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First-order phase transitions in rotating hybrid stars and pulsar glitches *

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Abstract The first order deconfinement phase transitions in rotating hybrid stars are studied and it is found that if the surface tension is sufficiently large, the transition from metastable hadron matter to stable mixed hadron-quark matter during the spin-down history of a hybrid star can cause a glitch.

Key words: dense matter — stars: rotation — pulsars: general

1 INTRODUCTION

Glitches are characterized by a sudden increase of pulsar rotation frequency (ν), sometimes followed by an interval of approximately exponential recovery or relaxation back towards the pre-glitch state. Of the 1750 known pulsars, about 170 glitches in 54 pulsars have been observed so far (Zou et al. 2008). Detected glitches are generally of magnitude $10^{-9} < \Delta\nu/\nu < 10^{-6}$, and the largest, with $\Delta\nu/\nu \approx 2.05 \times 10^{-5}$, was observed in PSR B2334+61 (Yuan et al. 2010).

The glitch events and the associated relaxation are usually explained in terms of superfluid vortex dynamics. The standard model describes a glitch as an event in which a significant number of vortices are suddenly unpinned from the crust nuclei, and angular momentum is transferred to the crust (Alpar 1977; Alpar et al. 1989; Ruderman 1991; Andersson et al. 2003). However, Link (2003) showed that the coexistence of superfluid and a superconductor in a neutron star core would quickly damp out the precession and is inconsistent with the observations of freely precessing pulsars (Stairs et al. 2000; Shabanova et al. 2001). Link (2003) and Horvath (2004) suggested that one possible way out from this problem is that an exotic core is present. Zhou et al. (2004) studied the quakes in quark stars and found that the general behaviors of a glitch could be reproduced if the cold strange quark matter is solid.

In this paper, we give an alternative way to understand the glitch phenomenon. We investigate the first order deconfinement phase transitions in rotating hybrid stars and find that the transition from metastable hadron matter to stable mixed hadron-quark matter can cause a glitch. This paper is organized as follows. In Section 2, we review early works about the first order hadron-quark matter phase transition and describe our models of the equation of state in hybrid stars. In Section 3, we briefly give our rotating compact star model. In Section 4, we calculate the value of spin up induced by deconfinement transitions in hybrid stars. In Section 5, we conclude with a discussion.

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2 HADRON-QUARK MATTER PHASE TRANSITION

Quark deconfinement phase transition is expected to occur in neutron matter at densities above the nuclear saturation density $\rho_B = 0.16\,\mathrm{fm}^{-3}$. Many theoretical calculations have suggested that the deconfinement transition should be of first order in a low-temperature and high density area (Pisarski & Wilczek 1984; Gavai et al. 1987). In early studies, the first order deconfinement phase transitions were described using a Maxwell construction (MC), as in the liquid-vapor phase transition of water (Baym & Chin 1976).

Glendenning (1992) first realized that for a first order hadron-quark phase transition, charge neutrality can be achieved by positively charged hadron matter and negatively charged quark matter, while both of them must have neutral charge in the MC. As a result, one may expect a mixed phase during the transition. However, he simply considered a mixed phase consisting of two sections of bulk matter separated by a sharp boundary without any surface tension or Coulomb interaction, which was named "bulk Gibbs" (Endo et al. 2006).

This "bulk Gibbs" is too simple for studying the mixed phase, since non-uniform structures should be considered and the mixed phase should have various geometrical structures. Heiselberg et al. (1993) studied a geometrical structure in the mixed phase by considering the spherical quark droplets embedded in hadron matter. They treated the surface tension σ as a free parameter and found that the mixed region became smaller if σ was bigger, but the region of the mixed phase cannot exist if σ was too large ($\sigma \geq 90\,\mathrm{MeV\,fm}^{-2}$). Glendenning & Pei (1995) calculated more geometrical structures, "droplets," "rods," "slabs," "tubes" and "bubbles," and suggested the mixed phase had a certain connection with glitch behavior. However, in their treatment, the rearrangement effect of charged particles in the presence of the Coulomb interaction (the charge screening effect) was completely disregarded. Voskresensky et al. (2002) and Tatsumi et al. (2003) studied the charge screening effect using a linear approximation to analytically solve the Poisson equation. Endo et al. (2006) pointed out that the linear approximation was inapplicable if the Coulomb interaction effect was large and they numerically studied the charge screening effect on the structured mixed phase in a self-consistent way. They found that both the surface tension effects and the charge screening effects would restrict the region of the mixed phase, so the equation of state for the mixed phase became similar to that given by MC. They came to the conclusion that the MC would again effectively gain the associated physical meaning in a first order hadron-quark transition.

In the present paper, we use the equation of state (EOS) in relativistic mean-field theory (RMF) for the description of hadronic matter and EOS in the MIT bag model for quark matter. For hadronic matter, we consider the simple n, p, and e components, and choose one group of parameters given in Glendenning (1997): the coupling constants $g_\sigma/m_\sigma = 9.927\,\mathrm{fm}^2$, $g_\omega/m_\omega = 4.820\,\mathrm{fm}^2$, and $g_\rho/m_\rho = 4.791\,\mathrm{fm}^2$ and the scalar self interaction coefficients $b = 0.008659$ and $c = -0.002421$ (these parameters are obtained by fitting nuclear saturation values $\rho_0 = 0.153\,\mathrm{fm}^{-3}$, $B/A = 16.3\,\mathrm{MeV}$, $a_{\mathrm{sym}} = 32.5$, $K = 240\,\mathrm{MeV}$ and $m^* = 0.78 m_N$). For quark matter, we take $m_u = m_d = 0$, $m_s = 150\,\mathrm{MeV}$, Bag constant $B^{1/4} = 175\,\mathrm{MeV}$ and QCD structure constant $\alpha_s = 0$.

We show the EOS in Figure 1. There appears to be no region of the mixed phase shown by calculation with MC, but there is a wide region of the mixed phase shown by the "bulk Gibbs" calculation. Both the dotted line and the dashed line in Figure 1 are not the exact first order phase transition curve when the surface tension effects and the charge screening effects are considered and, as studied by Endo et al. (2006), the exact first phase transition line should be located between them (see fig. 12 in Endo et al. 2006). The exact transition curve can be very close to the dotted line given by MC if the surface tension parameter (σ) is large enough (σ is treated as a free parameter since the hadron quark interface is not clearly understood).

Since our interest lies with the hadron-quark phase transition during the spin down of the hybrid star and not the surface tension effects and the charge screening effects themselves, we take the

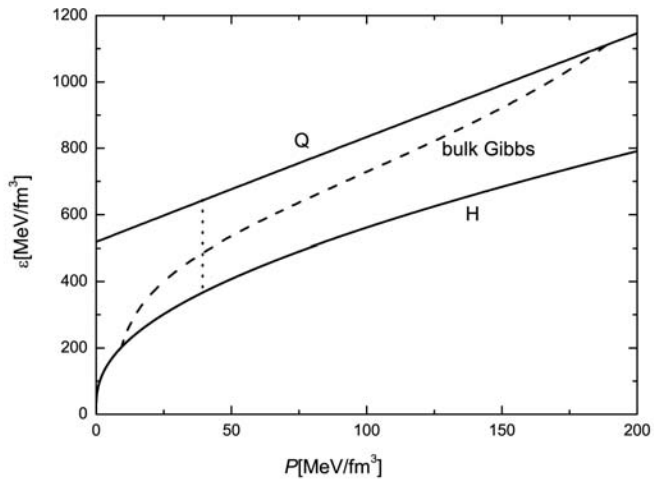

Fig. 1 Energy density as a function of pressure. The dotted line shows the critical pressure of the first order phase transition in the MC method. The dashed line corresponds to the mixed phase and is given by the "bulk Gibbs" method.

following simple process to get the exact phase transition line. We first fit the "bulk Gibbs" curve linearly, and the resulting expression is $\epsilon = 4.403P + 279.363$. The intersection point between the fitted line and the MC phase transition curve is (39.938, 455.213). We assume that the exact phase transition curve is a straight line passing (39.938, 455.213) with slope k larger than 4.403. Now, we have the parameter k that describes the location of the exact phase transition curve: a larger k means the exact phase transition line is closer to the MC one, and corresponds to a larger surface tension (σ). Of course, this treatment is a rough one, but it is reasonable especially for a large k (or a large σ).

3 ROTATING COMPACT STAR MODEL

Using Hartle's perturbation theory (Hartle 1967), Chubarian et al. (2000) and Kang & Zheng (2007) studied the change of the internal structure of the hybrid stars due to rotation. In this paper, we also apply Hartle's approach to investigate the structure and the moment of inertia of rotating hybrid stars following Kang & Zheng (2007) and Benhar et al. (2005).

Hartle's formalism is based on the treatment of a rotating star as a perturbation on a non-rotating star, expanding the metric of an axially symmetric rotating star in even powers of the angular velocity Ω. The metric of a slowly rotating star to second order in the angular velocity Ω can be written as

$$ds^2 = -e^{\nu(r)}[1 + 2(h_0 + h_2 P_2)]dt^2 + e^{\lambda(r)}\left[1 + \frac{2(m_0 + m_2 P_2)}{(r - 2M(r))}\right]dr^2$$
$$+r^2[1 + 2(v_2 - h_2)P_2]\{d\theta^2 + \sin^2\theta[d\phi - w(r,\theta)dt]^2\} + O(\Omega^3), \quad (1)$$

where $e^{\nu(r)}$, $e^{\lambda(r)}$ and $M(r)$ are functions of r and describe the non-rotating star solution of the Tolman-Oppenheimer-Volkov (TOV) equations. $P_2 = P_2(\theta)$ is the $l = 2$ Legendre polynomials. Here ω is the angular velocity of the local inertial frame and is proportional to the star's angular velocity Ω, whereas the perturbation functions h_0, h_2, m_0, m_2, and v_2 are proportional to Ω^2. We assume that matter in the star is described by a perfect fluid with energy momentum tensor

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}. \quad (2)$$

The energy density and pressure of the fluid are affected by the rotation because the rotation deforms the star. In the interior of the star at given (r, θ), in a reference frame that is momentarily moving with the fluid, the variations in pressure and energy density are respectively

$$\delta P(r,\theta) = [\epsilon(r) + P(r)][p_0^* + p_2^* P_2(\theta)], \tag{3}$$

$$\delta\epsilon(r,\theta) = \frac{d\epsilon}{dP}[\epsilon(r) + P(r)][p_0^* + p_2^* P_2(\theta)], \tag{4}$$

where p_0^* and p_2^* are dimensionless functions of r, proportional to Ω^2, which describe the pressure perturbation. The rotational perturbations of the star's structure are described by the functions $h_0, m_0, p_0^*, h_2, m_2, v_2$, and p_2^*. These functions are calculated from Einstein's field equations. The effect of rotation described by the metric on the shape of the star can be divided into two contributions: One is a spherical expansion which changes the radius of the star, which is described by the functions h_0 and m_0. The other part is a quadrupole deformation, described by functions h_2, v_2 and m_2.

As a consequence of these contributions, the difference between the gravitational mass of the rotating star and the non-rotating star with the same central pressure is

$$\delta M_{\rm grav} = m_0(R) + \frac{J^2}{R^3}. \tag{5}$$

The change in the radius of the star is given by

$$\delta R = \xi_0(R) + \xi_2(R) P_2(\theta). \tag{6}$$

The expansion of total baryon numbers in powers of Ω is

$$N_{\rm B} = N_{\rm B}^0 + \delta N_{\rm B} + O(\Omega^4), \tag{7}$$

where

$$N_{\rm B}^0 = \int_0^R n_{\rm B}(r)[1 - 2M(r)/r]^{-1/2} 4\pi r^2 dr \tag{8}$$

is the total number of baryons in a non-rotating star and

$$\delta N_{\rm B} = \frac{1}{m_N}\int_0^R \left(1 - \frac{2M(r)}{r}\right)^{-1/2} \left\{\left[1 + \frac{m_0(r)}{r - 2M(r)} + \frac{1}{3}r^2\left[\Omega - \omega(r)\right]^2 e^{-\nu}\right] n_{\rm B}(r) + \frac{dn_{\rm B}(r)}{dP}(\epsilon + P) p_0^*(r)\right\} 4\pi r^2 dr, \tag{9}$$

where $n_{\rm B}(r)$ is baryon number density.

The expansion of the moment of inertia is

$$I = I^0 + \delta I + O(\Omega^4) = \frac{J}{\Omega} + \frac{\delta J}{\Omega} + O(\Omega^4), \tag{10}$$

where J is the angular momentum to the first order of Ω and δJ is the angular momentum to the third order. Our calculation of J and δJ follows Benhar et al. (2005).

4 FIRST ORDER DECONFINEMENT TRANSITIONS DURING THE SPIN DOWN OF HYBRID STARS

As the star spins down under electromagnetic torque, the centrifugal force continuously decreases, resulting in the continuous increase of its internal density and pressure (Kang & Zheng 2007; Glendenning et al. 1997). Considering the spin down of a hybrid star, the increase of internal density means more and more neutron matter converts into strange quark matter through the first order phase transition. However, the conversion can be more complicated. In a first order phase transition, the new phase can appear only via nucleation. Therefore, the star first acquires a metastable shell in the normal phase, in which an exotic phase nucleates (Schaeffer et al. 1983; Zdunik et al. 1987; Bejger et al. 2005). Since the MC equation of state is a metastable one compared with the EOS of the exact first order phase transition, including the surface tension effects and the charge screening effects (Endo et al. 2006), we suppose the phase transition occurs at the critical pressure of the MC phase transition.

We use Figures 2 and 3 to explicitly depict the first order transition in a spinning-down hybrid star. Let us consider a hybrid star with rotation frequency ν_0, its EOS is the solid line in Figure 2(a) and the structure of its interior is shown in Figure 3(a). The pressure of matter at the boundary of the grey area ($r = R_{\rm C}^1$) in Figure 3(a) is $P_{\rm H}^{\rm (M)}$, which means the grey core of the star consists of pure strange quark matter and mixed hadron-quark matter, and the rest is filled with pure stable hadron matter. As the star spins down, the matters in its interior are compressed, and the pressure of pure hadron matter in the vicinity of $r = R_{\rm C}^1$ increases and becomes metastable hadron matter (hadron matter with pressure larger than $P_{\rm H}^{\rm (M)}$). When the star continuously spins down, the shell of the metastable hadron matter become larger and larger. The deconfinement phase transition occurs at the inner boundary of this metastable hadron shell once its pressure increases to P_0. The occurrence of a phase transition destroys the hydrostatic equilibrium of the star and quickly results in the whole metastable hadron matter shell's transition into the stable hadron-quark mixed phase (Bejger et al. 2005). The transition is accompanied by a core-quake and a sudden spin-up of rotation, which is one possible reason for the glitch phenomenon. Let us assume the rotation frequency of the star is ν

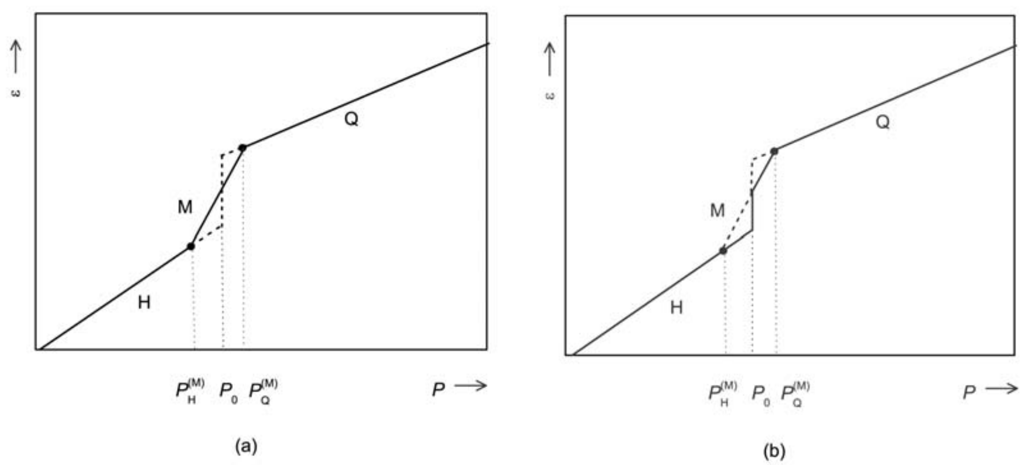

Fig. 2 Schematic diagram of EOS in rotating hybrid stars. P_0 is the critical pressure of the first phase transition in the MC method. The solid line between $P_{\rm H}^{\rm (M)}$ and $P_{\rm Q}^{\rm (M)}$ of (a) is the mixed phase line including the surface tension effects and the charge screening effects.

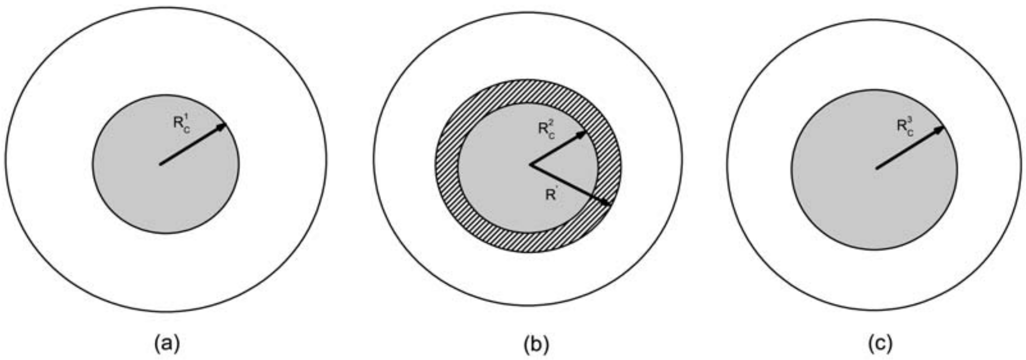

Fig. 3 Schematic diagram of structure of the stars. The components of the grey region are pure strange quark matter and hadron-quark mixed matter, the components of the blank region of the star are stable hadron matter, and the dashed shell is filled with metastable hadron matter.

($\nu < \nu_0$) when the phase transition occurs, the EOS is the solid curve in Figure 2(b) and the structure is as shown in Figure 3(b). Immediately after the phase transition, the EOS returns to the solid curve in Figure 2(a), the structure becomes Figure 3(c), and the spin frequency can be regarded as ν (since we aim to explain glitches, the value of this spin-up causing the star-quake is very small relative to ν).

We numerically study the evolution of the $1.4\,M_\odot$ (non-rotating gravitational mass) star to show whether the phase transition mentioned above can induce a glitch. First, the TOV equations are solved using the EOS of the solid line in Figure 2(b) at fixed k (each phase-curve is given using the model in Section 2), and we can get the total baryon number of the $1.4M_\odot$ star for each k. Then, for an assigned value of spin frequency ν, the equations of the rotating star are solved in order to get the moment of inertia I of hybrid star configurations as shown in Figure 3(b) and Figure 3(c). Of course, in doing so, we select the proper pressure for the star's center to make sure the star configurations have the same total baryon number as the $1.4\,M_\odot$ non-rotating star. The moment of inertia of these two configurations at fixed k and fixed ν has a difference and this difference is caused by the sudden transition of the metastable hadron shell into a hadron-quark mixed phase and the shrinking of the whole star. We show the difference in moment of inertia ΔI in Table 1. Due to the conservation of

Table 1 ΔI (in units of $10^{45}\,\mathrm{g\,cm^2}$) for different k and different rotation frequency ν. The non-rotating star mass is taken as $1.4M_\odot$.

k	$\nu = 100\,\mathrm{Hz}$	$\nu = 10\,\mathrm{Hz}$	$\nu = 1\,\mathrm{Hz}$	$\nu = 0.1\,\mathrm{Hz}$
5×10^2	-1.3×10^{-3}	-1.4×10^{-3}	-1.4×10^{-3}	-1.4×10^{-3}
5×10^3	-1.3×10^{-4}	-1.4×10^{-4}	-1.5×10^{-4}	-1.5×10^{-4}
5×10^4	-6.1×10^{-6}	-1.3×10^{-5}	-1.6×10^{-5}	-1.7×10^{-5}

Table 2 $\frac{\Delta\Omega}{\Omega}$ for different k and different rotation frequency ν. The non-rotating star mass is taken as $1.4\,M_\odot$.

k	$\nu = 100\,\mathrm{Hz}$	$\nu = 10\,\mathrm{Hz}$	$\nu = 1\,\mathrm{Hz}$	$\nu = 0.1\,\mathrm{Hz}$
5×10^2	1.4×10^{-3}	1.5×10^{-3}	1.5×10^{-3}	1.5×10^{-3}
5×10^3	1.4×10^{-4}	1.5×10^{-4}	1.7×10^{-4}	1.7×10^{-4}
5×10^4	6.7×10^{-6}	1.4×10^{-5}	1.8×10^{-5}	1.9×10^{-5}

stellar angular momentum, the sudden decrease in moment of inertia results in the increase of spin frequency,

$$\frac{\Delta\Omega}{\Omega} = -\frac{\Delta I}{I}. \tag{11}$$

Table 2 shows our results of $\frac{\Delta\Omega}{\Omega}$.

5 CONCLUSIONS AND DISCUSSION

As shown in Table 2, if k is as large as 5×10^4 (which means the surface tension σ must be sufficiently large), the star quake caused by the sudden transition of the metastable hadron shell into the hadron-quark mixed phase can generate an increase of spin frequency on the order of 10^{-5}, which is in agreement with the largest glitch observed in PSR B2334+61.

We summarize our discussion in the following:

First, the essential idea that spinning-down neutron stars may undergo a hadron-quark phase transition was provided by Ma & Xie more than ten years ago (Ma & Xie 1996; Ma 1998). However, we propose a totally different scheme of the deconfinement transition from metastable hadron matter to hadron-quark mixed matter. Moreover, only exceptionally large glitches can be generated by mechanisms described in those early works. In addition, considering the recently studied surface tension effects and charge screening effects of hadron-quark mixed matter, we found that the largest observed glitch can be explained if the surface tension is sufficiently large.

Secondly, our star quake mechanism for pulsar glitches can also work in a hybrid star accreting matter in a close binary system, where the internal pressure increases due to the gravity of accumulated layers of accreted matter.

Thirdly, if the surface tension is not sufficiently large as we supposed in this paper, the mixed phase region will be larger. The larger mixed phase region in the EOS will result in a large mixed phase area in the $1.4\,M_\odot$ hybrid star under our hadron and quark matter model. Since the mixed phase contains geometrical structures and has a large shear modulus, a normal star quake without a deconfinement phase transition can occur. This star quake can be the reason for glitches as shown by Zhou et al. (2004).

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