sight, especially for the sample selected based on brightest components (especially for BL Lacs), are high (see Hovatta et al. 2009; Yang et al. 2012); however, Gopal-Krishna et al. (2006) showed that, allowing for the conical shape of ultra relativistic blazar jets with opening angles of a few degrees on parsec-scales, the actual viewing angles of these conical jets from the line-of-sight can be substantially larger than the values usually inferred by combining their flux-variability and proper motion measurements (Jorstad et al. 2005). This jet geometry also implies that de-projected jet opening angles will typically be significantly underestimated from VLBI measurements. Also, our estimated Lorentz factors are in general lower than those shown in table 3 of Hovatta et al. (2009), except for galaxies, though their sample was obtained at 22 and 37 GHz instead of 5 GHz for the CJF sample. Kellermann et al. (2004) noted that there is a systematic decrease in apparent velocity with increasing wavelength which indicates that observation at different wavelengths may be sampling a different part of the jet structure. Moreover, observations at lower frequency have lower angular resolution and thus are more sensitive to the lower surface brightness structure located downstream.

Further analysis of our result showed that for the sample selected based on brightest components, the apparent speed, component size, and component distance from the assumed stationary core are on average less than those of the sample selected based on fastest components, but with brighter apparent luminosity (see Table 1). They may be expected to have smaller and apparently brighter components, but if their motion is unimpeded, they should also, on average, have higher apparent speed. This is an indication that their apparent luminosity might be more of a projection effect/motion through bends than being relativistically Doppler boosted. Thus, the Lorentz factor calculated based on brightest components for a given sample will most likely represent the lower limit

		$\beta_{\rm a}$	Component Size (mas)	Component Luminosity $(W m^{-2} Hz^{-1}) \times 10^{25}$	Component Distance (pc)
Quasars	fastest	9.43	1.14	4.16	24.59
	brightest	5.40	0.69	6.87	13.65
Galaxies	fastest	4.43	1.30	1.45	25.26
	brightest	2.63	0.95	2.28	20.00
BL Lacs	fastest	4.52	1.35	2.40	10.09
	brightest	3.45	0.44	3.33	3.73

 Table 1
 Comparison of the Mean Values of some Parameters. (Between Sample selected based on Fastest Component and that selected based on Brightest Components)

of the bulk expansion speed, and the upper limit of the viewing angle, while that estimated from the fastest components will represent the most probable values (Gopal-Krishna et al. 2006; Wiita et al. (2008); Marshall et al. 2011). In Hovatta et al. (2009), their figure 11 shows that the $\gamma \sin \theta$ distribution for their sources peaks around ~1, though a number of sources have $\gamma \sin \theta$ being larger or smaller, thus the assumption of $\sin \theta = 1/\gamma$ seems reasonable.

Cohen et al. (1988) used a sample of compact radio quasars with measured redshift z and proper motion μ to obtain a result which indicated that the assumption of a relativistic beaming model gives a reasonable fit to their data with an upper bound of bulk Lorentz factor lying in the range $9 \le \gamma \le 18$. Pelletier & Roland (1989) believed that superluminal motion in powerful radio sources can be explained by a two-fluid model. In their modeling they estimated that relativistic electrons responsible for the synchrotron emission of VLBI jets are moving with Lorentz factor $3 < \gamma < 10$. In the analysis of flux variations in BL Lac objects, Mutel (1992) indicated that the Lorentz factor lies in the range $2 \le \gamma \le 4$. For a sample of 100 sources with published VLBI measurements of the core angular size, Ghisellini et al. (1993), assuming Synchrotron Self-Compton formalism, estimated the Doppler factor of these sources independent of superluminal motion. Using the estimated Doppler factor and observed superluminal speed β_a they constrained $\gamma = 10$. In VLBI observations of ~12 Flat Spectrum Radio Quasars (FSRQs) in ~2 Jy sample, Vermeulen & Cohen (1994) obtained a distribution of γ for the fastest superluminal components of FSRQs in the range of 5 – 35. Urry & Padovani (1995) showed that for their derived luminosity function of FSRQs to match the observed FSRQ luminosity function requires a distribution of Lorentz factor in the range of $5 \le \gamma \le 40$ with a mean value of $\langle \gamma \rangle = 11$. However, for BL Lac objects, a range of $2 \leq \gamma \leq 20$ and an average of $\langle \gamma \rangle = 3$ was needed for their luminosity function to match the observed value. They also obtained a critical angle value for maximum beaming of $\theta_c = 14^\circ$ for FSRQs and $\theta_c = 12^\circ$ or $\theta_c = 19^\circ$ for BL Lacs depending on the assumed model. These results generally agree with our result, especially the result obtained with the sample selected based on fastest components. The seemingly contradictory result for the BL Lac sample based on the brightest component may be interpreted as motion through bends with the increase in brightness being due to projection effects. The data are however too poor for any definite conclusion.

Miller-Jones et al. (2004) pointed out that for a tangled magnetic field with $B \propto R^{-1}$, having both synchrotron losses and adiabatic loses due to expansion, as the component moves out, size increases, thus we expect component luminosity to anti-correlate with both component size and component distance, except when being modified by projection/environmental effects. Homan et al. (2002) found that flux density of jet components fades from the core as $D^{-1.3}$. This supports our argument that time for component expansion should correlate with the time it takes for the components to move a radial distance away from the core. We should expect that an inverse correlation between component size R and component luminosity L, which is our underlying assumption, is correct. For the selection based on fastest components with a correlation coefficient, the results are r = -0.3, -0.2 and -0.3 for quasars, galaxies and BL Lacs respectively. These results indicate that selections based on fastest components are a good indicator of the bulk speed, though without any high level of significance.

We have shown that the expected increase in size as jet components move away from the core can be used to characterize the kinematics of these sources. This is true for free expanding jets beyond the collimation point. Jets, observed at a high frequency, may be sampling the upstream part (see Kellermann et al. 2004), thus a re-analysis of VLBI sources observed in a different frequency range may be used to place a limit on the point of collimation.

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