

# Cosmological constraints on the DGP braneworld model with gamma-ray bursts \*

Nan Liang and Zong-Hong Zhu

Department of Astronomy, Beijing Normal University, Beijing 100875, China; [liangn@bnu.edu.cn](mailto:liangn@bnu.edu.cn); [zhuzh@bnu.edu.cn](mailto:zhuzh@bnu.edu.cn)

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**Abstract** We investigate observational constraints on the Dvali, Gabadadze and Porrati (DGP) model with Gamma-ray bursts (GRBs) at high redshift obtained directly from the Union2 Type Ia supernovae data set. With cosmology-independent GRBs, the Union2 set, as well as the cosmic microwave background observation from the WMAP7 result, the baryon acoustic oscillation, the baryon mass fraction in clusters and the observed  $H(z)$  data, we obtain that the best-fit values of the DGP model are  $\{\Omega_{M0}, \Omega_{r_c}\} = \{0.235^{+0.015}_{-0.014}, 0.138^{+0.051}_{-0.048}\}$ , which favor a flat universe, and the transition redshift of the DGP model is  $z_T = 0.67^{+0.03}_{-0.04}$ . These results lead to more stringent constraints than the previous results for the DGP model.

**Key words:** Gamma rays: bursts — cosmology: cosmological parameters

## 1 INTRODUCTION

The accelerating expansion of the current universe has been confirmed by recent cosmological observations, such as Type Ia supernovae (SNe Ia; Riess et al. 1998; Perlmutter et al. 1999; Amanullah et al. 2010), cosmic microwave background (CMB; Bennett et al. 2003; Spergel et al. 2003; Komatsu et al. 2010), large scale structure (LSS, Tegmark et al. 2004; Eisenstein et al. 2005), as well as the X-ray gas mass fraction of clusters (Allen et al. 2004). By assuming a basis in General Relativity, a dark energy component with negative pressure in the universe has been invoked as the most feasible mechanism for the acceleration. In addition to the cosmological constant (the  $\Lambda$ CDM model), many candidates for dark energy have been taken into account. Examples include the scalar field models with a dynamical equation of state [e.g., quintessence (Ratra & Peebles 1988; Caldwell et al. 1998; Choudhury & Padmanabhan 2005), phantom (Caldwell 2002; Wu & Yu 2006), quintom (Feng et al. 2005; Guo et al. 2005; Liang et al. 2009), k-essence (Armendariz-Picon et al. 2001; Chiba 2002), tachyon (Padmanabhan 2002; Frolov et al. 2002)], the Chaplygin gas (Kamenshchik et al. 2001) and the generalized Chaplygin gas model (GCG, Bento et al. 2002; Zhu 2004), the holographic dark energy (Cohen et al. 1999; Li 2004), the agegraphic dark energy (Cai 2007; Wei & Cai 2008), the Ricci dark energy (Gao et al. 2009) and so on.

On the other hand, many alternatives to dark energy in which gravity is modified have been proposed as a possible explanation for the acceleration. Examples include the  $f(R)$  theory in which the Einstein-Hilbert action has been modified (Capozziello & Fang 2002; Vollick 2003; Carroll et al.

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2004), the Cardassian expansion model in which the Friedmann equation is modified by adding an extra Cardassian term (Freese & Lewis 2002; Wang et al. 2003; Zhu & Fujimoto 2002, 2003) as well as the braneworld models, in which our observable universe is considered to be a brane embedded in a higher dimensional bulk spacetime and the leakage of gravitational force propagating into the bulk can lead to the current accelerated expansion of the universe (Randall & Sundrum 1999).

In 2000, Dvali, Gabadadze and Poratti proposed a 5-dimensional brane world model in which a self-accelerating branch is included (the so-called DGP model, Dvali et al. 2000). The dynamics of gravity is governed by a competition between a Ricci scalar term in the 4-dimensional brane and an Einstein-Hilbert action in the 5-dimensional bulk. The Friedmann equation of the DGP model is modified as

$$H^2 = H_0^2 \left[ \Omega_K(1+z)^2 + \left( \sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_{M0}(1+z)^3} \right)^2 \right], \quad (1)$$

where  $H$  is the Hubble parameter as a function of redshift  $z$ ,  $\Omega_{M0}$  and  $\Omega_K$  represent the fractional contribution of matter and curvature, respectively, and  $\Omega_{r_c} = 1/4r_c^2 H_0^2$  is the bulk-induced term with respect to the crossover radius  $r_c$ . For scales below  $r_c$ , the induced 4-dimensional Ricci scalar dominates and the gravitational force is the usual  $1/r^2$  behavior; whereas for distance scales larger than  $r_c$ , the gravitational force follows the 5-dimensional  $1/r^3$  behavior. The normalization condition can be given by  $\Omega_K + (\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_{M0}})^2 = 1$ ; for a spatially flat scenario,  $\Omega_{r_c} = (1 - \Omega_{M0})^2/4$ .

The DGP model is a testable scenario with the same number of parameters as the standard  $\Lambda$ CDM model and has been constrained from many observational data, such as SNe Ia (Deffayet et al. 2002; Avelino & Martins 2002; Zhu & Alcaniz 2005; Maartens & Majerotto 2006; Barger et al. 2007; Rebouças 2008), the angular size of compact radio sources (Alcaniz 2002), the baryon mass fraction in clusters of galaxies (CBF) from the X-ray gas observation (Zhu & Alcaniz 2005; Alcaniz & Zhu 2005), CMB (Lazkoz et al. 2006; Rydbeck et al. 2007; He et al. 2007), large scale structures (Multamäki et al. 2003; Lue et al. 2004; Koyama & Maartens 2006; Song et al. 2007) the baryon acoustic oscillation (BAO) peak (Guo et al. 2006), the observed Hubble parameter  $H(z)$  data (Wan et al. 2007), gravitational lensing surveys (Jain et al. 2002; Zhu & Sereno 2008), age measurements of high- $z$  objects (Alcaniz et al. 2002) and the lookback time to galaxy clusters (Pires et al. 2006), as well as some different combined data (Bento et al. 2006; Davis et al. 2007; Movahed et al. 2009; Xia 2009; Li et al. 2010). See Lue (2006) for a review of the related DGP phenomenology.

Recently, gamma-ray bursts (GRBs) have been proposed as distance indicators and regarded as a complementary cosmological probe to the universe at high redshift (Schaefer 2003, 2007; Bloom et al. 2003; Dai et al. 2004; Ghirlanda et al. 2004; Friedman & Bloom 2005; Firmani et al. 2005, 2006; Liang & Zhang 2005; Bertolami & Silva 2006; Ghirlanda et al. 2006; Wright 2007; Wang et al. 2007; Amati et al. 2008; Basilakos & Perivolaropoulos 2008; Mosquera Cuesta et al. 2008a,b; Qi et al. 2008a,b). For constraints on the DGP model from GRBs with their associated joint observations, see some recent works, e.g., Wang et al. (2009a), Wei (2010a) and Xu & Wang (2010). However, the empirical luminosity relations of GRBs have usually been calibrated by assuming a certain cosmological model with particular model parameters, due to the lack of a low-redshift sample. Therefore, the calibrations are always cosmology-dependent and the so-called circularity problem occurs in GRB cosmology. The circularity problem cannot be avoided completely by means of statistical approaches (Schaefer 2003; Li et al. 2008; Wang 2008; Samushia & Ratra 2010; Xu 2010), because an input cosmological model is still required. Liang et al. (2008) presented a new method to calibrate GRB luminosity relations in a completely cosmology-independent way: GRB samples in the redshift range of SNe Ia are enough to calibrate GRB relations and their luminosity distances can be obtained directly from SNe Ia by the interpolation method or by another similar approach (Liang & Zhang 2008; Kodama et al. 2008; Cardone et al. 2009; Gao et al. 2010; Capozziello & Izzo 2010). Following the cosmology-independent GRB calibration method, the derived GRB data

at high redshift can be used to constrain cosmological models by using the standard Hubble diagram method (Capozziello & Izzo 2008; Izzo et al. 2009; Wei & Zhang 2009; Wei 2009; Qi et al. 2009; Wang et al. 2009a,b; Wang & Liang 2010; Liang et al. 2010a,b, 2011; Wei 2010a,b; Freitas et al. 2010; Demianski et al. 2010).

Very recently, Liang et al. (2010b) calibrated GRB data at high redshift directly from the Union2 data set containing a compilation of 557 SNe Ia (Amanullah et al. 2010) and constrained the Cardassian model (Liang et al. 2010b) and the generalized Chaplygin gas (GCG) model (Liang et al. 2011) by combining the updated GRB data with the joint observations, such as the Union2 set of SNe Ia, the CMB observation from the seven-year data of the Wilkinson Microwave Anisotropy Probe (WMAP7; Komatsu et al. 2010) result and the BAO observation from the spectroscopic Sloan Digital Sky Survey (SDSS) galaxy sample (Eisenstein et al. 2005).

In this paper, we investigate observational constraints on the DGP model, including the updated distance moduli of the GRBs at high redshift obtained directly from the Union2 set. We combine the GRB data with the joint observations such as the Union2 set, the CMB observation from the WMAP7 result; the BAO observation from the spectroscopic SDSS galaxy sample (Eisenstein et al. 2005), the baryon mass fraction in clusters of galaxies from the X-ray gas observation (Allen et al. 2004), and the observed Hubble parameter data ( $H(z)$ ; Simon et al. 2005; Gaztañaga et al. 2009). We also obtain the transition redshift  $z_T$  of the DGP model. We find that the combination of these recent data sets tightens constraints on the DGP model, which favors a flat universe. The paper is organized as follows. In Section 2, we introduce the analysis of the observational data including the updated cosmology-independent GRBs, as well as the Union2 SNe Ia set, the CMB observations from the WMAP7 result, in addition to the BAO, CBF and  $H(z)$  data. In Section 3, we present results which put constraints on the DGP model from the joint observations. Conclusions and discussions are given in Section 4.

## 2 OBSERVATIONAL DATA ANALYSIS

The recent Union2 compilation consists of a 557 SNe Ia data set (Amanullah et al. 2010). In this paper, we use the updated distance moduli of the 42 GRBs at  $z > 1.4$  (Liang et al. 2010b), which are obtained by the five luminosity relations (Schaefer 2007) calibrated with the sample at  $z \leq 1.4$  by using the linear interpolation method from the Union2 set. For more details about the calculation, see (Liang et al. 2008, 2010a). Constraints from SNe Ia and GRB data can be obtained by fitting the distance moduli  $\mu(z)$ . A distance modulus can be calculated as

$$\mu = 5 \log \frac{d_L}{\text{Mpc}} + 25 = 5 \log_{10} D_L - \mu_0, \quad (2)$$

where  $\mu_0 = 5 \log_{10}[H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})] + 42.38$ , and the luminosity distance  $D_L$  is calculated by

$$D_L \equiv H_0 d_L = (1+z) \Omega_K^{-1/2} \text{sinn} \left[ \Omega_K^{1/2} \int_0^z \frac{dz'}{E(z')} \right], \quad (3)$$

where  $\text{sinn}(x)$  is  $\sinh$  for  $\Omega_K > 0$ ,  $\sin$  for  $\Omega_K < 0$ , and  $x$  for  $\Omega_K = 0$ . The  $\chi^2$  value of the observed distance moduli can be calculated by

$$\chi_\mu^2 = \sum_{i=1}^N \frac{[\mu_{\text{obs}}(z_i) - \mu(z_i)]^2}{\sigma_{\mu,i}^2}, \quad (4)$$

where  $\mu_{\text{obs}}(z_i)$  are the observed distance moduli for SNe Ia and/or GRBs at redshift  $z_i$  with error  $\sigma_{\mu,i}$ ;  $\mu(z_i)$  are the theoretical values of distance modulus from cosmological models. Following an

effective approach (Nesseris & Perivolaropoulos 2005), we marginalize the nuisance parameter  $\mu_0$  by minimizing

$$\hat{\chi}_\mu^2 = C - B^2/A, \quad (5)$$

where  $A = \sum 1/\sigma_{\mu,i}^2$ ,  $B = \sum [\mu_{\text{obs}}(z_i) - 5 \log_{10} D_L]/\sigma_{\mu,i}^2$ , and  $C = \sum [\mu_{\text{obs}}(z_i) - 5 \log_{10} D_L]^2/\sigma_{\mu,i}^2$ .

For the CMB observation from the WMAP7 result (Komatsu et al. 2010), the shift parameter is constrained to be  $R = 1.725 \pm 0.018$ , which can be expressed as (Bond et al. 1997)

$$R = \Omega_{M0}^{1/2} \Omega_K^{-1/2} \text{sinn} \left[ \Omega_K^{1/2} \int_0^{z_{\text{rec}}} \frac{dz}{E(z)} \right], \quad (6)$$

where  $z_{\text{rec}}$  is the redshift of recombination which is given by (Hu & Sugiyama 1996)

$$z_{\text{rec}} = 1048 [1 + 0.00124(\Omega_b h^2)^{-0.738} (1 + g_1(\Omega_{M0} h^2)^{g_2})], \quad (7)$$

where  $g_1 = 0.0783(\Omega_b h^2)^{-0.238}(1 + 39.5(\Omega_b h^2)^{-0.763})^{-1}$  and  $g_2 = 0.560(1 + 21.1(\Omega_b h^2)^{1.81})^{-1}$ . From the WMAP7 result (Komatsu et al. 2010),  $z_{\text{rec}} = 1091.3$ . The  $\chi^2$  value of the shift parameter can be calculated by

$$\chi_{\text{CMB}}^2 = \frac{(R - 1.725)^2}{0.018^2}. \quad (8)$$

For the BAO observation from the SDSS spectroscopic sample of luminous red galaxies, the distance parameter is measured to be  $A = 0.469(n_s/0.98)^{-0.35} \pm 0.017$  (Eisenstein et al. 2005), with the scalar spectral index  $n_s = 0.963$  from the WMAP7 result (Komatsu et al. 2010). The distance parameter can be expressed as

$$A = \Omega_{M0}^{1/2} z_{\text{BAO}}^{-2/3} E(z_{\text{BAO}})^{-1/3} \Omega_k^{-1/2} \text{sinn} \left[ \Omega_K^{1/2} \int_0^{z_{\text{BAO}}} \frac{dz}{E(z)} \right]^{2/3}, \quad (9)$$

where  $z_{\text{BAO}} = 0.35$ . The  $\chi^2$  value of the distance parameter can be calculated by

$$\chi_{\text{BAO}}^2 = \frac{(A - 0.467)^2}{0.017^2}. \quad (10)$$

The baryon mass fraction in clusters of galaxies from the X-ray gas ( $f_{\text{gas}}$ ) observation can be used to constrain cosmological parameters. On the assumption that the gas mass fraction in clusters is a constant and thus independent of redshift, Allen et al. (2004) obtained 26 observational  $f_{\text{gas}}$  data values. The baryon gas mass fraction  $f_{\text{gas}}$  can be presented as

$$f_{\text{gas}}(z) = \lambda \left[ \frac{d_A^{\text{SCDM}}(z)}{d_A(z)} \right]^{2/3}, \quad (11)$$

where  $d_A \equiv d_L/(1+z)^2$  is the theoretical value of the angular diameter distance from cosmological models,  $d_A^{\text{SCDM}}$  is the angular diameter distance corresponding to the standard cold dark matter model (SCDM,  $\Omega_{M0} = 1$  for a flat universe),  $\lambda = [b\Omega_b(2h)^{3/2}]/[(1+a)\Omega_{M0}]$ ,  $a = 0.19\sqrt{h}$ , and  $b$  is a bias factor motivated by dynamical gas simulations. The  $\chi^2$  value of a cluster's baryon gas mass fraction (CBF) is

$$\chi_{\text{CBF}}^2 = \sum_{i=1}^{N=26} \frac{[f_{\text{gas}}^{\text{obs}}(z_i) - f_{\text{gas}}(z_i)]^2}{\sigma_{f_{\text{gas}},i}^2}. \quad (12)$$

The parameter  $\lambda$  can be treated as a nuisance parameter by minimizing (Nesseris & Perivolaropoulos 2007)

$$\hat{\chi}_{\text{CBF}}^2 = C - B^2/A, \quad (13)$$

where  $A = \sum [\tilde{f}_{\text{gas},i}/\sigma_{f_{\text{gas},i}}]^2$ ,  $B = \sum [\tilde{f}_{\text{gas},i} f_{\text{gas},i}]/\sigma_{f_{\text{gas},i}}^2$ ,  $C = \sum [f_{\text{gas},i}/\sigma_{f_{\text{gas},i}}]^2$ , and  $\tilde{f}_{\text{gas},i} = [d_A^{\text{SCDM}}(z)/d_A(z)]^{2/3}$ .

The Hubble parameter  $H(z)$  can be derived by

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}. \quad (14)$$

From the Gemini Deep Deep Survey (GDDS; Abraham et al. 2004) observations of differential ages of passively evolving galaxies and other archival data (Nolan et al. 2003; Treu et al. 2001, 2002; Spinrad et al. 1997; Dunlop et al. 1996), Simon et al. (2005) have obtained the  $H(z)$  data at nine different redshifts ( $0.09 \leq z \leq 1.75$ ). Recently,  $H(z) = 83.2 \pm 2.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$  at  $z = 0.24$ , and  $H(z) = 90.3 \pm 2.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$  at  $z = 0.43$  have been obtained by using the BAO peak position as a standard ruler in the radial direction (Gaztañaga et al. 2009). The  $\chi^2$  value of the 11  $H(z)$  data is

$$\chi_H^2 = \sum_{i=1}^{N=11} \frac{[H_{\text{obs}}(z_i) - H(z_i)]^2}{\sigma_{H,i}^2}. \quad (15)$$

The nuisance parameter  $H_0$  is also marginalized following the procedure used in calculating  $\hat{\chi}_\mu^2$ .

### 3 CONSTRAINTS FROM COMBINING GRBS, SNE IA, CMB, AND BAO

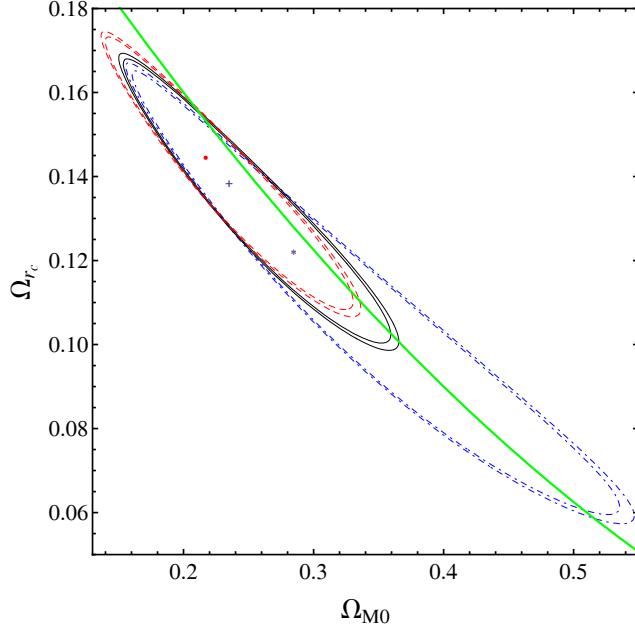
In order to combine GRB data with the SNe Ia data to constrain cosmological models, we follow a simple way that avoids any correlation between the SNe Ia data and the GRB data (Liang et al. 2010a): The 40 SNe points used in the interpolation procedure to calibrate GRBs are excluded from the Union2 SNe Ia sample used to calculate the joint constraints. Since the reduced 517 SNe Ia, 42 GRBs, CMB, and BAO, as well as CBF and  $H(z)$  are all effectively independent, we can combine the results by simply multiplying the likelihood functions. The best fit values for model parameters from the distance moduli of GRBs at high redshift obtained directly from the Union2 set, and SNe Ia, as well as the other joint observations (CMB+BAO+CBF+ $H(z)$ ) can be determined by minimizing

$$\chi^2 = \hat{\chi}_{\mu, \{42\text{GRBs}+517\text{SNe}\}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2 + \hat{\chi}_{\text{CBF}}^2 + \hat{\chi}_H^2. \quad (16)$$

In order to show the contribution of GRBs to the joint cosmological constraints, we also consider the  $\chi^2$  value from the joint data (SNe + CMB + BAO + CBF +  $H(z)$ ) without GRBs:  $\chi_S^2 = \hat{\chi}_{\mu, \{557\text{SNe}\}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2 + \hat{\chi}_{\text{CBF}}^2 + \hat{\chi}_H^2$ ; and the joint constraints with GRBs + CMB + BAO + CBF +  $H(z)$  without the SNe Ia contribution is:  $\chi_G^2 = \hat{\chi}_{\mu, \{42\text{GRBs}\}}^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2 + \hat{\chi}_{\text{CBF}}^2 + \hat{\chi}_H^2$ .

The joint confidence regions in the  $\{\Omega_{M0}, \Omega_{r_c}\}$  plane with the combined observational data for the DGP model are shown in Figure 1. For comparison, fitting results from the joint data without GRBs are also given in Figure 1. We present the best-fit value of  $\{\Omega_{M0}, \Omega_{r_c}\}$  with  $1-\sigma$  uncertainties and the corresponding  $\Omega_K$ , as well as  $\chi_{\text{min}}^2$  and  $\chi_{\text{min}}^2/\text{dof}$  for the DGP model in Table 1. In addition, we investigate the deceleration parameter for the DGP model. The deceleration parameter  $q(z)$  can be calculated by  $q = -1 + (1+z)E(z)^{-1}dE(z)/dz$ , where  $E(z) = H/H_0$ . We could derive the transition redshift at which the universe obeying the DGP model switches from deceleration to acceleration (Zhu & Alcaniz 2005; Guo et al. 2006)

$$z_T = -1 + 2 \left( \frac{\Omega_{r_c}}{\Omega_{M0}} \right)^{1/3}. \quad (17)$$



**Fig. 1** Joint confidence regions in the  $\{\Omega_{M0}, \Omega_{r_c}\}$  plane for the DGP model. The contours correspond to  $1-\sigma$  and  $2-\sigma$  confidence regions. The black solid lines, red dashed lines, and blue dash-dotted lines (color online) represent the results of SNe+GRBs+Others (CMB+BAO+CBF+ $H(z)$ ), SNe+CMB+Others, and GRBs+Others, respectively. The black plus, red point, and blue star symbols correspond to the best-fit values of SNe+GRBs+Others, SNe+Others and GRBs+Others, respectively. The green line represents a flat universe which can be given by  $(1 - \Omega_{M0})^2 - 4\Omega_{r_c} = 0$ .

**Table 1** Best-fit Value of the DGP Model Parameters

	The DGP Model		
	SNe+GRBs+Others	SNe+Others	GRBs+Others
$\Omega_{M0}$	$0.235^{+0.125}_{-0.074}$	$0.217^{+0.126}_{-0.073}$	$0.285^{+0.252}_{-0.066}$
$\Omega_{r_c}$	$0.138^{+0.031}_{-0.036}$	$0.144^{+0.032}_{-0.035}$	$0.122^{+0.044}_{-0.062}$
$\Omega_K$	0.033	0.037	0.024
$\chi^2_{\text{min}}$	595.95	606.37	77.93
$\chi^2_{\text{min}}/\text{dof}$	1.07	1.09	0.99
$z_T$	$0.67^{+0.03}_{-0.04}$	$0.74^{+0.05}_{-0.07}$	$0.51^{+0.14}_{-0.16}$

The best-fit values of  $z_T$  of the DGP model are also summarized in Table 1.

With SNe Ia + GRBs + CMB + BAO + CBF +  $H(z)$ , the best-fit values at  $1-\sigma$  confidence level are  $\{\Omega_{M0}, \Omega_{r_c}\} = \{0.235^{+0.125}_{-0.074}, 0.138^{+0.031}_{-0.036}\}$ , which are near the line of a flat universe ( $(1 - \Omega_{M0})^2 - 4\Omega_{r_c} = 0$ ) and the corresponding  $\Omega_K \simeq 0$ ; with SNe Ia + CMB + BAO + CBF +  $H(z)$ , the best-fit values are  $\{\Omega_{M0}, \Omega_{r_c}\} = \{0.217^{+0.126}_{-0.073}, 0.144^{+0.032}_{-0.035}\}$ ; while with GRBs + CMB + BAO + CBF +  $H(z)$ , the best-fit values are  $\{\Omega_{M0}, \Omega_{r_c}\} = \{0.285^{+0.252}_{-0.066}, 0.122^{+0.044}_{-0.062}\}$ . These results lead to more stringent constraints than previous results for constraining the DGP model with GRBs

and/or other combined observations (Wang et al. 2009a; Wei 2010a; Bento et al. 2006; Davis et al. 2007; Rebouças 2008; Li et al. 2010). We also obtain the transition redshift  $z_T = 0.67^{+0.03}_{-0.04}$  ( $1\sigma$ ) with the joint data including GRBs, which is more stringent and the former result ( $z_T = 0.86^{+0.07}_{-0.08}$ ) in Guo et al. (2006).

From comparing the joint constraints with and without GRBs, we can see that the contribution of GRBs to the joint cosmological constraints of the DGP model is a shift between the best fit values near the line which represents a flat universe, towards a higher matter density Universe ( $\Delta\Omega_{M0} > 0$ ). This situation has also been noted by Liang et al. (2010a,b), who compared the joint constraints with GRBs and without GRBs using the  $\Lambda$ CDM model,  $w$ CDM model, and Cardassian model. Also, a shift towards a later transition redshift can be found by comparison to the joint constraints of the DGP model with and without GRBs. It is shown that GRBs can give strong constraints on the DGP model when combined with CMB and BAO observations without SNe Ia, which has also been noted by Liang et al. (2010b, 2011).

#### 4 CONCLUSIONS AND DISCUSSION

In this paper, we investigate observational constraints on the DGP model including the cosmology-independent GRBs obtained directly from SNe Ia. Combining the GRBs at high redshift with the Union2 set, the WMAP7 result, the BAO observation, the clusters' baryon mass fraction, and the observed Hubble parameter data, we obtain  $\{\Omega_{M0}, \Omega_{r_c}\} = \{0.235^{+0.125}_{-0.074}, 0.138^{+0.031}_{-0.036}\}$ , with the corresponding  $\Omega_K = 0.033$ , which favors a flat universe. We also obtain the transition redshift of the DGP model  $z_T = 0.67^{+0.03}_{-0.04}$ . These results break the degeneracies between the model parameters and lead to more stringent constraints than the previous results for constraints on the DGP model with GRBs and/or other combined observations. It is shown that GRBs can give strong constraints on the DGP model when combined with CMB and BAO observations. We conclude that GRBs could be used as an optional choice to set tighter constraints at high redshift on cosmological models.

Zhu & Alcaniz (2005) tested the DGP model with the baryon mass fractions in clusters of galaxies and the SNe Ia data to find that  $\{\Omega_{M0}, \Omega_{r_c}\} = \{0.29^{+0.04}_{-0.02}, 0.21^{+0.08}_{-0.08}\}$ , and  $\Omega_K = -0.36^{+0.31}_{-0.35}$  at 99.73% confidence level. Guo et al. (2006) also obtained a spatially closed DGP universe with  $\Omega_K = -0.350^{+0.080}_{-0.083}$  by using SNe Ia + BAO data. Zhu & Sereno (2008) used gravitational lensing statistics to find that the likelihood peaks at  $\{\Omega_{M0}, \Omega_{r_c}\} \simeq \{0.29, 0.12\}$ , just slightly in the region of open models. These results seem to be in contradiction with the most recent WMAP results indicating a flat universe. However, constraints on the DGP model of the joint data including GRBs in this work are consistent with those obtained by Bento et al. (2006) using SNe Ia + CMB + BAO, and by Rebouças (2008) using SNe Ia + CMB, which favor a flat universe.

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