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The neutrino energy loss by electron capture of nuclides ^{55}Co and ^{56}Ni in explosive stellar environments

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Abstract Using the Shell-Model Monte Carlo method and the Random Phase Approximation theory, we carry out an estimation of neutrino energy loss (NEL) for ^{55}Co and ^{56}Ni by electron capture. We find that the NEL rates increase greatly at some typical stellar conditions, and can even exceed five orders of magnitude (e.g. $T_9 = 38.5$, $Y_{\rm e} = 0.42$ for ^{56}Ni). On the other hand, the error factor C shows that the fit is fairly good for two results at higher density and lower temperature, and the maximum error is $\sim 1.2\%$. However, the maximum error is $\sim 55.60\%$ (e.g. $T_9 = 18.5$, $Y_{\rm e} = 0.45$) at lower density and higher temperature.

Key words: physical data and processes: neutrinos, nuclear reactions — stars: supernovae — stars: evolution

1 INTRODUCTION

In the environment of supernova explosions, the neutrinos readily escape from stars and carry off a large amount of energy. The cooling rates from neutrino energy loss (NEL) by electron capture (EC) play a key role. The NEL rates of ^{55}Co and ^{56}Ni are very important in supernova explosions. Their cooling rates and weak interactions were investigated by Fuller et al. (1980, 1982) (hereafter FFN); Aufderheide et al. (1994); Later Heger et al. (2001); Langanke & Martinez-Pinedo (1998) and Nabi & Rahman (2005); Nabi & Sajjad (2008); Nabi (2010); Liu & Luo (2007a,b, 2008a,b); Liu et al. (2007a,b, 2011); Liu (2010) also discussed the weak interaction reactions of ^{55}Co and ^{56}Ni in stellar environments. Due to the importance of ^{55}Co and ^{56}Ni, we focus on these nuclides and reinvestigate NEL according to the Shell-Model Monte Carlo (hereafter SMMC) method, which is discussed in detail by Dean et al. (1998). We also discuss the electron capture cross section (ECCS) with the theory of Random Phase Approximation (RPA). We compared the results of $\lambda_{\rm SMMC}$, calculated by using the method of SMMC, with those of $\lambda_{\rm FFN}$, calculated by using the method of FFN.

2 THE NEL RATES IN AN EXPLOSIVE STELLAR ENVIRONMENT

The NEL rates by stellar EC for the kth nucleus (Z, A) in thermal equilibrium at temperature T is given by a sum over the initial parent states i and the final daughter states f (Fuller et al. 1980, 1982)

$$\lambda_k^{\nu} = \sum \frac{(2J_i + 1)e^{\frac{-E_i}{kT}}}{G(Z, A, T)} \sum_f \lambda_{\rm if}^{\nu} , \tag{1}$$

where J_i and E_i are the spin and excitation energies of the parent states, and $G(Z, A, T)$ is the nuclear partition function. The NEL rates from one of the initial states to all possible final states are $\lambda^{\nu}_{\rm if}$.

Based on the theory of RPA (Dean et al. 1998) with a global parameterization of the single particle numbers, the NEL rates are related to ECCS $\sigma_{\rm ec}$ by

$$\lambda^{\nu} = \frac{1}{\pi^2 \hbar^3} \sum_{\rm if} \int_{\varepsilon_0}^{\infty} p_{\rm e}^2 (\varepsilon_n - \xi) \sigma_{\rm ec}(\sigma_n, \sigma_i, \sigma_f) f(\sigma_n, U_F, T) d\varepsilon_n \,, \tag{2}$$

where $\varepsilon_0 = \max(Q_{\rm if}, 1)$, $p_{\rm e} = \sqrt{\varepsilon_n - 1}$ is the momentum of the incoming electron with energy ε_n and U_F is the electron chemical potential; T is the electron temperature. Note that in this paper, all of the energies and the momenta are respectively expressed in units of $m_{\rm e}c^2$ and $m_{\rm e}c$, where $m_{\rm e}$ is the electron mass and c is the light speed.

The electron chemical potential is found by inverting the expression for the lepton number density

$$n_{\rm e} = \frac{\rho}{\mu_{\rm e}} = \frac{8\pi}{(2\pi)^3} \int_0^{\infty} p_{\rm e}^2 (f_{-e} - f_{+e}) dp_{\rm e} \,, \tag{3}$$

where ρ is the density in g cm^{-3} and $\mu_{\rm e}$ is the average molecular weight. $f_{-e} = [1 + \exp(\frac{\varepsilon_n - U_F - 1}{kT})]^{-1}$ and $f_{+e} = [1 + \exp(\frac{\varepsilon_n + U_F + 1}{kT})]^{-1}$ are the electron and positron distribution functions respectively, and k is the Boltzmann constant. The phase space factor is defined as

$$f(\varepsilon_n, U_F, T) = \left[1 + \exp\left(\frac{\varepsilon_n - U_F}{kT}\right)\right]^{-1} \,, \tag{4}$$

where an electron with energy ε_n combines with a proton in the single particle state with energy ε_i to form a neutron in the single particle state with energy ε_f. Due to the energy conservation, the electron, proton and neutron energies are related to the neutrino energy, and Q–value for the capture reaction (Cooperstein & Wambach 1984)

$$Q_{\rm if} = \varepsilon_{\rm e} - \varepsilon\nu = \varepsilon_n - \varepsilon_{\nu} = \varepsilon_f^n - \varepsilon_i^p \,, \tag{5}$$

and we have

$$\varepsilon_f^n - \varepsilon_i^p = \varepsilon^*_{\rm if} + \hat{\mu} + \Delta_{np} \,, \tag{6}$$

where $\hat{\mu} = \mu_n - \mu_p$ is the difference between the chemical potentials of the neutron and proton in the nucleus and $\Delta_{np} = M_n c^2 - M_p c^2 = 1.293$ MeV is the mass difference for the neutron and proton. $Q_{00} = M_f c^2 - M_i c^2 = \hat{\mu} + \Delta_{np}$, with M_i and M_f being the masses of the parent nucleus and the daughter nucleus, respectively; $\varepsilon^*_{\rm if}$ corresponds to the excitation energies in the daughter nucleus at the states with zero temperature.

The total cross section for EC is (Dean et al. 1998)

$$\begin{aligned}
\sigma_{\rm ec} = \sigma_{\rm ec}(\varepsilon_n) \; &= \sum_{\rm if} \frac{(2J_i + 1)\exp(-\beta E_i)}{Z_{\rm A}} \sigma_{fi}(Ee) = \sum_{\rm if} \frac{(2J_i + 1)\exp(-\beta E_i)}{Z_{\rm A}} \sigma_{fi}(En) \\
&= \; 6 g_{\rm wk}^2 \int d\xi (\varepsilon_n - \xi)^2 \frac{G_{\rm A}^2}{12\pi} S_{\rm GT+}(\xi) F(Z, \varepsilon_n) \,,
\end{aligned} \tag{7}$$

where $g_{\rm wk} = 1.1661 \times 10^{-5}$GeV^{-2} is the weak coupling constant and $G_{\rm A}$ is the axial vector's form-factor, which at zero momentum is $G_{\rm A} = 1.25$. $S_{\rm GT+}$ is the total amount of Gamow-Teller (GT) strength available for an initial state, which is given by summing over a complete set of final states in the GT transition matrix elements $|M_{\rm GT}|^2_{\rm if}$. The ε_n is the sum of the total rest mass and kinetic energies; $F(Z, \varepsilon_n)$ is the Coulomb wave correction which is the ratio of the square of the electron wave function distorted by the Coulomb scattering potential to the square of the wave function of the free electron.

The NEL rates by EC are given by integrating the total cross section with the flux of a degenerate relativistic electron gas

$$\lambda^{\nu} = \frac{\ln 2}{6163} \int_0^{\infty} d\xi S_{\mathrm{GT}} \frac{c^3}{(m_{\mathrm{e}}c^2)^5} \int_{p_0}^{\infty} dp_{\mathrm{e}} p_{\mathrm{e}}^2 (-\xi + \varepsilon_n)^3 F(Z, \varepsilon_n) f(\varepsilon_n, U_F, T) \,. \tag{8}$$

The p_0 is defined as

$$p_0 = \begin{cases} \sqrt{Q_{\mathrm{if}}^2 - 1}, & (Q_{\mathrm{if}} < -1), \\ 0, & (\mathrm{otherwise}). \end{cases} \tag{9}$$

In order to compare our results of λ_{SMMC}, which are calculated by using the SMMC method, with those of λ_{FFN}, which are calculated by using FFN's method, the error factor C is defined as

$$C = \frac{\lambda_{\mathrm{SMMC}} - \lambda_{\mathrm{FFN}}}{\lambda_{\mathrm{SMMC}}} \,. \tag{10}$$

3 NUMERICAL CALCULATIONS OF NEL RATES AND DISCUSSION

The NEL rates of ^{55}Co and ^{56}Ni are shown in Figures 1 and 2. We find that the NEL rates are affected greatly by astrophysical conditions and can increase in some typical cases. The NEL rates can be enhanced by more than four orders of magnitude at relatively higher temperature (e.g., $T_9 = 38.5, Y_{\mathrm{e}} = 0.42$ for ^{55}Co), and can even exceed five orders of magnitude for ^{56}Ni. This is because the higher the temperatures and densities are, the higher electron energies and electron chemical potentials are. So, the NEL rates would dramatically rise due to large numbers of electrons joining the EC process. On the other hand, the different effects of the NEL in different nuclides are caused by different Q values and transition orbits in the EC process.

Figures 3 and 4 show the effect on the error factor C, which is the comparison of λ_{SMMC} with those of λ_{FFN} by ρ_7 and T_9, respectively. Figure 3 shows at lower density the maximum error is $\sim 55.6\%$ (e.g. $T_9 = 18.5, Y_{\mathrm{e}} = 0.45$). And the maximum error is $\sim 37.10\%$ at higher temperature in Figure 4 (e.g. $\rho_7 = 3.34, Y_{\mathrm{e}} = 0.48$).

In summary, we find that at lower temperature and higher density, the fit is fairly good for two results and the maximum error is $\sim$ 1.20% (e.g. $\rho_7 = 106, Y_{\mathrm{e}} = 0.42$ and $T_9 = 1$ in Fig. 4). However, the error factor would reach its maximum at higher temperature and lower density (e.g. $\rho_7 = 3.34, Y_{\mathrm{e}} = 0.48$ and $T_9 = 20$ in Fig. 4). It is because the GT transition may not be dominant at lower densities and temperatures but would be dominate in higher temperatures and densities due to

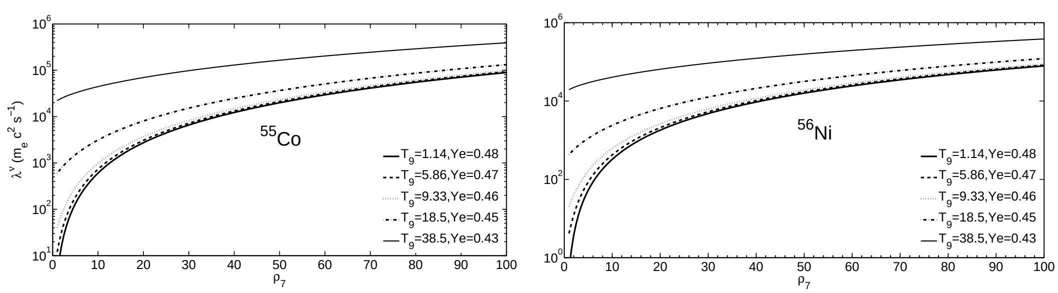

Fig. 1 The NEL rates for nuclides ^{55}Co and ^{56}Ni as a function of the density ρ_7 at some typical astrophysical conditions.

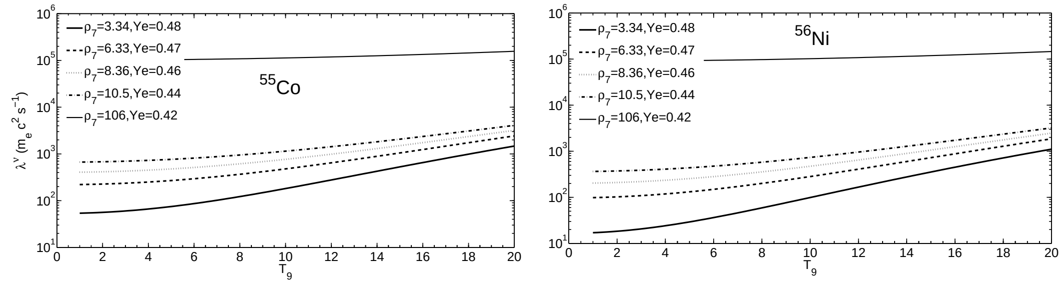

Fig. 2 The NEL rates for nuclides ^{55}Co and ^{56}Ni as a function of the temperature T_9 at some typical astrophysical conditions.

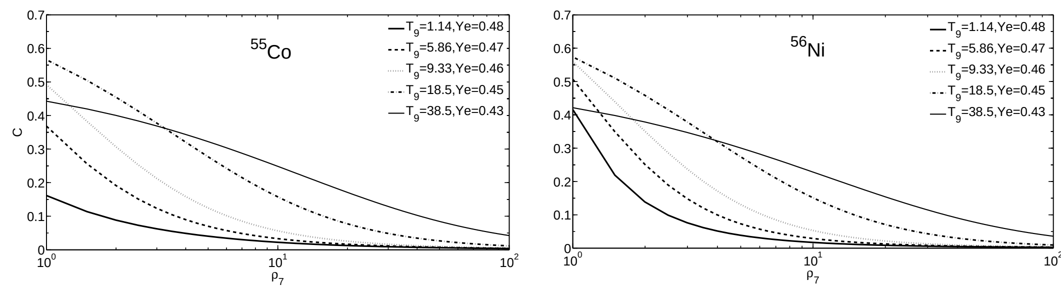

Fig. 3 The factor C for nuclides ^{55}Co and ^{56}Ni as a function of the density ρ_7 at some typical astrophysical conditions.

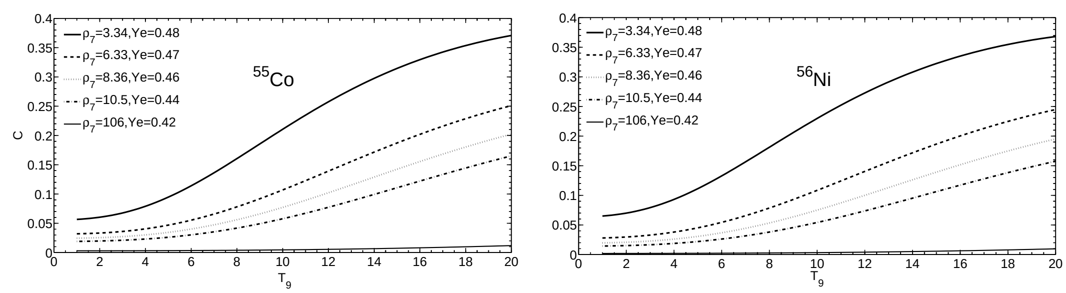

Fig. 4 The factor C for nuclides ^{55}Co and ^{56}Ni as a function of the temperature T_9 at some typical astrophysical conditions.

higher electron energy and electron chemical potential. So, the NEL rates would be enhanced greatly due to large numbers of electrons joining the EC process. On the other hand, experimental data on the GT distribution of nuclear excited states are now becoming available. These data show that the GT strength disappears in the daughter nucleus and splits into several excitations. FFN's works (Fuller et al. 1980, 1982), which use GT contribution parameters, have a few errors. SMMC is in fact obtained as an average of the GT strength distribution and has improved FFN's computational method. A reliable replication of the GT distribution in the nucleus is carried out and detailed analysis is made by using an amplification of the electronic shell model. Thus the method is relatively accurate.

4 CONCLUDING REMARKS

By using the SMMC method and the RPA theory, we carry out an estimation of the NEL for ^{55}Co and ^{56}Ni. We find the different effects on NEL for different nuclides at some typical stellar conditions, which even increase by five orders of magnitude (e.g. $T_9 = 38.5, Y_{\rm e} = 0.42$ for ^{56}Ni). On the other hand, the error factor C shows that the fit is fairly good for two results at higher density and lower temperature and the maximum error is ~ 0.12%. However, the maximum error is ~ 55.6% (e.g. $T_9 = 18.5, Y_{\rm e} = 0.45$) at lower density and higher temperature. The NEL rates strongly influence the stellar cooling rates and evolutionary timescale. Thus, the NEL and EC play key roles at the late stage of stellar evolution, especially for the process involved in a core-collapse supernova. Therefore, the results we obtained may have significant influence on further research of supernova explosions and stellar cooling mechanisms.

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