Research in Astron. Astrophys. **2011** Vol. **11** No. **8**, 895–901
http://www.raa-journal.org *http://www.iop.org/journals/raa*

***R**esearch in*
***A**stronomy and*
***A**strophysics*

Testing the distance-duality relation with data from galaxy clusters and type Ia supernovae *

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Received 2011 March 25; accepted 2011 May 6

Abstract We test the distance-duality (DD) relation by combining the angular diameter distance $D_{\rm A}$ provided by two galaxy cluster samples compiled by De Filippis et al. (the elliptical β model) and Bonamente et al. (the spherical β model), and the luminosity distance $D_{\rm L}$ from Constitution and Union2 type Ia supernova (SNe Ia) datasets. To obtain $D_{\rm L}$ associated with the observed $D_{\rm A}$ at the same redshift, we smooth the noise of the SNe Ia in a model-independent way, obtain the evolutionary curve of $D_{\rm L}$ and, finally, test the DD relation. We find that the elliptical β model, when compared with the SNe Ia from the Constitution compilation, is only consistent with the DD relation at the 3σ confidence level (CL), while the spherical β model is incompatible with the DD relation at the 3σ CL. For the Union2 compilation, the De Filippis and Bonamente samples are marginally compatible with the validity of the DD relation at the 1σ and 2σ CLs, respectively.

Key words: galaxies: distances and redshifts — galaxies: clusters: general — supernovae: general

1 INTRODUCTION

Etherington (1933) proved, using the requirements that photons travel along null geodesics and that the geodesic deviation equation holds, a reciprocity relation which relates the source angular distance and the observer area distance. If the conservation of the photon number is further assumed, the reciprocity relation reduces to the well-known distance-duality (DD) relation, which is a relation between the luminosity distance $D_{\rm L}$ and the angular diameter distance (ADD) $D_{\rm A}$,

$$\frac{D_{\rm L}}{D_{\rm A}}(1+z)^{-2}=1, \tag{1}$$

where z is the redshift.

* Supported by the National Natural Science Foundation of China.
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It is easy to prove this equation in the context of Friedmann-Lemaître-Roberson-Walker (FLRW) cosmology (Ellis 2007). Being independent of the Einstein field equation and the nature of matter, this equation is valid for all cosmological models based on Riemannian geometry.

The DD relation plays an essential role in astronomical observation and cosmology, such as in the gravitational lensing studies (Schneider 1999) and the analysis of cosmic microwave background radiation (CMBR) observations and galaxy and galaxy cluster observations (Cunha et al. 2007; Mantz et al. 2010). Even the cosmological interpretation of galaxy number count distribution and the optical theorem showing that the surface brightness of an extended source does not depend on the angular diameter distance between the observer and source are both consequences of this relation (Ribeiro & Stoeger 2003).

Due to the fundamental importance of the DD relation, any observational deviation from it would be a theoretical catastrophe (Ellis 2007). So, the verification of the observational validity of this relation is one of the major under-appreciated open problems in observational cosmology. Until now, some works have been done to test this DD relation. For example, Uzan et al. (2004) took a test for it from the observations of the Sunyaev-Zeldovich effect (SZE) and the X-ray surface brightness of galaxy clusters from the Reese et al. (2002) sample and found no deviation from the DD relation at the 1σ confidence level (CL). Using the ADD from galaxy clusters from the Bonamente et al. (2006) sample, de Bernardis et al. (2006) also obtained a non-violation of the DD relation in the framework of the concordance ΛCDM model. Avgoustidis et al. (2010) employed a parametrization $d_{\rm L} = d_{\rm A}(1+z)^{2+\epsilon}$ for the DD relation, in a flat ΛCDM model, to place constraints on cosmic opacity through combining the Constitution Type Ia Supernovae (SNe Ia) data (Kowalski et al. 2008) with the latest measurements of the Hubble expansion within the redshift regions $0 < z < 2$ (Stern et al. 2010), and found $\epsilon = -0.04^{+0.08}_{-0.07}$ at the 2σ CL.

Recently, by proposing the DD relation to be a function of redshift z

$$\frac{D_{\rm L}}{D_{\rm A}}(1+z)^{-2} = \eta(z)$$

with $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1+z)$, and comparing the ADD from the elliptical and spherical β models (used to describe the galaxy clusters) (Bonamente et al. 2006; De Filippis et al. 2005) with the value of $D_{\rm L}$ obtained from the ΛCDM model (Komatsu et al. 2011), Holanda et al. (2010) found that the DD relation is consistent with the elliptical and spherical β models at the 1σ and 3σ CLs, respectively. In addition, the possibility to test new physics based on the validity of the DD relation was done by Bassett & Kunz (2004).

More recently, Holanda et al. (2011) proposed a model-independent cosmological test for the DD relation by employing two sub-samples of SNe Ia carefully chosen from the Constitution data (Hicken et al. 2009) and ADD samples from galaxy clusters with different assumptions concerning the geometry used to describe the clusters: elliptical and spherical β models (Bonamente et al. 2006; De Filippis et al. 2005). They showed that the elliptical β model (De Filippis et al. 2005) is marginally compatible with the validity of the DD relation at the 2σ CL, however, the spherical β model (Bonamente et al. 2006) indicates a strong deviation from this relation. Later, with the latest Union2 SNe Ia data, Li et al. (2011) also tested the DD relation using the same model-independent method and obtained that the DD relation is compatible with the results from the De Filippis et al. and the Bonamente et al. samples at the 1σ and 2σ CLs, respectively. In these two works (Holanda et al. 2011; Li et al. 2011), the selection criterium ($\Delta z = |z_{\rm Cluster} - z_{\rm SNe\ Ia}| < 0.005$) is used to obtain associated SNe Ia data because it is almost impossible to obtain the ADD data and SNe Ia data at the same redshift. With this selection criterium, some ADD data points are discarded when the Constitution SNe Ia data are used. In order to obtain D_L associated with the ADD from the galaxy cluster at the same redshift to test the DD relation, we propose a different method in this paper. We will apply a Gaussian function to smooth the SNe Ia data from the Constitution and Union2 compilations to reconstruct the luminosity distance $D_{\rm L}(z)$ as a smooth function of z so that for each ADD we can find a $D_{\rm L}(z)$, and we then test the validity of the DD relation.

2 THE METHOD AND SAMPLES

Following a procedure used frequently in the analysis of the cosmic large scale structure (Martínez & Saar 2002), Shafieloo et al. (2006); Shafieloo (2007) proposed a model-independent method to extract information of some cosmological parameters, including the expansion rate H, the present matter density parameter $\Omega_{\rm m0}$ and the equation of state of dark energy w, and so on, by using a Gaussian smoothing function to smooth the SNe Ia data in redshift space. This method was generalized by Wu & Yu (2008) to eliminate the impact of H_0. Here, we use this generalized method given in (Wu & Yu 2008) to obtain the evolutionary curve of the luminosity distance to get $D_{\rm L}$ data points associated with the observed $D_{\rm A}$ from galaxy clusters, and then test the DD relation. In order to reconstruct the luminosity distance $D_{\rm L}(z)$, we firstly obtain the variable $\ln f(z) = \ln D_L(z) - \ln h$ by the following iterative method

$$\ln f(z)^{\rm s}_n = \ln f(z)^{\rm s}_{n-1} + N(z)\sum_i \left[\ln f^{\rm obs}(z_i) - \ln f(z)^{\rm s}_{n-1}\right] \exp\left[-\frac{\ln^2\left(\frac{1+z}{1+z_i}\right)}{2\Delta^2}\right] , \quad (2)$$

where $h = H_0/100$ and $N(z)$ is defined as a normalization parameter

$$N(z)^{-1} = \sum_i \exp\left[-\frac{\ln^2\left(\frac{1+z}{1+z_i}\right)}{2\Delta^2}\right] . \quad (3)$$

The effect of the parameter Δ in Equations (2) and (3) has been discussed in detail in (Shafieloo et al. 2006; Shafieloo 2007) and it was found that the results are not sensitive to the chosen value of Δ. Following Shafieloo (2007) and Wu & Yu (2008), we choose $\Delta = 0.6$. Using $f(z)^{\rm s}_n$ in Equation (2), we can get the smoothed luminosity distance at any redshift z after n iterations. When $n = 1$

$$\begin{aligned}\ln f(z)^{\rm s}_1 &= \ln f(z)^{\rm s}_0 + N(z)\sum_i \left[\ln f^{\rm obs}(z_i) - \ln f(z)^{\rm s}_0\right] \exp\left[-\frac{\ln^2\left(\frac{1+z}{1+z_i}\right)}{2\Delta^2}\right] \\ &= \ln D_{\rm L}(z)^{\rm s}_0 + N(z)\sum_i \left[\ln f^{\rm obs}(z_i) - \ln D_{\rm L}(z)^{\rm s}_0\right] \exp\left[-\frac{\ln^2\left(\frac{1+z}{1+z_i}\right)}{2\Delta^2}\right] , \end{aligned} \quad (4)$$

where $D_{\rm L}(z)^{\rm s}_0$ is the luminosity distance of the suggested background model. Shafieloo (2007) found that the results are independent of the assumed background model. Here, the wCDM model with $w = -0.9$ and $\Omega_{\rm m0} = 0.28$ is used as the assumed background model. The variable $\ln f^{\rm obs}(z_i)$ relates to the corresponding quantity $\ln D^{\rm obs}_{\rm L}(z_i)$ which is the observed one from the SNe Ia and can be expressed as

$$\ln f^{\rm obs}(z_i) \equiv \frac{\ln 10}{5}\left[\mu^{\rm obs}(z_i) - 42.38\right] = \ln D^{\rm obs}_{\rm L}(z_i) - \ln h . \quad (5)$$

Here $\mu^{\rm obs}$ is the observed distance modulus of SNe Ia. To determine whether we obtain a best fit value after some iterations, we calculate, after each iteration, χ^2_s

$$\chi^2_{s,n} = \sum_i \frac{[\mu(z_i)_n - \mu^{\rm obs}(z_i)]^2}{\sigma^2_{\mu_{{\rm obs},i}}} . \quad (6)$$

When the value of $\chi^2_{s,n}$ reaches its minimum value, we get the best fit result. For the Constitution and Union2 SNe Ia data, $\chi^2_{s,n}$ reaches its minimum values at $n = 22$ and 28, respectively, as shown

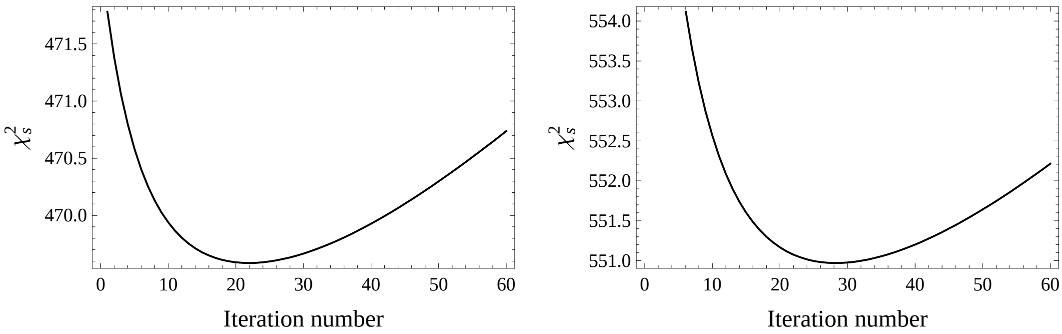

Fig. 1 Computed χ_s^2 for the reconstructed results at each iteration for the SNe Ia data from Constitution (*left* panel) and Union2 (*right* panel).

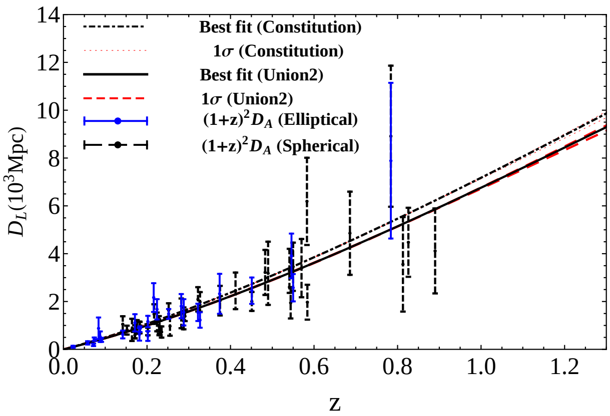

Fig. 2 Galaxy cluster data and the smoothed luminosity distance $D_{\rm L}$ from the Constitution and Union2 SNe Ia. The *solid* line and *dashed* line error bars with the associated middle points stand for the ADD data from the elliptical (De Filippis et al. 2005) and spherical (Bonamente et al. 2006) β models, respectively. The *dot-dashed* line and *solid* line denote the smoothed luminosity distance $D_{\rm L}$ as a function of z from the Constitution and Union2 SNe Ia, respectively.

in Figure 1. So, we can obtain the best fit smoothed luminosity distance at any redshift z. The 1σ corresponds to $\Delta\chi^2_{s,n} = 1$.

In order to test the DD relation by assuming $\eta(z) = D_{\rm L}(z)(1+z)^{-2}/D_{\rm A}(z)$ with $\eta(z) = 1+\eta_0 z$ (Model a) and $\eta(z) = 1 + \eta_0 z/(1 + z)$ (Model b), we use the ADD from the galaxy cluster samples obtained by combining SZE and X-ray surface brightness observations and the luminosity distance $D_{\rm L}$ obtained by smoothing the SNe Ia data compilation. The first galaxy cluster sample contains 25 galaxy clusters (De Filippis et al. 2005), described by an isothermal elliptical β model. The second includes 38 galaxy clusters (Bonamente et al. 2006), whose plasma and matter distributions were analyzed by assuming a hydrostatic equilibrium model and spherical symmetry. For the galaxy cluster samples, the statistical and systematic errors amount to about 20% and 24% (Reese et al. 2002; Reese 2004; Mason et al. 2001; Bonamente et al. 2006) and they are combined in quadrature for the ADD (Holanda et al. 2010; D'Agostini 2004; Bonamente et al. 2006). The distributions of these two ADD samples and the smoothed luminosity distance $D_{\rm L}^{\rm s}(z)$ are shown in Figure 2. From this figure, one can see that the errors of $D_{\rm L}$ at the 1σ CL are much smaller than the ones of the ADD from the galaxy cluster samples. So, we will neglect the errors of the SNe Ia data in our following analysis.

3 ANALYSIS AND RESULTS

To place constraints on η_0, we must firstly obtain $\eta_{\rm obs}(z)$ with $\eta(z) = D_{\rm A}^{\rm cluster}(z)(1+z)^2/D_{\rm L}(z)$ from the galaxy cluster samples and the smoothed luminosity distance, where $D_{\rm A}^{\rm cluster}(z) = D_{\rm A}(z)\eta^2$ is considered (Sunyaev & Zeldovich 1972; Cavaliere & Fusco-Femiano 1978). Thus, using the following equation

$$\chi_D^2 = \sum_z \frac{\left[\eta^2(z) - \eta^2_{\rm obs}(z)\right]^2}{\sigma^2_{\eta_{\rm obs}}}\,, \tag{7}$$

we can obtain the constraints on η_0. Here $\sigma^2_{\eta_{\rm obs}}$ is the error of $\eta_{\rm obs}$. Our results are shown in Figures 3 and 4.

From Figure 3 where the SNe Ia data from Constitution are used, we obtain that for the elliptical β model (De Filippis et al. 2005), $\eta_0 = -0.33^{+0.48}_{-0.48}$ (Model a) and $\eta_0 = -0.49^{+0.67}_{-0.67}$ (Model b) at the 3σ CL, while for the spherical β model (Bonamente et al. 2006), $\eta_0 = -0.33^{+0.29}_{-0.29}$ (Model a) and $\eta_0 = -0.52^{+0.43}_{-0.44}$ (Model b) at the 3σ CL. Thus, we find that the test between ADD from

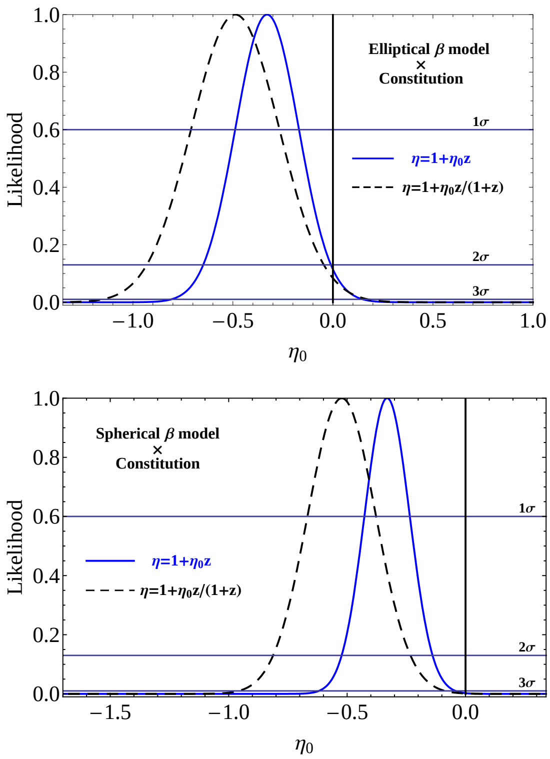

Fig. 3 Likelihood distribution functions from the combination of the ADD from the elliptical (De Filippis et al. 2005) (*upper* panel) and spherical (Bonamente et al. 2006) (*bottom* panel) β models, and the Constitution SNe Ia data.

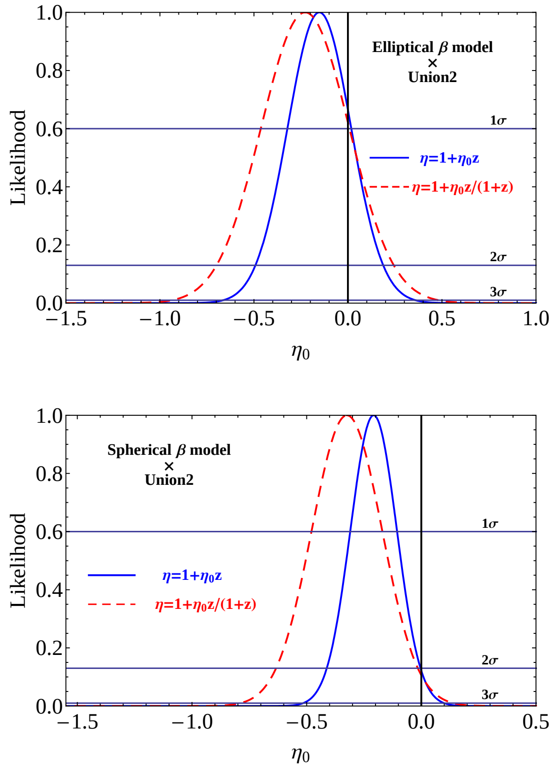

Fig. 4 Likelihood distribution functions from the combination of the ADD from the elliptical (De Filippis et al. 2005) (*upper* panel) and spherical (Bonamente et al. 2006) (*bottom* panel) β model, and the Union2 SNe Ia data.

the elliptical model with the SNe Ia from Constitution indicates the DD relation is only consistent with the observational data at the 3σ CL. However, for the spherical β model, the DD relation is not obeyed even at the 3σ CL.

Our results suggest a stronger violation than that obtained by Holanda et al. (2011), where the result from the elliptical model is compatible with the DD relation at the 2σ CL. In contrast, our result is consistent with that obtained by Li et al. (2011).

The results for the latest Union2 SNe Ia data are shown in Figure 4. We find that, for the elliptical β model, $\eta_0 = -0.15^{+0.17}_{-0.17}$ (Model a) and $\eta_0 = -0.23^{+0.24}_{-0.24}$ (Model b) at the 1σ CL, and for the spherical β model, $\eta_0 = -0.21^{+0.21}_{-0.20}$ (Model a) and $\eta_0 = -0.31^{+0.31}_{-0.31}$ (Model b) at the 2σ CL. Obviously, the elliptical and spherical samples are marginally compatible with the validity of the DD relation at the 1σ and 2σ CLs, which are consistent with the results given by Li et al. (2011).

4 CONCLUSIONS

In this paper, we test the distance-duality (DD) relation by considering the angular diameter distances given by two samples of galaxy clusters (Bonamente et al. 2006; De Filippis et al. 2005) together with the luminosity distance $D_{\rm L}$ reconstructed by smoothing the noise of Constitution and Union2 datasets over redshift with the Gaussian smoothing function. For the Constitution SNe Ia data, we find that the elliptical β model (De Filippis et al. 2005) indicates that this relation is only consistent with the observational data set at the 3σ CL, while the spherical β model (Bonamente et al. 2006) is incompatible with it even at the 3σ CL. However, for the Union2 data set, the elliptical and spherical samples are marginally compatible with the validity of the DD relation at the 1σ and 2σ CLs, respectively. Our results are consistent with those obtained by Li et al. (2011).

Acknowledgements This work was supported in the part by the National Natural Science Foundation of China (Grants Nos. 10935013 and 11075083), the Zhejiang Provincial Natural Science Foundation of China under Grant No. Z6100077, the FANEDD under Grant No. 200922, the National Basic Research Program of China (Grant No. 2010CB832803), the NCET under Grant No. 09-0144, and the PCSIRT under Grant No. IRT0964.

References

Avgoustidis, A., Burrage, C., Redondo, J., Verde, L., & Jimenez, R. 2010, J. Cosmol. Astropart. Phys., 10, 24
Bassett, B. A., & Kunz, M. 2004, Phys. Rev. D, 69, 101305
Bonamente, M., Joy, M. K., LaRoque, S. J., et al. 2006, ApJ, 647, 25
Cavaliere, A., & Fusco-Femiano, R. 1978, A&A, 70, 677
Cunha, J. V., Marassi, L., & Santos, R. C. 2007, International Journal of Modern Physics D, 16, 403
D'Agostini, G. 2004, arXiv:physics/0403086
de Bernardis, F., Giusarma, E., & Melchiorri, A. 2006, International Journal of Modern Physics D, 15, 759
De Filippis, E., Sereno, M., Bautz, M. W., & Longo, G. 2005, ApJ, 625, 108
Ellis, G. F. R. 2007, General Relativity and Gravitation, 39, 1047
Etherington, I. M. H. 1933, Philosophical Magazine, 15, 761
Hicken, M., Wood-Vasey, W. M., Blondin, S., et al. 2009, ApJ, 700, 1097
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B. 2010, ApJ, 722, L233
Holanda, R. F. L., Lima, J. A. S., & Ribeiro, M. B. 2011, A&A, 528, L14
Komatsu, E., Smith, K. M., Dunkley, J., et al. 2011, ApJS, 192, 18
Kowalski, M., Rubin, D., Aldering, G., et al. 2008, ApJ, 686, 749
Li, Z., Wu, P., & Yu, H. 2011, ApJ, 729, L14
Mantz, A., Allen, S. W., Rapetti, D., & Ebeling, H. 2010, MNRAS, 406, 1759
Martínez, V. J., & Saar, E. 2002, Statistics of the Galaxy Distribution (London: Chapman and Hall)
Mason, B. S., Myers, S. T., & Readhead, A. C. S. 2001, ApJ, 555, L11
Reese, E. D. 2004, in Measuring and Modeling the Universe, ed. W. L. Freedman (Cambridge: Cambridge Univ. Press), 138
Reese, E. D., Carlstrom, J. E., Joy, M., et al. 2002, ApJ, 581, 53
Ribeiro, M. B., & Stoeger, W. R. 2003, ApJ, 592, 1
Schneider, P., Ehlers, J., & Falco, E. E. 1999, Gravitational Lenses (New York: Springer)
Shafieloo, A. 2007, MNRAS, 380, 1573
Shafieloo, A., Alam, U., Sahni, V., & Starobinsky, A. A. 2006, MNRAS, 366, 1081
Stern, D., Jimenez, R., Verde, L., Kamionkowski, M., & Stanford, S. A. 2010, J. Cosmol. Astropart. Phys., 2, 8
Sunyaev, R. A., & Zeldovich, Y. B. 1972, Comments on Astrophysics and Space Physics, 4, 173
Uzan, J. P., Aghanim, N., & Mellier, Y. 2004, Phys. Rev. D, 70, 083533
Wu, P., & Yu, H. 2008, J. Cosmol. Astropart. Phys., 2, 19