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Neutron star mass-radius relation with gravitational field shielding by a scalar field *

Bo-Jun Zhang[1,2], Tian-Xi Zhang[2,3], Padmaja Guggilla[2] and Mostafa Dokhanian[2]

[1] Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA; *bojun.zhang@vanderbilt.edu*
[2] Department of Physics, Alabama A & M University, Normal, AL 35762, USA; *tianxi.zhang@aamu.edu*
[3] National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China

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Abstract The currently well-developed models for equations of state (EoSs) have been severely impacted by recent measurements of neutron stars with a small radius and/or large mass. To explain these measurements, the theory of gravitational field shielding by a scalar field is applied. This theory was recently developed in accordance with the five-dimensional (5D) fully covariant Kaluza-Klein (KK) theory that has successfully unified Einstein's general relativity and Maxwell's electromagnetic theory. It is shown that a massive, compact neutron star can generate a strong scalar field, which can significantly shield or reduce its gravitational field, thus making it more massive and more compact. The mass-radius relation developed under this type of modified gravity can be consistent with these recent measurements of neutron stars. In addition, the effect of gravitational field shielding helps explain why the supernova explosions of some very massive stars (e.g., 40 $M_\odot$ as measured recently) actually formed neutron stars rather than black holes as expected. The EoS models, ruled out by measurements of small radius and/or large mass neutron stars according to the theory of general relativity, can still work well in terms of the 5D fully covariant KK theory with a scalar field.

Key words: stars: neutron — gravitation — black hole physics — equation of state

1 INTRODUCTION

A neutron star is a compact object, resulting from a supernova explosion of a massive star at the end of its stellar evolution. As the compact remnant of a supernova explosion, a neutron star has a mass of approximately 1.4 $M_\odot$ and a radius approximately within the range 10–20 km. It is formed from a progenitor star with a mass between 8 and 20 $M_\odot$. The existence of neutron stars in nature was theoretically predicted in 1934 (Baade & Zwicky 1934), but not observationally discovered until 1965 (Hewish & Okoye 1965). Some recent measurements of neutron star masses and radii are, however, outside of these normal ranges. These results challenge the currently well-developed models of neutron stars and theories of black hole formation.

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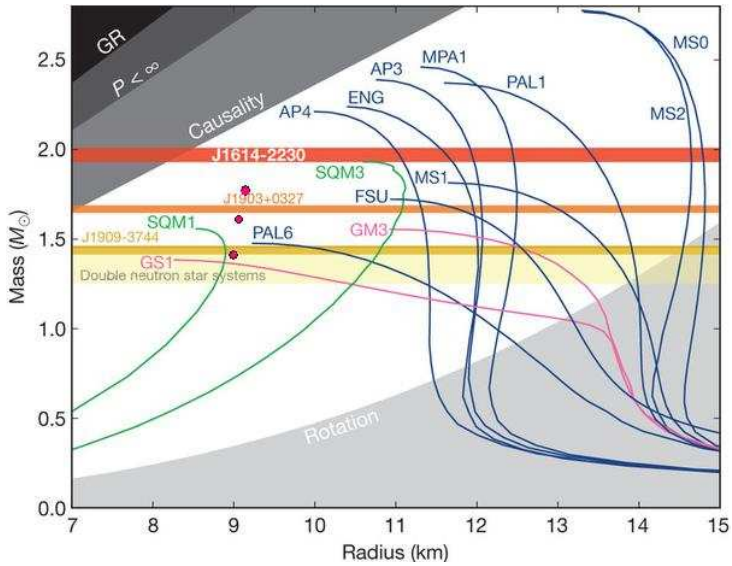

Fig. 1 Mass-radius relations of neutron stars from various EoS models. This figure is taken from Demorest et al. (2010) with three red dots, which were newly added to represent the recent measurements of the three neutron stars with a radius of about 9 km and a mass of about 1.4, 1.58 and 1.74 $M_\odot$, respectively, from Guillot et al. (2011), Güver et al. (2010a,b). The neutron stars with a mass of about 1.58 and 1.74 $M_\odot$ cannot fit any current EoS model.

A recent measurement of the Shapiro delay of light from the binary millisecond pulsar J1614–2230 has shown the mass of the neutron star to be about 2 $M_\odot$ (Demorest et al. 2010). This remarkable discovery of a two-solar-mass neutron star has ruled out, as declared by them, almost all currently proposed hyperon or boson condensate equations of state (EoSs) for neutron stars (Fig. 1). The models that have been ruled out by the two-solar-mass neutron star, as shown in Figure 1, include GS1, SQM1, PAL6, GM3, FSU, and MS1. For the models AP4, ENG, AP3, MPA1, PAL1, MS2 and MS0 to work, the radii of neutron stars must be in the range 11–15 km. Quark matter can support a compact object this massive only if the quarks strongly interact as shown by the model SQM3. However, asymptotic freedom is a property of quantum chromodynamics (QCD). The interaction between quarks should become arbitrarily small at such a short distance. In addition, in most theoretical models, a quark star should have already collapsed into a black hole before it ever reaches 2 $M_\odot$.

The spectral and timing analysis of data from some low-mass X-ray binaries has precisely measured the radii of some neutron stars to be only about 9 km (Guillot et al. 2011; Güver et al. 2010a,b), which challenges almost all current EoS models (see the three red dots that were newly added to Fig. 1). The radius of a 1.4 $M_\odot$ neutron star that represent part of U24 in NGC 6397 was measured to be about 8.9 km (Guillot et al. 2011); the radius of a 1.58 $M_\odot$ neutron star in 4U 1820–30 was measured to be about 9.1 km (Güver et al. 2010a); and the radius of a 1.74 $M_\odot$ neutron star in 4U 1608–52 was measured to be about 9.3 km (Güver et al. 2010b). The densities of these measured neutron stars are all very high, around 10^{18} kg m^{-3}. This is about one-eighth higher than that of a free neutron due to the strong gravitational compression, which was recently measured with a radius of about 0.826×10^{-15} m (Storti & Desiato 2009). These measurements of neutron stars with a small radius and a mass of about 1.58 and 1.74 $M_\odot$ cannot be explained by any model that has been developed for neutron stars.

The observations of supernova remnants and progenitors have shown that some very massive stars over 40 $M_{\odot}$ actually collapsed to form neutron stars rather than conventionally expected black holes (Muno et al. 2006; Figer et al. 2006; Ritchie et al. 2010). According to the theory of stellar evolution and Einstein's general theory of relativity, at the end of its life, a star with 0.5–8 $M_{\odot}$ will form a white dwarf, a star with 8–20 $M_{\odot}$ will form a neutron star, and a star above 20 $M_{\odot}$ will form a Schwarzschild black hole. This observational discovery strongly implies that nature forms black holes with more difficulty than previously thought, and so challenges the currently well-developed models of neutron stars and theories about the formation of black holes.

In this paper, we propose a possible explanation for all these observational phenomena involved in strong field astrophysics. We develop a new mass-radius relation for neutron stars by applying the five-dimensional (5D) fully covariant Kaluza-Klein (KK) theory with a scalar field to describe the gravity of neutron stars, rather than modifying the existing EoS models of neutron stars. The significant improvement that we have made in the development of the neutron star mass-radius relation is that the gravitational field shielding by a scalar field, found recently to be significant in the case of strong fields, is included. By including the effect of the gravitational field shielding, we will demonstrate that the previously well-developed EoS models of neutron stars can still work well and the recently measured masses and radii of some neutron stars can also be explained.

2 GRAVITATIONAL FIELD SHIELDING AND NEUTRON STAR MASS-RADIUS RELATION

According to the 5D fully covariant KK theory with a scalar field, in the Einstein frame, the gravitational field of a static spherically symmetric object with mass M and charge Q is given by Zhang (2010, 2011a),

$$g = \frac{c^2}{2\phi^2}\left(\frac{d\phi}{dr} + \phi\frac{d\nu}{dr}\right)\mathrm{e}^{\nu-\lambda}\,. \tag{1}$$

Here the scalar field ϕ and the components of the metric 00 and 11 (i.e. tt- and rr-components), e^{ν} and e^{λ}, were solved as Zhang (2006 and references therein)

$$\phi^2 = -\alpha^2\psi^4 + (1+\alpha^2)\psi^{-2}\,, \tag{2}$$

$$\mathrm{e}^{\nu} = \psi^2\phi^{-2}, \qquad \mathrm{e}^{\lambda} = \left(1 - \frac{B^2}{r^2}\right)^2\psi^{-2}\,, \tag{3}$$

in the Jordan frame, where ψ, B, and α are given by

$$\psi = \left(\frac{r-B}{r+B}\right)^{1/\sqrt{3}}, \quad B = \frac{GM}{\sqrt{3(1+\alpha^2)}c^2}, \quad \alpha = \frac{Q}{2\sqrt{G}M}\,. \tag{4}$$

This exact solution does not include any unknown parameter and thus completely determines the field properties of a static spherically symmetric object. In the Einstein frame, it reduces to the Schwarzschild solution of Einstein's general relativity when the object considered is neutral and the fields of the object are weak. Therefore, the four fundamental weak field tests of Einstein's general relativity are also the tests of the 5D fully covariant KK theory with a scalar field. For massive compact objects such as neutron stars, the fields are strong. The field solution of the 5D fully covariant KK theory with a scalar field gives results that are significantly different from those derived from the Schwarzschild solution of Einstein's general relativity, such as space polarization (Nodvik 1985; Dragilev 1990), gravitational field shielding (Zhang 2010), gravitationless black holes (Zhang 2011a), and electric redshifts (Zhang 2006). All these differences are results of the strong scalar field, which significantly shields or reduces the gravity, or in other words, decreases the equivalent gravitational constant (Zhang 2011b; Zhang et al. 2013).

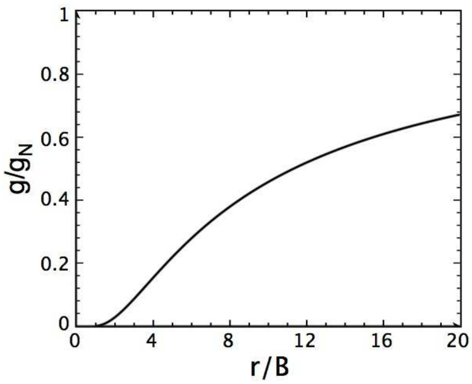

Fig. 2 The KK and Newtonian gravitational field ratio, $g/g_{\rm N}$, of a neutral object versus the normalized radial distance r/B.

To compare the gravitational field given by Equation (1) with the Newtonian gravitational field defined by $g_{\rm N} = GM/r^2$, in Figure 2 we plot the gravitational field (normalized by $g_{\rm N}$) as a function of the radial distance (normalized by B, which is the KK singular radius). It is seen that the gravitational field is significantly reduced (or shed) by the scalar field when r is comparable to B. For the radii and masses of the three neutron stars measured by Guillot et al. (2011), Güver et al. (2010a,b), the ratio R/B is about 7.4, 6.7, and 6.2, respectively, and thus, from Figure 2, the gravitational fields at their surfaces are weakened by sixty to seventy percent.

Under this type of modified gravity with a scalar field, the mass-radius relation of neutron stars can be simply determined according to the hydrostatic equilibrium (Oppenheimer & Volkoff 1939; Duric 2004),

$$P_i = -P_{\rm g}\,, \tag{5}$$

where P_i is the matter pressure of neutron stars and $P_{\rm g}$ is the gravitational pressure of neutron stars. As shown from the discussion, Equation (5) derives a fairly accurate mass-radius relation for neutron stars, especially those with a small radius and large mass in the 5D fully covariant KK theory with a scalar field.

The matter pressure chiefly depends on the composition of neutron stars and the nature of strong interactions. According to the Pauli exclusion principle (Pauli 1925), the extraordinarily dense matter of neutron stars should be degenerate. Various models for the EoSs of neutron stars or the expressions of P_i have been well developed according to the physics of nuclei, particles, and quarks. The fundamental model is the degenerate free neutron gas model (Chandrasekhar 1931), which gives

$$\begin{aligned} P_i &= \frac{1}{10 m_{\rm n}}\left(\frac{3h^3}{8\pi}\right)^{2/3} n^{5/3} \quad \text{for nonrelativistic,} \\ &= \frac{1}{8}\left(\frac{3c^3h^3}{\pi}\right)^{1/3} n^{4/3} \quad \text{for relativistic.} \end{aligned} \tag{6}$$

where $m_{\rm n}$ is the mass of the neutron, h is the Planck constant, and n is the number density of the neutron star. Considering that, at extremely high density, the neutrons are largely transformed into protons and hyperons, which interact strongly through the nuclear force, Skyrme modeled the EoS of neutron stars as (Skyrme 1959; Cameron 1959),

$$P_i = 5.32\times10^9\rho^{5/3} + 1.632\times10^{-5}\rho^{8/3} - 1.381\times10^5\rho^2\,, \tag{7}$$

where the matter pressure P_i and density ρ (only in this equation) are represented in the cgs unit system. Later, many other neutron star EoS models with a strong interaction among particles and nuclei have been developed, such as the AP model (Akmal & Pandharipande 1997), the MS model (Müller & Serot 1996), the PAL model (Prakash et al. 1988), the GM model (Glendenning & Moszkowski 1991), the strange quark matter (SQM) model (Farhi & Jaffe 1984; Prakash et al. 1990), and others.

The gravitational pressure $P_{\rm g}$ is usually determined in terms of Einstein's general relativity or a Newtonian gravitational field, by

$$P_{\rm g,E} = \frac{3GM^2}{8\pi R^4}\,. \tag{8}$$

To explain the recent observations shown above, we must either develop a better neutron star EoS model to have a more appropriate P_i or modify the theory of gravitation to have a new expression of $P_{\rm g}$. In this study, we use the gravitational pressure newly derived from the 5D fully covariant KK theory with a scalar field (Zhang 2011a)

$$P_{\rm g} \equiv -\int_0^R \rho g(r)dr = \frac{\sqrt{3}\rho c^2}{95312}\left[(R-B)^{7/\sqrt{3}-2}(R+B)^{-7/\sqrt{3}-2}\times\right.$$
$$\left.\left(3653\sqrt{3}R^4 + 3486BR^3 + 828\sqrt{3}B^2R^2 + 378B^3R + 27\sqrt{3}B^4\right) - 3653\sqrt{3}\right]. \tag{9}$$

Here Equation (9) is an exact integration that can be performed by substituting Equation (1).

To see the difference between Equation (8) and Equation (9), in Figure 3 we plot the ratio of gravitational pressure derived between the 5D fully covariant KK theory with a scalar field to that derived from Einstein's general relativity, $P_{\rm g}/P_{\rm g,E}$, as a function of the radius. The mass of the neutron stars is chosen to be $1.5\,M_\odot$ (top line) and $2\,M_\odot$ (bottom line). It is seen that the gravitational pressure obtained from the 5D fully covariant KK theory with a scalar field is only $\sim 50\% - 70\%$ of that obtained from Einstein's general relativity for a neutron star with a mass of 1.5–2 $M_\odot$ and a radius of 9–12 km. The greater mass the neutron star has, the higher the percentage of gravity that is shed. Since a large percentage of its gravitational field is shed or reduced by the strong scalar field, a neutron star with the same matter pressure can be more massive (e.g., by a factor of 1.4) than that previously modeled without the effect of gravitational field shielding. Therefore, the EoS models GS1, SQM1, PAL6, GM3, FSU, and MS1 can also support a neutron star with $2\,M_\odot$ if gravitational field shielding is considered. On the other hand, the smaller the radius a neutron star has, the higher the percentage of its gravity is shed. This implies that a large mass neutron star can be balanced at a small radius without collapsing into a black hole. For a non-compact star ($R \gg B$, the case of weak field), Equation (9) reduces to Equation (8) (i.e., $P_{\rm g} \longrightarrow P_{\rm g,E}$). In this case, the effect of gravitational field shielding is negligible.

Figure 4 plots the mass-radius relation of neutron stars according to the hydrostatic equilibrium Equation (5). Here, the gravitational pressure $P_{\rm g}$ is chosen from the 5D fully covariant KK theory with a scalar field as shown in Equation (9). The matter pressure P_i is chosen from two neutron star models, which are the non-relativistic degenerate neutron gas model (left solid line) and the Skyrme model (right solid line). The dashed line is the causality of the KK black hole, which has been shown to be gravitationless at the surface and smaller in radius than the Schwarzschild black hole (Zhang 2011a). The three dots are the recent measurements of the three neutron stars that represent part of U24 in NGC 6397, 4U 1820–30, and 4U 1608–52, respectively (Guillot et al. 2011; Güver et al. 2010a,b). It is seen that the measurements of mass and radius for the three neutron stars are consistent with the mass-radius relation derived from the 5D fully covariant KK theory with a scalar field and the Skyrme model for the EoS of neutron stars. Since, from Equation (9), the gravitational pressure approaches zero (i.e., $P_{\rm g} \longrightarrow 0$) at $r \longrightarrow B$, the neutron star, modeled in the 5D fully covariant KK theory with a scalar field, is stable.

A previous study by Zhang (2011a) has shown that a non-relativistic degenerate gas core of free neutrons will collapse into a KK black hole if its mass exceeds about $2.7\,M_\odot$. A relativistic

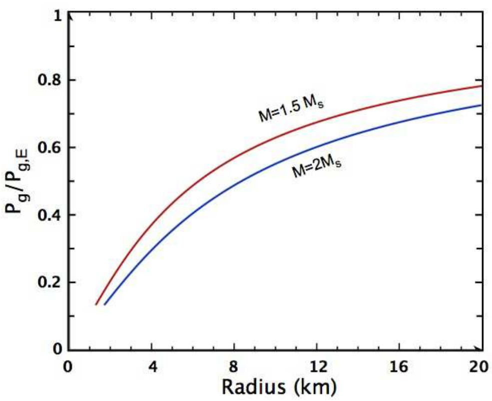

Fig. 3 The ratio of the gravitational pressures of neutron stars versus the radius of neutron stars between the 5D fully covariant KK theory with a scalar field and the 4D Einstein general relativity. The top line is for a neutron star with 1.5 $M_\odot$, while the bottom line is for a neutron star with 2 $M_\odot$.

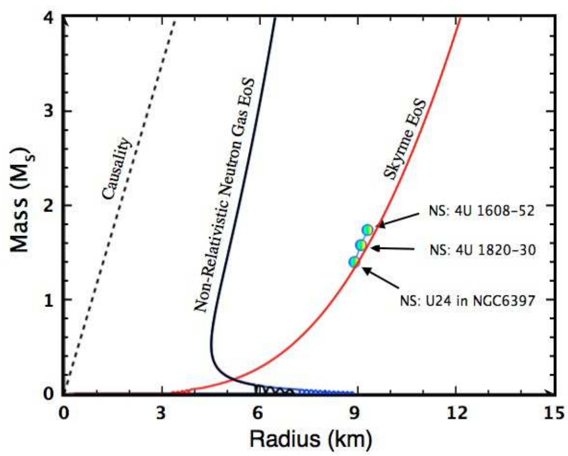

Fig. 4 The mass-radius relation of neutron stars derived from the 5D fully covariant KK theory with a scalar field that can partially shield the gravity of neutron stars. The dashed line is the causality of the KK gravitationless black hole. The blue line is the mass-radius relation of neutron stars governed by the non-relativistic neutron degenerate gas model, while the red line refers to the mass-radius relation of neutron stars governed by the Skyrme model. The three light blue dots show the mass and radius measurements of the three neutron stars found in 4U 1608–52, 4U 1820–30, and U24 in NGC 6397, respectively. Here M_s is the mass of the Sun.

degenerate gas core of free neutrons, or a core with a strong nuclear force such as the Skyrme model, needs more mass to form a KK black hole. Therefore, a neutron star with mass under 2.7 $M_\odot$ is stable under this type of modified gravity with a scalar field.

3 DISCUSSIONS AND CONCLUSIONS

The densities of these three neutron stars (represented by the three dots in Figs. 1 and 4) are all about 10^{18} kg m^{-3}. The matter pressure from the Skyrme model (Eq. (7)) at this density is about 10^{34} Pa.

Therefore, any neutron star model that has this amount of matter pressure at this density can also use Equation (9) to explain the three neutron star measurements with the models SQM2, PS, WFF3, and the GM3 models of Lattimer & Prakash (2001). Other neutron star models of P_i, such as the models WFF1, WFF2, AP3, AP4, GS1, GS2, MS1 etc., can use our $P_{\rm g}$, derived from the 5D fully covariant KK theory with a scalar field, to explain measurements of other neutron stars with different masses and radii. Since different neutron stars may have different structures and compositions, one cannot explain the mass-radius relations of all neutron stars by using a single model of EoS. In this paper, the Skyrme model of EoS has been applied to explain the recent measurements of the three neutron stars with small radii. We will model other EoSs with gravitational field shielding by applying a scalar field to explain measurements of other neutron stars.

The hydrostatic equilibrium expressed by Equation (5) is an approximate condition. The gravitational pressure defined by Equation (9) is an exact result from the integration. The error from using Equation (5) in the development of the mass-radius relation under the KK theory cannot be quantitatively estimated because the Tolman-Oppenheimer-Volkoff (TOV) equation has not yet been modified according to the KK theory. The TOV equation was derived from Einstein's general relativity and the Schwarzschild solution. Similarly, from the KK field equation and solution, one should be able to derive a modified TOV equation, but this has not yet been done. Fortunately, if the effects of finite pressure and singular radius on the hydrostatic equilibrium are negligible (i.e., $P_i/(\rho c^2) \ll 1$ and $r_{\rm g}/R \ll 1$), the TOV equation and also the modified TOV equation should both simply reduce to Equation (5). Here $r_{\rm g}$ is the singular radius and R is the radius of the neutron star. For the density of the three neutron stars with small radii that were observed recently and considered by us, where the pressure was determined from the Skyrme model, we have $P_i/(\rho c^2) \sim 0.086 \ll 1$. This indicates that the effect of pressure on the hydrostatic equilibrium is not significant and is thus negligible. According to the Schwarzschild solution, the singular radius is given by $r_s = 2GM/c^2 \sim 4.68\,\rm km$ for a compact object with a mass of 1.58 $M_\odot$. This gives $r_s/R \sim 0.51$ for a neutron star with mass of 1.58 $M_\odot$ and radius of 9.1 km, and thus indicates that the effect of a finite singular radius on the hydrostatic equilibrium is significant under the gravity of Einstein's general relativity. That is why the TOV equation (rather than Eq. (5)) must be applied to derive the neutron star mass-radius relation under the gravity of Einstein's general relativity. In our well-developed 5D fully covariant KK theory with a scalar field, however, the singular radius is given by $r_s \equiv B = GM/(\sqrt{3}c^2) \sim 1.35\,\rm km$ for an object with the same mass as considered above. This gives $r_s/R \sim 0.14 \ll 1$ for a neutron star with the same mass and radius as considered above and thus indicates that the effect of a finite singular radius on the hydrostatic equilibrium is not significant under the gravity of the KK theory and is thus negligible. Therefore, Equation (5) is a good approximation to develop a fairly accurate mass-radius relation of neutron stars under the gravity of the KK theory. The small error from this approximation can only be estimated after the modified TOV equation is derived from the KK theory and solution. This will be studied in the future.

Since the KK black hole is gravitationless at the surface, and especially because its radius is smaller than that of a Schwarzschild black hole (Zhang 2011a), naturally forming a gravitationless KK black hole will be much more difficult than forming a Schwarzschild black hole. This qualitatively explains the observations that some very massive stars over 40 $M_\odot$ formed neutron stars rather than black holes as expected. In addition, a quark star, when it reaches 2 $M_\odot$, should have already collapsed into a Schwarzschild black hole, but should not yet have collapsed into a KK gravitationless black hole.

As a consequence, we have studied the role of a scalar field on the mass-radius relation of neutron stars, in accordance with the 5D fully covariant KK theory with a scalar field that unifies the 4D Einstein general relativity and Maxwell electromagnetic theory. The mass-radius relation obtained from this study is consistent with recent measurements of neutron stars with a small radius (e.g. 9 km) and large mass (e.g. 1.74 $M_\odot$), that have ruled out almost all the previously developed models of a neutron star's mass-radius relations. We have applied the field solution of the 5D fully covariant

KK theory with a scalar field to describe the gravity of neutron stars, rather than modify the existing models or EoSs describing neutron stars. The significance of this work is that the gravitational field shielding by a strong scalar field is included. With the effect of gravitational field shielding, we can allow a neutron star to be more massive and more compact, consistent with what has been measured (Guillot et al. 2011; Güver et al. 2010a,b). We can also understand why neutron stars were actually formed from the supernova explosions of some very massive stars over 40 $M_\odot$, as was recently measured (Muno et al. 2006; Figer et al. 2006; Ritchie et al. 2010). The models that use an EoS of neutron stars, which are ruled out by cases with a small radius and large mass under the 4D Einstein general theory of relativity, can still work well under the 5D fully covariant KK theory with a scalar field. Therefore, the gravitational field shielding by a scalar field may play an essential role in the formation of neutron stars.

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References

Akmal, A., & Pandharipande, V. R. 1997, Phys. Rev. C, 56, 2261
Baade, W., & Zwicky, F. 1934, Physical Review, 46, 76
Cameron, A. G. 1959, ApJ, 130, 884
Chandrasekhar, S. 1931, ApJ, 74, 81
Demorest, P. B., Pennucci, T., Ransom, S. M., Roberts, M. S. E., & Hessels, J. W. T. 2010, Nature, 467, 1081
Dragilev, V. M. 1990, Theoretical and Mathematical Physics, 84, 887
Duric, N. 2004, Advanced Astrophysics (Cambridge: Cambridge Univ. Press)
Farhi, E., & Jaffe, R. L. 1984, Phys. Rev. D, 30, 2379
Figer, D. F., MacKenty, J. W., Robberto, M., et al. 2006, ApJ, 643, 1166
Glendenning, N. K., & Moszkowski, S. A. 1991, Physical Review Letters, 67, 2414
Guillot, S., Rutledge, R. E., & Brown, E. F. 2011, ApJ, 732, 88
Güver, T., Wroblewski, P., Camarota, L., & Özel, F. 2010a, ApJ, 719, 1807
Güver, T., Özel, F., Cabrera-Lavers, A., & Wroblewski, P. 2010b, ApJ, 712, 964
Hewish, A., & Okoye, S. E. 1965, Nature, 207, 59
Lattimer, J. M., & Prakash, M. 2001, ApJ, 550, 426
Müller, H., & Serot, B. D. 1996, Nuclear Physics A, 606, 508
Muno, M. P., Clark, J. S., Crowther, P. A., et al. 2006, ApJ, 636, L41
Nodvik, J. S. 1985, Physical Review Letters, 55, 2519
Oppenheimer, J. R., & Volkoff, G. M. 1939, Physical Review, 55, 374
Pauli, W. 1925, Zeitschrift fur Physik, 31, 765
Prakash, M., Lattimer, J. M., & Ainsworth, T. L. 1988, Physical Review Letters, 61, 2518
Prakash, M., Baron, E., & Prakash, M. 1990, Physics Letters B, 243, 175
Ritchie, B. W., Clark, J. S., Negueruela, I., & Langer, N. 2010, A&A, 520, A48
Skyrme, T. 1959, Nucl. Phys. A, 9, 615
Storti, R. C., & Desiato, T. J. 2009, Physics Essays, 22, 27
Zhang, T. X. 2006, ApJ, 636, L61
Zhang, T. X. 2010, ApJ, 725, L117
Zhang, T. X. 2011a, Ap&SS, 334, 311
Zhang, B. J. 2011b, Research Report (AAMU, Physics)
Zhang, B. J., Zhang, T. X., Guggilia, P., & Dohkanian, M. 2013, Progress in Physics, 1, 69