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Polynomial regression calculation of the Earth's position based on millisecond pulsar timing *

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Abstract Prior to achieving high precision navigation of a spacecraft using X-ray observations, a pulsar rotation model must be built and analysis of the precise position of the Earth should be performed using ground pulsar timing observations. We can simulate time-of-arrival ground observation data close to actual observed values before using pulsar timing observation data. Considering the correlation between the Earth's position and its short arc section of an orbit, we use polynomial regression to build the correlation. Regression coefficients can be calculated using the least square method, and a coordinate component series can also be obtained; that is, we can calculate Earth's position in the Barycentric Celestial Reference System according to pulse arrival time data and a precise pulsar rotation model. In order to set appropriate parameters before the actual timing observations for Earth positioning, we can calculate the influence of the spatial distribution of pulsars on errors in the positioning result and the influence of error source variation on positioning by simulation. It is significant that the threshold values of the observation and systematic errors can be established before an actual observation occurs; namely, we can determine the observation mode with small errors and reject the observed data with big errors, thus improving the positioning result.

Key words: autonomous positioning — millisecond pulsar — pulsar timing — regression analysis

1 INTRODUCTION

Pulsars are distant celestial sources which are spinning neutron stars that have jets of particles moving at almost the speed of light streaming out above their magnetic poles. The magnetic and rotational axes of a pulsar are often misaligned, so the beams of light from the jets sweep around as the pulsar rotates, just as in the spotlight in a lighthouse. Like a ship in the ocean that sees only regular flashes of light, we see pulsars turn on and off as the beam sweeps over the Earth[1] (GSFC, 2006). The observer records regular flashes or pulses of radio emission appearing periodically as time-of-arrival (TOA) observations.

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[1] NASA's Goddard Space Flight Center Imagine Team, 2006, *http://imagine.gsfc.nasa.gov/docs/science/know_l1/pulsars.html*

Since the discovery of pulsars by Bell and Hewish, early observers recognized the potential of the stable and periodic signals from pulsars to provide a high quality celestial clock. Using pulsars as accurate beacons in space for vehicle navigation was therefore investigated.

A proposal to use pulsar signals as a clock for Earth-based systems was pointed out, and timing observations of 11 pulsars were performed (Reichley et al. 1970, 1971). Several pulsars matching the quality of atomic clocks were demonstrated in presentations (Allan 1987; Matsakis et al. 1997). Thus it was soon conjectured that pulsars could also be used as clocks for navigation.

A method of navigation for orbiting spacecraft based on radio signals from a pulsar was presented (Downs 1974; Downs & Reichley 1980). This method proposed to develop omnidirectional antennae (2 m) to be placed on a spacecraft to record pulsar signal phases and create a three-dimensional position fix. This introductory paper on pulsar navigation provided the original basis for subsequent research.

Issues related to using celestial sources that produce radio emission, including pulsars, was discussed for navigation applications on the Earth (Wallace 1988). It was expected that radio-based systems would require large antennae to detect weak pulsar signals, which would be impractical for most spacecraft. Furthermore, the low signal intensity from radio pulsars would require long signal integration times for an acceptable signal-to-noise ratio.

During the 1970s, astronomical observations within the X-ray band yielded pulsars with X-ray signatures. Using pulsars emitting in the X-ray band as an improved option for Earth satellite navigation was proposed (Chester & Butman 1981). X-ray emitting sources present a significant benefit to spacecraft applications, primarily through their utilization of smaller-sized detectors.

Researchers proposed developing a comprehensive approach to X-ray navigation covering attitude, position and time, as part of the NRL-801 experiment for the Advanced Research and Global Observation Satellite (ARGOS) (Wood 1993). The accuracy of a vehicle's position on the order of tens of meters was forecast. Timekeeping accuracy approaching 30 μs over different timescales was believed possible. As part of the Naval Research Laboratory (NRL) development effort for this experiment, a thesis on autonomous timekeeping using X-ray sources was presented, including the implementation of a phase-locked loop to maintain accurate time aboard a spacecraft (Hanson 1996).

NRL's Unconventional Stellar Aspect experiment onboard the ARGOS satellite provided a platform for pulsar-based spacecraft navigation experimentation (Wood 2001). An external estimation of the navigation system onboard ARGOS had errors between 5 and 15 km. Factors that limited the position offset calculation included pulsar timing model inaccuracies, calibration errors in the experiment timing system, photon time binning of 32 μs in the data collection mode and pulsar position errors.

NRL's research efforts were continuing to demonstrate position determination and timekeeping using the recorded data from this flight experiment (Sheikh 2005). For a $0.1\,\mathrm{m}^2$ detector, the estimated range error was 0.1 km for the Crab Pulsar after 500 seconds of observation.

Pulsar-based navigation systems, which use pulsars at great distance from the Earth, remain attractive for complementing existing near-Earth navigation systems and for developing future navigation systems that could operate in an autonomous mode. Researchers not only became convinced of the superiority of using pulsars in the field of spacecraft autonomous navigation (Sala et al. 2004[2]), but also expected to better calculate the relation between the radio reference frame and the ephemeris reference frame (Kovalevsky et al. 1989), as well as compute the ephemeris reference frame via accurate observations of the pulsar reference frame, and to extend and apply spacecraft autonomous navigation to Earth positioning. The theoretical basis of this research was formulated in the 1980s.

Pulsar timing and its relativistic effects were analyzed (Hellings 1986; Backer & Hellings 1986), providing the conversion equation from TOA to the virtual solar system barycenter arrival time (BAT).

[2] Sala, J., Urruela, A., Villares, X., Estalella, R., & Paredes, J. M., 2004, *http://www.esa.int/gsp/ACT/publications/index.htm*

Accurate cataloging of the coordinates of pulsars can provide a pulsar reference frame and give a very accurate positioning of the ecliptic with respect to inertial directions (Souchay & Cognard 2004).

The theoretical framework of Earth positioning based on pulsar timing was independently presented (Zhao 2007). Zhao & Huang (2009) pointed out that we should not use the pulsar timing conversion equation provided by Sheikh in his doctoral dissertation (Sheikh 2005) which is similar to very long baseline interferometry (VLBI) Sheikh attempted to calculate the differential TOA as well as the differential gravitational delay between the observer and the true solar system barycenter (SSB). In fact, VLBI is the differential between two real radio telescopes, and it contains the differential of the two telescopes' gravitational delay described by the same model; moreover, since pulsar-timing-based autonomous positioning is a single station observation, there is no observational connection with true arrival time at SSB which is introduced by Sheikh, and so there is no gravitational delay between the observer and the true SSB among observed values. Observed TOAs are therefore irrelevant to the arrival time of the pulses at a true SSB. The BAT computed in a vacuum and without a gravitational field should be used as a time reference. The model of pulsar rotation can be obtained through long term monitoring of the BAT. Zhao further derived the basic principle of calculating a spacecraft's position (or the Earth position) via a pulsar rotation model.

Ruggiero et al. (2011), in Italy, simulated the arrival times of the signals from four pulsars at the Parkes radio telescope in Australia and recalculated the trajectory of the observatory in space. Their method is relative positioning. The origin is assumed to coincide with the starting point of the positioning process, and the location of that origin with respect to a global reference frame, such as the International Celestial Reference System (ICRS), must be independently defined.

All these topics represent the research background. Due to the fact that a radio telescope is unable to observe several pulsars simultaneously, the observation series is obtained through the observation of a few pulsars over a period of time. Our work focuses on Earth positioning via timing of several pulsars at different epochs with the pulsar rotation model that is already available. The paper is arranged as follows: Section 2 is the outline of the theory and method, Section 3 provides the results and analyses of our simulations, and the conclusions are given in Section 4.

2 PRINCIPLE AND METHOD OF CALCULATING EARTH'S POSITION BASED ON MILLISECOND PULSAR TIMING

At present, the Earth's position in the BCRS is obtained by indirect methods, such as ground optical orientation observation of major planets and asteroids, ground ranging observations of Mercury and Venus, and LLR (lunar laser ranging). The calculated position is largely influenced by the orbit theory of planets and the error parameters of outer planets. Although there is a difference of just meters for the Earth's heliocentric distance between ephemerides EPM 2004 and DE410 (Pitjeva 2005), and a difference of just meters for the EMB (Earth–Moon barycenter) heliocentric distance between ephemerides VSOP 2002b and DE405, as well as INPOP06 and DE405 (Fienga & Simon 2005; Fienga et al. 2008), the precision of heliocentric distance is only the internal precision of the ephemeris. Contrastively, there is a big difference of kilometers for the Earth SSB distance between ephemerides DE414 and DE405.

As shown in Figure 1, an accurate Earth SSB position cannot be reached using a dynamics-related method. However, pulsar timing is able to calculate the Earth SSB coordinate directly for the first time on the basis of the pulsar's space-time reference. In order to describe the kinematic characteristics of the pulsar, Earth and the regularity of BAT at the SSB, the optimum reference system for positioning the Earth by pulsar timing is BCRS. The definition of BCRS from the nomenclature for fundamental astronomy is a system of barycentric space-time coordinates for the solar system within the framework of general relativity, with a metric tensor specified by the IAU 2000 Resolution, unless otherwise stated. The BCRS is assumed to be oriented according to the ICRS axes.

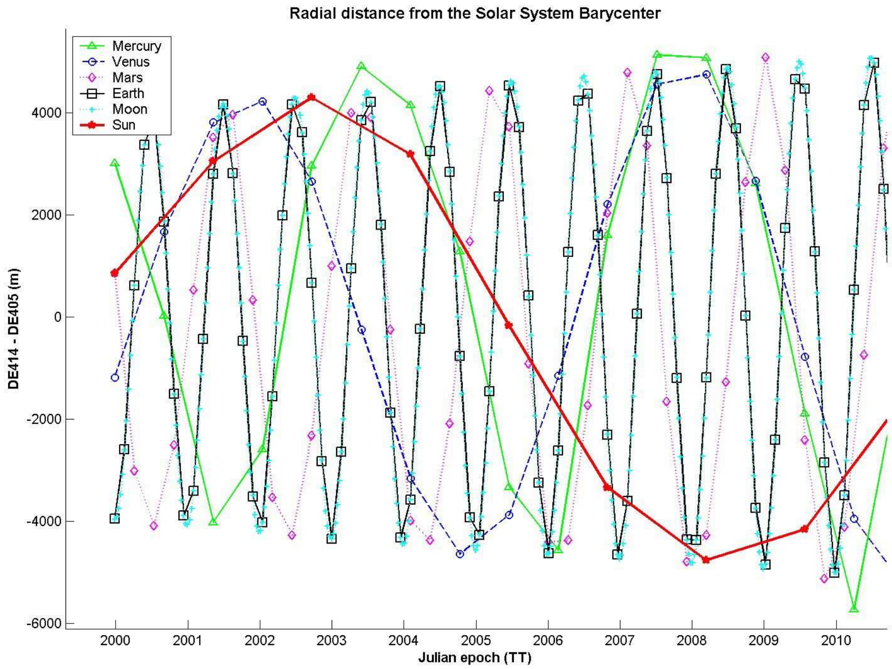

Fig. 1 Difference for the Earth's SSB distance between DE414 and DE405, which was 4 km from 2000 to 2010 (data provided by Jinling Li), is far greater than the internal precision of the ephemeris.

The SSB depends on the position and mass of the Earth, the Sun and other planets. This paper mainly deals with the SSB position of the Earth. The mass of the Earth is about 300 000 times smaller than the mass of the Sun, and the error of the Earth mass is tiny, so as a result Earth's ephemeris errors have no obvious influence on the position error of SSB, and there is no rank defect or paradox.

The pulsar rotation model is needed to calculate the position of the Earth. The predictions and reduction of the pulse's TOA is gained from long-term timing observations while the Earth moves around the Sun. Then the TOA is used to generate vacuum and without the gravitational field's virtual BAT. The BAT should be used as a time reference, and a model of pulsar rotation can be obtained through long term monitoring of the BAT.

The high-precision pulsar catalog can be gained from the VLBI phase-reference technique, i.e. background sources near to the pulsar in a celestial sphere are employed as a position reference to calculate the pulsar's position and proper motion (Guo et al. 2010). So the reference system of pulsars is established from the radio reference system (ICRS). Background radio sources are so far away from the Earth that their parallax and proper motion are almost zero, and as a result the Earth's position errors have no influence on the radio reference system.

The Earth ephemeris is required to construct the connection between the pulsar reference system and BCRS. Pulsars have parallax and proper motion, but the distances of pulsars are far enough away that Earth's position errors only have a tiny influence on a pulsar's position and parallax, as shown in Figure 2.

The position of the Earth can be calculated in BCRS according to pulse arrival time data. The arrival times of the pulses are shifted by the Doppler effect due to the relative speed of the Earth in its orbit, whose direction and amplitude vary during the year with respect to the line of sight. It

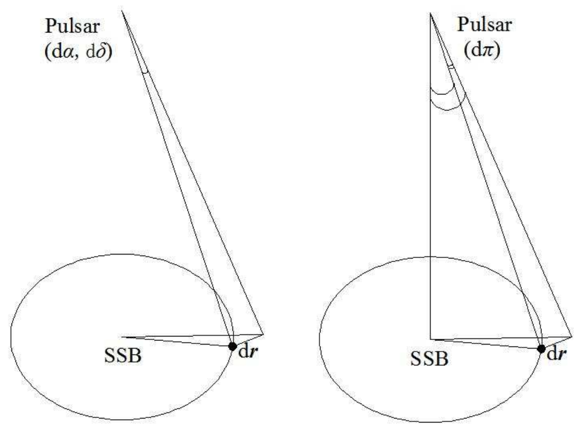

Fig. 2 Influence of Earth's ephemeris error on building the pulsar reference system for a pulsar distance of 0.2 kpc (parallax is 5 mas); the error of the Earth position vector ***dr*** of about 1000 km can only produce an error of α and δ (right ascension and declination) and an error of π (parallax) as low as 0.03 μas. Even if the Earth's ephemeris is improved, the pulsar's right ascension, declination and parallax will always be insensitive, i.e. the Earth ephemeris has no obvious contribution to the pulsar catalog.

therefore provides precious information about the location of the ecliptic. Pulsar timing can achieve accuracies of tens of microseconds, and millisecond pulsar timing achieves the level of a few hundred nanoseconds over long periods of time. Timing accuracies of a few hundred nanoseconds imply that the observer's position with respect to the solar system barycenter is known within about 100 meters. This is very close to the internal accuracy limit of the latest ephemerides (Fisher 1996[3]). However, the position of the SSB is generally obtained from a dynamic ephemeris and is relatively poor, if using an ephemeris to build a model of pulsar rotation (BAT), and considering that the BAT is sensitive to the position of SSB. As a result, the precision of the Earth's SSB distance calculated by pulsar timing will be influenced by the SSB error.

Data can be simulated as close as possible to the actual situation before dealing with the actual observation, then we can simulate and evaluate the reliability and precision of the instantaneous positioning of the Earth. The Earth's positions in the ephemeris of DE405 are used as initial orbit positions in this paper. The simulated true value is the uncertainty of the Earth's position plus its ephemeris position, then the inverse TOA observation value is calculated and the Gaussian distributed random noise is added. Next, the observed simulation value can be used to calculate the Earth coordinate position's correction value, and this is compared with the true value of the Earth ephemeris position's uncertainty. Finally, positioning precision is evaluated.

The pulsar rotation model must be built based on astrometry before calculating Earth's position in an actual observation, and integer cycle ambiguity can be obtained after the model is built. Nevertheless, the simulated calculation in this paper is concerned with positioning error more than building the model. If the pulsar rotation model is simulated to generate BAT (t^{B}), making inverse calculations of the observed value from t^{B} plus a time delay τ_i, then t^{B} will be subtracted from the observed value t_i to get a geometric delay, t^{B}, equivalent to being added and then subtracted. Moreover, there is no need to simulate the observed value in the form of phase and integer cycle ambiguity. Therefore, only the geometric delay of the observed value τ_i needs to be simulated, and

[3] NRAO's Charlottesville Operations. *http://www.cv.nrao.edu/ rfisher/*

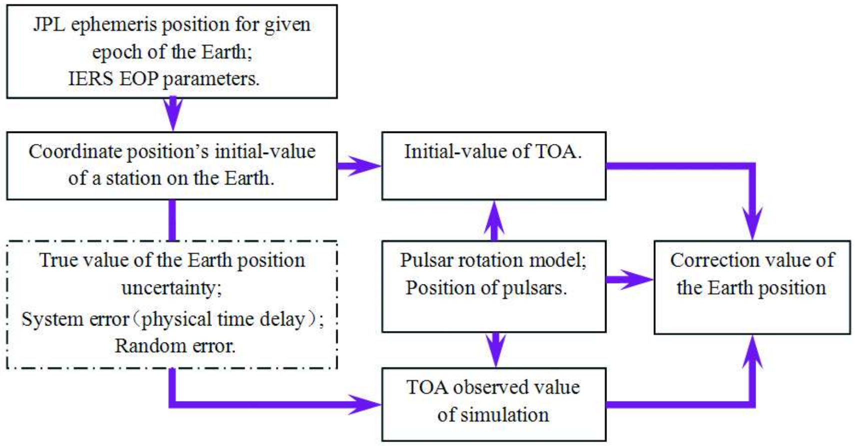

Fig. 3 Calculation flow chart of Earth positioning using simulated data.

this can be calculated inversely from the Earth ephemeris of JPL and the Earth Orientation Data from IERS (International Earth Rotation and Reference Systems Service).

Given a series of time points as the time argument, using JPL's subroutine PLEPH [4] (SSD, 2007) to read the ephemeris block of the JPL planetary ephemeris DE405 and give the position and velocity of the Earth with respect to the barycenter, the initial value of the coordinate position vector is $\boldsymbol{r}_{\mathrm{E}}^{\mathrm{c}}$.

Given the date t_i, the UT1 and the polar motion, we use the IAU 2000B precession-nutation model (McCarthy & Luzum 2003; Capitaine et al. 2003), and call this routine from the International Astronomical Union's Standards of Fundamental Astronomy software collection to obtain the celestial [C] to terrestrial [T] matrix [CtoT] and the inverse matrix [TtoC]. Then the station's geocentric vector $\boldsymbol{r}_{\mathrm{crs}}$ in ICRF at time t_i can be calculated from $\boldsymbol{r}_{\mathrm{trs}}$ in ITRF. Because the precision of the station coordinate in ITRF is better than centimeters, smaller by about 1000 orders of magnitude than the Earth positioning by timing observations, the station's geocentric vector can be directly used without error.

$$\boldsymbol{r}_{\mathrm{crs}} = [\mathrm{TtoC}] \cdot \boldsymbol{r}_{\mathrm{trs}}. \tag{1}$$

The Earth ephemeris position correction value $d\boldsymbol{r}_i$ is given as a true value. Calculated using the same coordinate system, the stationary barycenter vector's true value is

$$\boldsymbol{r}_i = (\boldsymbol{r}_{\mathrm{E}}^{\mathrm{c}} + d\boldsymbol{r}_i) + \boldsymbol{r}_{\mathrm{crs}}. \tag{2}$$

The total gravitational time delay (Shapiro delay) of the solar system is about 10^{-5}–10^{-7} s, and the mass uncertainty of the outer planet is about 10^{-7}–10^{-9} of its mass; therefore, the influence of planet mass error on gravitational delay is tiny for the precision of recent timing observations, and simulation of the gravitational delay error is not necessary. Gravitational delay and its error are not taken into account in this paper. As a result, the simulated TOA does not contain any gravitational information about the Sun and the other planets of the solar system, and consequently Earth's position could only be defined with respect to a reference system that constitutes observed pulsars. According to the timing model, the simulated observed value of geometric delay τ_i is

$$\tau_i = t_i - t^{\mathrm{B}} = -\frac{1}{c}(\boldsymbol{u}_j \cdot \boldsymbol{r}_i) + \frac{1}{2cB_0}[(\boldsymbol{r}_i)^2 - (\boldsymbol{u}_j \cdot \boldsymbol{r}_i)^2]. \tag{3}$$

[4] JPL's Solar System Dynamics Group, Pasadena, CA: JPL. *ftp://ssd.jpl.nasa.gov/pub/eph/planets*

The coordinate direction of the pulsar is $\boldsymbol{u}_j$. If the pulsar's parallax and proper motion from reference moment to present moment are taken into account, symbol B_0 is the light time distance of a reference pulse from pulsar to SSB. The proper motion of the pulsar has no great influence on the simulated calculation's conclusion in this paper, so we do not take it into account.

The initial-value of the station's barycenter vector is

$$\boldsymbol{r}_i^{\mathrm{c}} = \boldsymbol{r}_{\mathrm{E}}^{\mathrm{c}} + \boldsymbol{r}_{\mathrm{crs}}. \tag{4}$$

The initial value of the geometric delay is

$$\tau_i^{\mathrm{c}} = -\frac{1}{c}(\boldsymbol{u}_j \cdot \boldsymbol{r}_i^{\mathrm{c}}) + \frac{1}{2cB_0}[(\boldsymbol{r}_i^{\mathrm{c}})^2 - (\boldsymbol{u}_j \cdot \boldsymbol{r}_i^{\mathrm{c}})^2]. \tag{5}$$

Consequently, the error equation of calculating the Earth's position is

$$\Phi(\tau)_i - \Phi(\tau^{\mathrm{c}})_i = \frac{\partial \Phi}{\partial \boldsymbol{r}_i}\delta \boldsymbol{r}_i + \nu_i, \tag{6}$$

$$\delta \boldsymbol{r}_i = [\mathrm{C}]\{\delta x_i \quad \delta y_i \quad \delta z_i\}. \tag{7}$$

This paper presents the Earth positioning solution of timing several pulsars in several continuous time intervals using one radio telescope. It is a geometric method, not an orbit determination method of Kalman filtering.

Theoretically, the Earth's position can be fixed once a single radio telescope makes a timing observation of three or more pulsars simultaneously, just like the point positioning theory of GPS. However, a radio telescope is unable to time several pulsars at the same second. In order to calculate the Earth's position at the moment of TOA in BCRS, a nonlinear relation can be established between the Earth's coordinate components and the time point within a few hours. This is because the coordinate points in a short arc section of an Earth orbit less than one day are correlated and can be described approximately by nonlinear functions. From this premise, according to the theory of multivariate nonlinear regression analysis, the coordinate components need to be expressed by the quadratic or cubic polynomial functions of the time points. The coordinate position's calculation can be converted to the polynomial coefficients' calculation of the short arc section. Nine unknown parameters $(a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2)$ will be calculated if a quadratic polynomial is used to describe the coordinate components. Instantaneous positioning can be performed by timing these pulsars more than nine times in one day. The correction values of coordinate components are

$$\begin{aligned} \delta x_i &= a_0 + a_1 t_i + a_2 t_i^2, \\ \delta y_i &= b_0 + b_1 t_i + b_2 t_i^2, \\ \delta z_i &= c_0 + c_1 t_i + c_2 t_i^2\,. \end{aligned} \tag{8}$$

The final error function is

$$\Phi(\tau)_i - \Phi(\tau^{\mathrm{c}})_i = \sum_{k=0}^{2} \frac{\partial \Phi}{\partial \boldsymbol{r}_i}\frac{\partial \boldsymbol{r}_i}{\partial x_i}\frac{\partial x_i}{\partial a_k}\delta a_k + \sum_{k=0}^{2} \frac{\partial \Phi}{\partial \boldsymbol{r}_i}\frac{\partial \boldsymbol{r}_i}{\partial y_i}\frac{\partial y_i}{\partial b_k}\delta b_k + \sum_{k=0}^{2} \frac{\partial \Phi}{\partial \boldsymbol{r}_i}\frac{\partial \boldsymbol{r}_i}{\partial z_i}\frac{\partial z_i}{\partial c_k}\delta c_k + \nu_i. \tag{9}$$

The least square method can be used to calculate $\delta \boldsymbol{r}_i$, and compare the calculated positioning result $(\boldsymbol{r}_{\mathrm{E}}^{\mathrm{c}} + \delta \boldsymbol{r}_i)$ with Earth's true position $(\boldsymbol{r}_{\mathrm{E}}^{\mathrm{c}} + d\boldsymbol{r}_i)$ in the simulation.

3 CALCULATION RESULTS AND INSTANTANEOUS EARTH POSITIONING ANALYSIS

A radio telescope is unable to time several pulsars at the same second, so the method of point positioning is unable to calculate the Earth's position in BCRS. This paper presents a solution using multivariate nonlinear regression analysis, considering the limitation of the actual observation method, and the short arc section of the Earth's orbit can be approximately described by nonlinear functions. Regression analysis methods use an approximate short arc section to describe the actual arc section of the orbit. This is the implementation of a the needed accuracy to calculate the Earth's position.

In this paper's simulation, the Sheshan 25 m radio telescope in Shanghai is set to be the observation station; the geocentric coordinates are (–2831686.7134, 4675733.6542, 3275327.6282) m. Assuming the timing observation is carried out on 2010 November 30, we select three or four pulsars, timing them in a fixed sequence, and obtain a TOA from a pulsar every 6 minutes. Then the next pulsar, with a total of three turns, can produce enough of an observed value to solve the nine unknown parameters. The actual size of the Earth is taken into account so that every pulsar selected can be seen above the horizon; the elevation angle is higher than 10°. We use the quadrant polynomial regression analysis method given in Section 2, and calculate every coordinate position sampled TOA in the short arc section of the Earth's orbit. We keep the pulsar's position in the same coordinate system as the ephemeris. The corresponding coordinate system of the positioning result is a celestial ecliptic coordinate system of the barycenter. Analyzing the reliability and precision of the positioning, and describing the relation between the error sources and the positioning errors, is of prime importance. All of these can be estimated through simulated calculations.

The TOA sampling rate is set as 6 minutes per TOA, considering that at least a few minutes of pulse profile folding can get a high precision TOA. We present the aim of a 100 m positioning precision in the paper. Two kinds of random errors are simulated: the timing measurement error limit is under 0.5 μs, and the pulsar direction error limit is under 1 mas (milli-arcsecond). Two kinds of systematic errors are simulated: systematic error of pulsar distance is set to be pulsar distance $\times$ uncertainty percentage of distance, the model error of the hydrostatic troposphere delay model is set to be $-\varepsilon \cdot Z/(\sin E \cdot \tan^2 E)$, where ε=0.00122 (Lanyi 1984), Z is the zenith delay and E is the elevation angle of a pulsar.

As a simulation, the following conditions are appropriate:

(1) The wet component of the troposphere propagation delay is highly variable and cannot be predicted accurately, but it is small, and there is no zenith wet delay information available in this simulation, so the effect is neglected.

(2) In any case, ionospheric dispersion is inseparable from and less than uncertainties in interplanetary and interstellar dispersion, so such an effect must be removed by fitting for a time-variable dispersion measure with the aid of multi-frequency observations (Hobbs et al. 2006; Edwards et al. 2006). As a calculated simulation of this paper, this effect is neglected.

The influence of the spatial distribution of the pulsars on the coordinate's three component errors in the positioning result are calculated in Section 3.1, and the influence of the variation of error sources on positioning are calculated in Sections 3.2 and 3.3.

3.1 Influence of Pulsars' Spatial Distribution on the Positioning Result

Earth positioning based on millisecond pulsar timing requires a calculation of the time delay of the Earth coordinate position vector's projection on the pulsar's coordinate direction, which is just a differential distance measurement. In addition, observing several pulsars for positioning is similar to resection distance measurement theory. According to resection distance measurement theory, in a space defined by a rectangular coordinate system *O-XYZ*, if the known reference point is only distributed in a small direction angle, then the constraint on this direction is strong, and the coordinate component precision of this direction is high. For instance, if an unknown point is near the origin point O, all reference points are distributed around the Z axis and the deviation angle from the Z axis is within 5 degrees. So, the unknown point's positioning result must be such that the coordinate component's precision in Z is higher than in X and Y.

For Earth positioning, in order to allocate the error of the X, Y and Z coordinate components well enough, one condition is that the selected pulsars should preferably be well-distributed over the celestial sphere. Pulsars only distributed in a small direction angle should be avoided.

The ideal configuration of the spatial distribution of pulsars should be tetrahedron-like, which may not be realized in the real world. Many restrictions account for this: there is no pulsar or they need to be discovered in the required direction, or the pulsar in the required direction does not have the necessary navigation characteristics, such as strong flux, stable rotation and short period. Hence, the spatial distribution of the selected pulsars may not be ideal.

According to the qualitative analysis above, the spatial distribution of the pulsars has a systematic effect on the coordinate's three component errors of the positioning result. Specific quantitative calculation is carried out as follows. The types of pulsar distribution are simulated in an ecliptic coordinate system or equatorial coordinate system, and the selected number of pulsars is four. There are three different spatial distributions, as shown in Figures 4, 5, and 6, and they are alternately observed for three turns to obtain 12 point coordinates. For every figure in Section 3.1, the abscissa axis is the time point (obtaining a TOA every six minutes), and the ordinate axis is the coordinate's three component errors of the positioning result in the ecliptic coordinate system. Every figure in Section 3.1 includes two panels: the left panel is the Gaussian distribution random error of 0.05 $\mu\mathrm{s}$ added to the timing measurement, with no other error sources, and the right panel is the Gaussian random error of 0.05 $\mu\mathrm{s}$ added to the timing measurement; with Gaussian random errors of 0.05 mas added to the direction of the pulsars, the systematic error of pulsar distance is simulated to be pulsar distance $\times$ 10%, and the model error of the hydrostatic tropospheric delay model is simulated to be $-\varepsilon \cdot Z/(\sin E \cdot \tan^2 E) \times 10\%$. Among the added random errors and systematic errors, timing measurement error plays a primary role, and there are more error sources in the right panel than in the left panel in Section 3.1. The positioning errors in the right panel are therefore slightly larger than in the left panel.

3.1.1 Positioning errors corresponding to a poor spatial configuration of selected pulsars

If pulsars used as spatial positioning reference are only distribute in a small direction angle, the resulting spatial configuration and constraints on numerical values will be very poor. Distribution type is calculated and the specifics are illustrated in the figures in this section. As shown in Figure 4, with regard to a poor pulsar spatial configuration, the minimum coordinate component error can only be obtained in the direction angle of these pulsars ($<$ 15 m), and the other directions are unable to be well constrained. The errors are amplified up to 100 m in the left panel, and even 250 m in the right panel, much larger than the order of magnitude of the positioning error (15 m) corresponding to the timing error (0.05 $\mu\mathrm{s}$).

3.1.2 Positioning errors corresponding to a good spatial configuration of selected pulsars

Three reference points determine a plane, and they can only be on one side of the unknown point. As a result by selecting three pulsars we cannot reach the distribution level realized by four pulsars for Earth positioning. However, it is feasible to improve precision by increasing the number of pulsars, as the positioning result will be better with four pulsars than with three. As shown in Figures 5 and 6, four pulsars with good spatial configurations are selected. If the pulsars, once selected, compose a

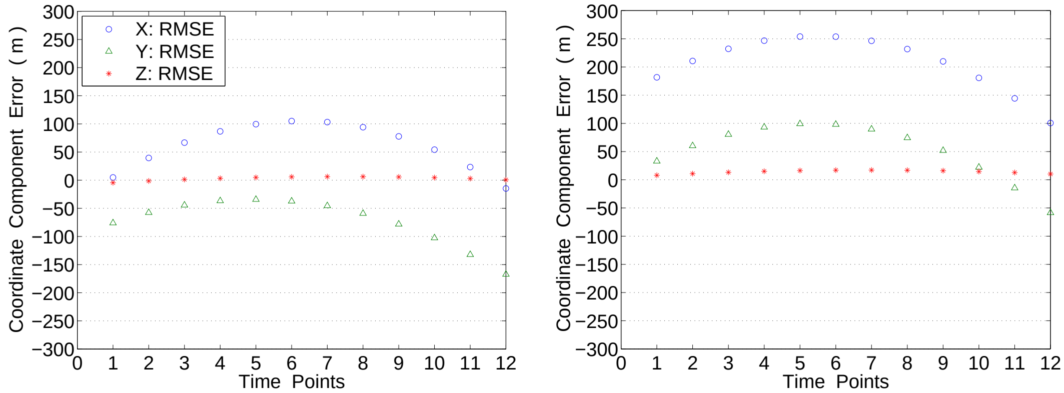

Fig. 4 Four pulsars are selected in an ecliptic coordinate system, assuming that they are distributed around the Z axis and that the deviation angle from the Z axis is 5 degrees. The constraint of the coordinate component Z is good in both the left and right panels, however, the constraints of coordinate components X and Y are poor.

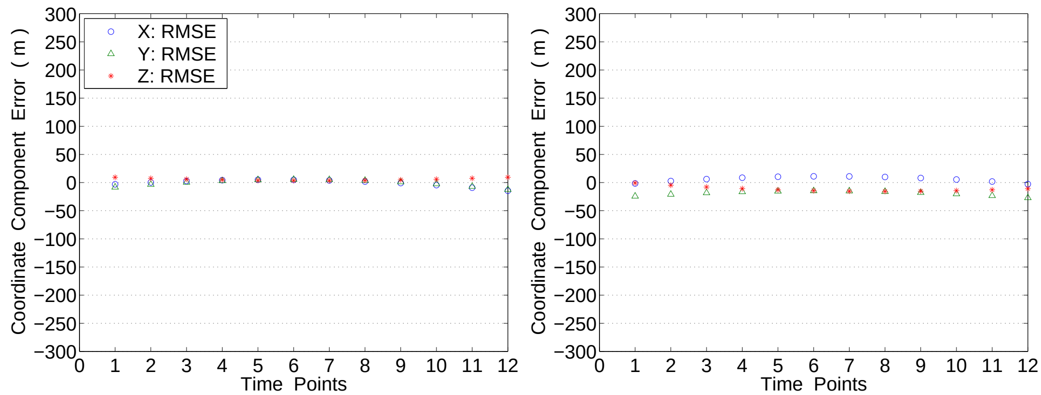

Fig. 5 Four pulsars are selected in an ecliptic coordinate system, assuming that they compose a tetrahedron. Three are under the ecliptic plane, one is over the ecliptic plane and one is near the Z axis, named the pulsar's ecliptic longitude and latitude: 0300–1928, 0800–1928, 1200–1928 and 0600+8500. The constraints of the three coordinate components are all good in the left and right panels.

good spatial configuration and are not distributed in a small direction angle, a good constraint for the Earth coordinate's three component errors will be provided. For instance, regular tetrahedrons of four pulsars, regular octahedrons of six pulsars and cubes of eight pulsars are all good spatial constraint configurations. However, the primary study of this paper is the positioning of the Earth through the timing of single radio telescope stations for different pulsars in different time intervals, where the rise and set of the pulsars must be taken into account. A regular polyhedron configuration cannot therefore be realized in observations from a single station. Timing different pulsars in different stations almost simultaneously may realize the configuration of a regular polyhedron. With regard to the single station, there are still many pulsars to be selected in the required observing time intervals, and we should then select pulsars which are able to compose preferable spatial configurations.

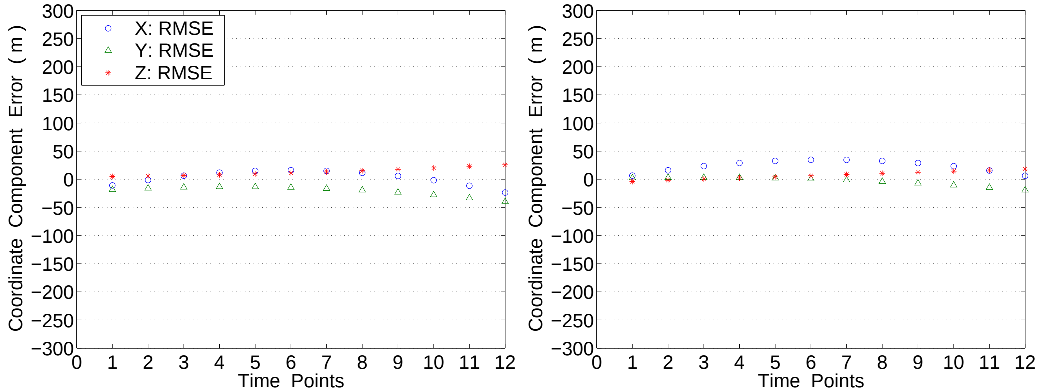

Fig. 6 Four pulsars are selected from the ATNF pulsar catalog with the middle and high declination in the equatorial coordinate system. The pulsars' right ascension and declination are as follows: PSR J0026+6320, J0700+6418, J1321+8323 and J2149+6329. The constraints of the three coordinate components are all good in the left and right panels.

As shown in Figures 4 and 5, supposing existed pulsars appearing in particular directions have previously been selected to compose the spatial configurations. However, in the real world there may be no pulsars or they need to be discovered in the required direction. Therefore, using cataloged pulsars to perform calculations is necessary for a simulation approaching actual observation. In the pulsar catalog version 1.40 of the ATNF (The Australia Telescope National Facility), there are 44 pulsars with rotation periods less than 10 ms, and pulsar distances obtained from the parallax method and dispersion method are equal (a distance uncertainty of 10% is credible).

Next, four pulsars in a higher declination belt are selected from these 44 pulsars to calculate the Earth position. In the ATNF pulsar catalog, there are 25 pulsars with a declination higher than $60°$, and pulsar distances obtained from the parallax and dispersion methods are equal. These high declination pulsars can be observed all day from the Sheshan station. Four pulsars are distributed evenly in the right ascension of these 44 pulsars, and the positioning error is shown in Figure 6.

As shown in the figures above, all the positioning results are better with a good spatial configuration (Sect. 3.1.2) than with a poor one (Sect. 3.1.1).

3.2 Influence of Error Sources on Positioning Results

Uncertainty always exists in measurement, and the order of magnitude of error sources has a critical influence on positioning results. We select the following pulsars from the ATNF pulsar catalog: PSR J1012+5307, J1455–3330, J1918–0642 and J2051–0827. Assuming we time four pulsars in sequence, we time one pulsar to obtain one TOA in 6 minutes, then obtain 12 TOAs in a total of three turns. Next, we increase the random errors or the systematic errors, and the relations between the error sources and the positioning error are shown in Figures 7 to 10. The ordinate axis is the position's standard deviation of 12 points, and each point corresponds to the root mean square of the three component errors of the positioning results.

3.2.1 Influence of random error on positioning results

The order of magnitude of random errors is increased, either from timing measurement errors or pulsar direction errors. Meanwhile, the other random errors are fixed to typical values, and another

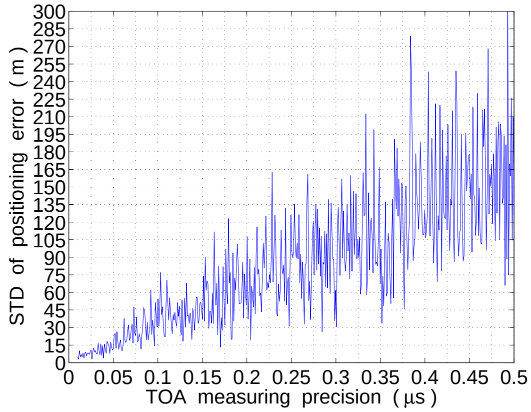

Fig. 7 Influence of TOA measurement precision on the positioning error's standard deviation calculated by regression analysis. When timing precision varies from 0.5 to 0.01 μs, the positioning precision gets better.

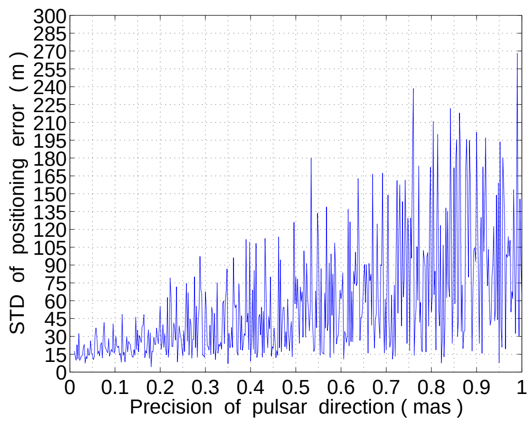

Fig. 8 Influence of the error of right ascension and declination on the positioning error's standard deviation calculated by regression analysis. When the pulsar's direction error varies from 1 to 0.01 mas, positioning precision gets better.

two systematic errors are set as follows: the simulated systematic error of pulsar distance is pulsar distance $\times 10\%$, and the simulated model error of the hydrostatic troposphere delay model is $-\varepsilon \cdot Z/(\sin E \cdot \tan^2 E) \times 10\%$. Then the relation between random error and positioning error is calculated in Sections 3.2.1.1 and 3.2.1.2. It is significant that the appropriate threshold of the observation error can be obtained for a given precision aim in an actual observation. That is to say, observed data with large errors can be rejected and observation modes with small errors can be built for higher positioning precision.

3.2.1.1 The influence of timing measurement precision on positioning Suppose that a station is capable of timing several pulsars at the same second for the point positioning method, then the positioning precision has a direct relation with timing measurement precision from Equation (6), i.e. a TOA measurement error of 0.1 μs corresponds to a positioning error of about 30 m. However, when using

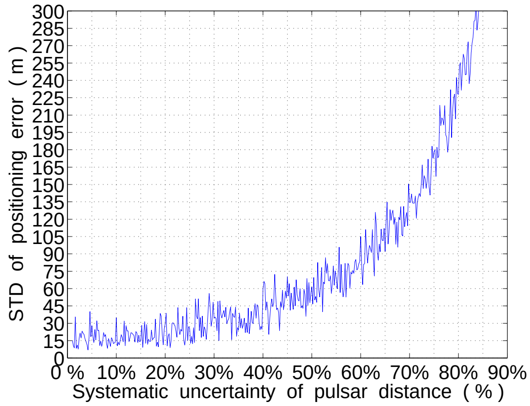

Fig. 9 Influence of systematic uncertainty of a pulsar's distance on the positioning error's standard deviation calculated by regression analysis. The variation trend from small to large is due to the parallax term's error becoming greater. The small curve fluctuation is caused by random errors composed of timing measurement error and pulsar direction error.

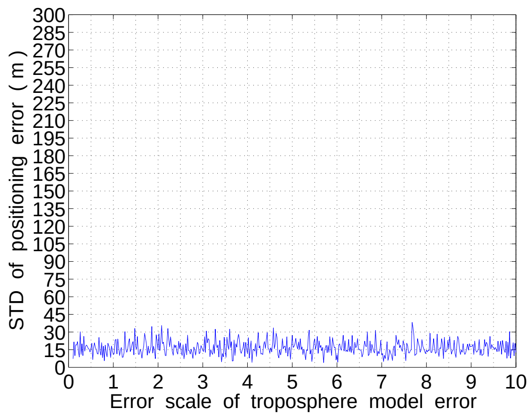

Fig. 10 Influence of the model error of hydrostatic troposphere delay on the positioning error's standard deviation is calculated by regression analysis. Due to modeling error, for an error value 10 times larger, which is still tiny compared to other error sources, the figure shows a flat variation trend. The small curve fluctuation is caused by random errors composed of timing measurement error and pulsar direction error, and the trend determined by the model error of the hydrostatic troposphere delay is covered up by random errors.

regression analysis with a short arc section to describe the actual arc section of the orbit, the influence from random errors is not a simple direct relation. When TOA measurement precision changes, the influence on the positioning result needs to be calculated quantitatively. The other random error is given as typical values, by adding Gaussian random errors of 0.05 mas to pulsar directions. According to the conditions above, the position error's standard deviation (STD) is calculated when a set of Gaussian distributed random timing measurement errors changes from 0.01 to 0.5 μs.

As shown in Figure 7, the abscissa axis is the value of timing precision. For a timing precision of 0.1 μs, the positioning error's standard deviation of 12 points, calculated by regression analysis, is less than 75 m. Therefore, even if we take into account the other error source's influences (set to a typical value), regression analysis is capable of achieving the positioning precision aim of 100 m for a timing precision of 0.1 μs.

The primary methods of increasing TOA measuring precision are improving equipment, technique, observational mode and data processing of the timing observation.

3.2.1.2 The influence of pulsar direction error on positioning Using estimations from the pulsar timing equation of the first order, a 0.1 mas pulsar direction error has a 0.242 μs influence on time delay τ. The estimated result is the same, and a direction error of 0.1 arcsecond can produce a timing error as high as 240 microseconds (Fisher 1996[5]).

The influence of pulsar direction error on positioning needs to be calculated quantitatively. The other random error is given typical values, where Gaussian random errors of 0.05 μs are added to the timing measurements. With the conditions above, the position error's standard deviation is calculated when the Gaussian distributed random errors of right ascension and declination change from 0.01 to 1 mas.

As shown in Figure 8, the abscissa axis is the value of right ascension and declination error, since its error of 0.25 mas and positioning error's standard deviation of 12 points is less than 75 m. Therefore, even if we take into account the other error source's influences (set to typical values), regression analysis is capable of achieving the positioning precision aim of 100 m for a pulsar direction error of 0.25 mas.

3.2.2 Influence of systematic errors on positioning results

The order of magnitude of systematic errors, either pulsar distance error or hydrostatic troposphere delay model error, is increased. Meanwhile, the other systematic error is fixed to typical values, and another two random errors are set as follows: add Gaussian random errors of 0.05 μs to the timing measurement, and add Gaussian random errors of 0.05 mas to pulsar direction, then the relation between systematic errors and positioning errors is calculated in Sections 3.2.2.1 and 3.2.2.2. It is significant that the appropriate systematic error correction model needs to be built for higher positioning precision.

3.2.2.1 The influence of the systematic uncertainty of pulsar distance on positioning Pulsar distance can be obtained from measuring annual parallax or interstellar dispersion, assuming that every pulsar of a spatial distribution has a measured distance value greater than or less than the true value, which can be seen as a kind of systematic error; then the parallax error term of the timing equation decreases or increases with it. Supposing these four pulsars to be positioning references with the measured distance value systematically less than the true value by $B \times e$%, the distance of a pulsar is B, and the systematic uncertainty of distance is e%. The parallax term error from the timing model increases with increasing uncertainty e%. Meanwhile, the other systematic error (the model error of the hydrostatic troposphere delay model) is given typical values: $-\varepsilon \cdot Z/(\sin\boldsymbol{E} \cdot \tan^2\boldsymbol{E}) \times 10\%$. With the conditions above, the position error's standard deviation is calculated when systematic uncertainty distance e% changes from 1% to 85%.

As shown in Figure 9, the abscissa axis is the value of the systematic uncertainty of pulsar distance. If the distance uncertainty is greater than 30%, the parallax term's error becomes larger, and the positioning error increases more quickly.

3.2.2.2 The influence of model error of hydrostatic troposphere delay on positioning The group velocity of radio waves in the ionosphere and troposphere differs from the vacuum speed of light,

[5] NRAO's Charlottesville Operations. *http://www.cv.nrao.edu/ rfisher/*

so the passage of the signal through the atmosphere induces a delay. The tropospheric propagation delay is separated into the so-called "hydrostatic" and "wet" components, and the hydrostatic component contributes approximately 90% of the total delay. The mean sea level value for the hydrostatic troposphere delay is ∼2.3 m at zenith, and the model error of the mapping function is set to be $-\varepsilon \cdot Z/(\sin E \cdot \tan^2 E) \times 10\%$. Consequently, if the pulsar's elevation angles are $6°, 10°$ and $20°$, the model error will be ∼2 m, ∼0.5 m and ∼0.06 m, respectively. The elevations of four pulsars in this section are all higher than $10°$ in the selected observation time; as a result, the influence of the model error of hydrostatic troposphere delay is small for a positioning precision aim of 100 m.

The atmosphere delay term's error increases with increasing model error of the mapping function, indicated as $-\varepsilon \cdot Z/(\sin E \cdot \tan^2 E) \times p$; the error scale is p, increasing from 0.1 to 10. Meanwhile, the systematic error of the pulsar distance is given as typical values: $B \times 10\%$. With the conditions above, the position error's standard deviation is calculated when the error scale p of the mapping function's model error changes from 0.1 to 10.

As shown in Figure 10, the abscissa axis is the value of the error scale of the mapping function's model error. Even if the model error of the hydrostatic troposphere delay becomes 10 times larger, the influence on positioning result is tiny, because when pulsar elevation becomes higher than $15°$, the model error of the hydrostatic troposphere delay decreases more quickly.

4 CONCLUSIONS

We calculate the Earth's coordinate position based on pulsar timing and employ regression analysis of a short arc section of the Earth's orbit. The regression analysis method is not the point positioning method which can simultaneously time different pulsars; it instead uses an approximate short arc section to describe the actual arc section of the orbit. This is a realization of a certain accuracy to calculate the Earth's position. Assuming there are no systematic or random errors, we conclude that regression analysis itself achieves the desired precision. If a sampled rate of TOAs remains unchanged, the number of TOAs will increase along with an increase in the observation time range. If the number of TOAs remains unchanged, the sampled point will become more sparse, while the observation time range will increase. For the two conditions above, if the arc section of the orbit calculated by regression analysis is extended too much, we cannot sufficiently describe the position information of the actual orbit, and the reliability of positioning is lower. For a short arc section within one day, under the required precision (100 m), a quadratic polynomial is able to describe the coordinate component well.

The spatial distribution of pulsars has a systematic effect on the error of the three positioning result components. The positioning result is better with a good spatial configuration than with a poor one. It is also concluded that the positioning result is able to be improved by increasing the number of pulsars with good spatial configuration.

The conclusion of the relation between error sources and positioning errors is made through quantitative calculations. For a timing precision of 0.1 μs, pulsar direction error is about 0.25 mas, or the distance uncertainty is less than the pulsar distance×30%, and regression analysis is able to achieve a positioning precision aim of 100 m. The reliability of the positioning solution of regression analysis is therefore verified. Meanwhile, an appropriate threshold of observation error and systematic error can be obtained.

We simulate a kind of systematic error of the selected pulsar distances. That is, the measured distance value is systematically less than the true value, and the systematic error of every pulsar's distance is equal. As a result, every pulsar in the spatial configuration becomes closer to the solar system, the parallax term's error of the timing model increases with increasing systematic uncertainty, and the positioning errors become greater.

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