

Maximum mass of a hot neutron star with a quark core

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Abstract We have considered a hot neutron star with a quark core, a mixed phase of quark-hadron matter, and a hadronic matter crust and have determined the equation of state of the hadronic phase and the quark phase. We have then found the equation of state of the mixed phase under the Gibbs conditions. Finally, we have computed the structure of a hot neutron star with a quark core and compared our results with those of the neutron star without a quark core. For the quark matter calculations, we have used the MIT bag model in which the total energy of the system is considered as the kinetic energy of the particles plus a bag constant. For the hadronic matter calculations, we have used the lowest order constrained variational formalism. Our calculations show that the results for the maximum gravitational mass of a hot neutron star with a quark core are substantially different from those of a neutron star without the quark core.

Key words: dense matter — equation of state — stars: fundamental parameters (masses, radii, temperatures)

1 INTRODUCTION

A hot neutron star is born following the gravitational collapse of the core of a massive star just after the supernova explosion. The interior temperature of a neutron star at its birth is on the order 20 – 50 MeV (Burrows & Lattimer 1986). Therefore, the high temperature of these stages cannot be neglected with respect to the Fermi temperature throughout the calculation of its structure. This shows that the equation of state of the hot dense matter is very important for investigating the structure of a newborn neutron star. Depending on the total number of nucleons, a newborn neutron star evolves to either a black hole or to a stable neutron star (Strobel & Weigel 2001). Hence, calculation of the maximum mass of a hot neutron star is of special interest in astrophysics.

As we go from the surface to the center of a neutron star, at sufficiently high densities, the matter is expected to undergo a transition from hadronic matter, where the quarks are confined inside the hadrons, to a state of deconfined quarks. Finally, there are up, down and strange quarks in the quark matter. Other quarks have high masses and do not appear in this state. Glendenning has shown that a proper construction of the hadron-quark phase transition inside the neutron stars implies the coexistence of nucleonic matter and quark matter over a finite range of the pressure. Therefore, a mixed hadron-quark phase exists in the neutron star and its energy is lower than that of the quark matter and nucleonic matter (Glendenning 1992). These show that we can consider a neutron star to

be composed of a hadronic matter layer, a mixed phase of quarks and hadrons and, in the core, quark matter. Recent Chandra observations also imply that the objects RX J185635–3754 and 3C 58 could be neutron stars with a quark core (Prakash et al. 2003).

Burgio et al. have investigated the structure of neutron stars with a quark core at zero (Burgio et al. 2002) and finite temperatures (Burgio et al. 2007), using the Brueckner-Bethe-Goldstone formalism to determine the equation of state of the hadronic matter. We have calculated the structural properties of the cold neutron star by considering a quark phase at its core (Bordbar et al. 2006) and compared the results with our previous calculations for the neutron star without the quark core (Bordbar & Hayati 2006). In these works, we have employed the lowest order constrained variational (LOCV) method for the hadronic matter calculations. In the present paper, we intend to extend these calculations to a hot neutron star with a quark core.

2 EQUATION OF STATE

As was mentioned in the previous section, we consider a neutron star composed of hadronic matter (hadron phase), a mixed phase of quarks and hadrons, and a quark core (quark phase). Therefore, we should separately calculate the equation of state of these phases as follows.

2.1 Hadron Phase

For this phase of the neutron star matter, we consider the total energy per nucleon as the sum of contributions from the leptons and nucleons,

$$E = E_{\text{lep}} + E_{\text{nucl}}. \quad (1)$$

The contribution from the energy of leptons (electrons and muons) is

$$E_{\text{lep}} = E_e + E_\mu, \quad (2)$$

where E_e and E_μ are the energies of electrons and muons, respectively,

$$E_i = \frac{m_i^4 c^5}{\pi^2 n \hbar^3} \int_0^\infty \frac{\sqrt{1+x^2}}{1 + \exp\{\beta[m_i c^2 \sqrt{1+x^2} - \mu_i]\}} x^2 dx, \quad (3)$$

where μ_i and m_i are the chemical potential and mass of particle i , $\beta = \frac{1}{k_B T}$ (T is the temperature), n is the total number density of nucleons ($n = n_p + n_n$), c is the speed of light and x is as follows,

$$x = \frac{\hbar k}{m_i c}. \quad (4)$$

In our calculations, the equation of state of hot nucleonic matter is determined using the LOCV method as follows (Bordbar & Modarres 1997, 1998; Modarres & Bordbar 1998; Bordbar & Bigdeli 2007a,b, 2008a,b; Bigdeli et al. 2009). We adopt a trial wave function as

$$\psi = F \phi, \quad (5)$$

where ϕ is the Slater determinant of the single-particle wave function and F is the correlation function which is taken to be

$$F = \mathcal{S} \prod_{i>j} f(ij), \quad (6)$$

\mathcal{S} is a symmetrizing operator. For the energy of nucleonic matter, we consider up to the two-body term in the cluster expansion,

$$E_{\text{nucl}} = E_1 + E_2. \quad (7)$$

The one body term E_1 for the hot asymmetrical nucleonic matter that consists of Z protons and N neutrons is simply the fermi gas kinetic energy,

$$E_1 = \sum_{i=1,2} \mathcal{E}_i, \quad (8)$$

where labels 1 and 2 are used instead of proton and neutron, respectively, and \mathcal{E}_i is

$$\mathcal{E}_i = \sum_k \frac{\hbar^2 k^2}{2m_i} f_i(k, T, n_i), \quad (9)$$

where $f(k, T, n_i)$ is the Fermi-Dirac distribution function (Fetter & Walecka 1971),

$$f(k, T, n_i) = \frac{1}{e^{\beta[\epsilon_i(k, T, n_i) - \mu_i(T, n_i)]} + 1}. \quad (10)$$

In the above equation, n_i are the number densities and ϵ_i are the single particle energies associated with the protons and neutrons,

$$\epsilon_i(k, T, n_i) = \frac{\hbar^2 k^2}{2m_i^*(T, n_i)}, \quad (11)$$

where m_i^* are the effective masses.

The two-body energy, E_2 , is

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij | \nu(12) | ij - ji \rangle, \quad (12)$$

where

$$\nu(12) = -\frac{\hbar^2}{2m} [f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12), \quad (13)$$

$f(12)$ and $V(12)$ are the two-body correlation and inter-nucleonic potential.

We note that the conditions of charge neutrality and beta stability impose the following constraints on the number densities and chemical potentials,

$$n_p = n_e + n_\mu, \quad (14)$$

$$\mu_n - \mu_p = \mu_e = \mu_\mu. \quad (15)$$

The procedure to calculate the nucleonic matter has been fully discussed in Bordbar & Modarres (1997, 1998).

2.2 Quark Phase

We use the MIT bag model for the quark matter calculations. In this model, the energy density is the kinetic energy of quarks plus a bag constant (\mathcal{B}) which is interpreted as the difference between the energy densities of non-interacting quarks and interacting ones (Farhi & Jaffe 1984),

$$\mathcal{E}_{\text{tot}} = \mathcal{E}_u + \mathcal{E}_d + \mathcal{E}_s + \mathcal{B}, \quad (16)$$

where \mathcal{E}_i is the kinetic energy per volume of particle i ,

$$\mathcal{E}_i = \frac{g}{2\pi^2} \int_0^\infty (m_i^2 c^4 + \hbar^2 k^2 c^2)^{1/2} f(k, T, n_i) k^2 dk. \quad (17)$$

In the above equation, g is the degeneracy number of the system and n_i is the number density of particle i ,

$$n_i = \frac{g}{2\pi^2} \int_0^\infty f(k, T, n_i) k^2 dk. \quad (18)$$

For the quark phase, the Fermi-Dirac distribution function, $f(k, T, n_i)$, is given by

$$f(k, T, n_i) = \frac{1}{\exp\{\beta[(m_i^2 c^4 + \hbar^2 k^2 c^2)^{1/2} - \mu_i]\} + 1}. \quad (19)$$

We assume that the up and down quarks are massless, the strange quark has a mass equal to 150 MeV and $\mathcal{B} = 90 \text{ MeV fm}^{-3}$.

Now, by applying the beta stability and charge neutrality conditions, we get the following relations for the chemical potentials and number densities,

$$\mu_d = \mu_u + \mu_l, \quad (20)$$

$$\mu_s = \mu_u + \mu_l, \quad (21)$$

$$\Rightarrow \mu_d = \mu_s, \quad (22)$$

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_l = 0, \quad (23)$$

$$n_B = \frac{1}{3}(n_u + n_d + n_s), \quad (24)$$

where n_l and μ_l are the leptonic number density and chemical potential, and n_B is the baryonic number density.

The pressure of the system is calculated from free energy using the following equation,

$$P = \sum_i n_i \frac{\partial \mathcal{F}_i}{\partial n_i} - \mathcal{F}_i, \quad (25)$$

where the Helmholtz free energy per volume (\mathcal{F}) is given by

$$\mathcal{F} = \mathcal{E}_{\text{tot}} - T\mathcal{S}_{\text{tot}}. \quad (26)$$

The entropy of quark matter (\mathcal{S}_{tot}) can be written as follows:

$$\mathcal{S}_{\text{tot}} = \mathcal{S}_u + \mathcal{S}_d + \mathcal{S}_s, \quad (27)$$

where \mathcal{S}_i is the entropy of particle i ,

$$\begin{aligned} \mathcal{S}_i(n_i, T) = & -\frac{3}{\pi^2} k_B \int_0^\infty \{f(k, T, n_i) \ln[f(k, T, n_i)] \\ & + [1 - f(k, T, n_i)] \ln[1 - f(k, T, n_i)]\} k^2 dk. \end{aligned} \quad (28)$$

2.3 Mixed Phase

For the mixed phase, where the fraction of space occupied by quark matter smoothly increases from zero to unity, we have a mixture of hadrons, quarks and electrons. In the mixed phase, according to the Gibbs equilibrium conditions, the temperatures, pressures and chemical potentials of the hadron phase (H) and quark phase (Q) are equal (Glendenning 1992). Here, for each temperature we let the pressure be an independent variable.

The Gibbs conditions imply that

$$\mu_n^Q = \mu_n^H, \quad (29)$$

$$\mu_p^Q = \mu_p^H, \quad (30)$$

where μ_n^H and μ_n^Q (μ_p^H and μ_p^Q) are the neutron (proton) chemical potentials in the hadron phase and the quark phase, respectively,

$$\mu_n = \frac{\partial \mathcal{E}}{\partial n_n}, \quad (31)$$

$$\mu_p = \frac{\partial \mathcal{E}}{\partial n_p}. \quad (32)$$

In the above equations, \mathcal{E} is the energy density of the system,

$$\mathcal{E} = n(E + mc^2). \quad (33)$$

To obtain μ_p^H and μ_n^H for the hadronic matter in the mixed phase, we use the semiempirical mass formula (Kutschera & Niemiec 2000; Lagaris & Pandharipande 1981; Wiringa et al. 1988),

$$E = T(n, x) + V_0(n) + (1 - 2x)^2 V_2(n), \quad (34)$$

where $x = \frac{n_p}{n}$ is the proton fraction. $T(n, x)$ is the kinetic energy contribution and the functions V_0 and V_2 represent the interaction energy contributions which are determined from the energies of the symmetric nuclear matter ($x = \frac{1}{2}$) and pure neutron matter ($x = 0$). We calculate V_0 and V_2 using our results for the LOCV calculation of nucleonic matter with the UV_{14} + TNI nuclear potential which is discussed in Section 2.1. Now, we can obtain the chemical potentials of neutrons and protons from Equations (31)–(34) as follows:

$$\begin{aligned} \mu_p^H = & T(n, x) + n \frac{\partial T(n, x)}{\partial n} + \frac{\partial T(n, x)}{\partial x} + V_0(n) + nV_0'(n) \\ & + (-3 + 8x - 4x^2)V_2(n) + (1 - 2x)^2 nV_2'(n) + mc^2, \end{aligned} \quad (35)$$

$$\begin{aligned} \mu_n^H = & T(n, x) + n \frac{\partial T(n, x)}{\partial n} - \frac{\partial T(n, x)}{\partial x} + V_0(n) + nV_0'(n) \\ & + (1 - 4x^2)V_2(n) + (1 - 2x)^2 nV_2'(n) + mc^2. \end{aligned} \quad (36)$$

For the quark matter in the mixed phase, we have

$$\mu_p^Q = 2\mu_u + \mu_d, \quad (37)$$

$$\mu_n^Q = \mu_u + 2\mu_d. \quad (38)$$

At a certain pressure, we calculate μ_u for different μ_d under the condition that the densities yield this certain pressure. By calculating μ_u and μ_d , we obtain μ_p^Q and μ_n^Q .

Now, we plot μ_p versus μ_n for both hadron and quark phases, where the crossing point of the two curves satisfies the Gibbs conditions. In the mixed phase, since the chemical potentials determine

the densities, the volume fraction occupied by quark matter, χ , can be obtained by the requirement of global charge neutrality,

$$\chi \left(\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s \right) + (1 - \chi)n_p - n_e = 0. \quad (39)$$

Finally, we can calculate the baryonic density of the mixed phase (M),

$$n_B = \chi n_Q + (1 - \chi)n_H, \quad (40)$$

and then the total energy density of the mixed phase is found,

$$\mathcal{E}_M = \chi \mathcal{E}_Q + (1 - \chi)\mathcal{E}_H. \quad (41)$$

2.4 Results

We have shown our results for the energy densities of the hadron phase, quark phase and mixed phase in Figures 1 and 2 at two different temperatures.

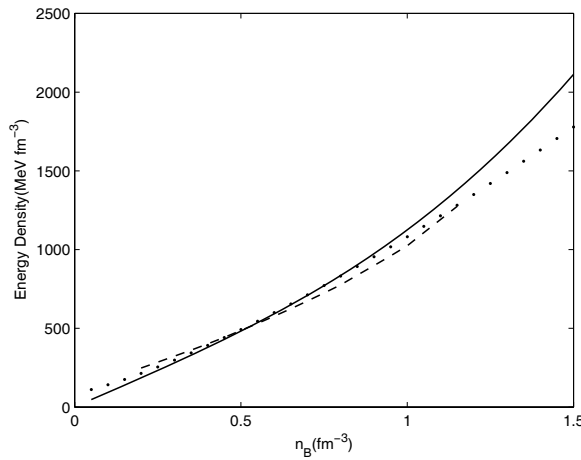


Fig. 1 Energy density versus the baryonic density at $T = 10$ MeV for the hadron phase (*solid line*), quark phase (*dotted line*) and mixed phase (*dashed line*).

Figures 1 and 2 show that at low densities the energy density of the hadronic matter is lower than those of other phases. However, as the density increases, at first the energy of the mixed phase and finally the energy of the quark phase is lower than those of other phases. We also see that there is a mixed phase for a range of densities. Below (beyond) this range, we have the pure hadron (quark) phase. By comparing Figures 1 and 2, we see that for a given value of the density, the energies of all phases increase by increasing the temperature.

Using the above calculated energy density, we can determine the equation of state and finally the structure of the hot neutron star with the quark core, which is discussed in the next section.

3 STRUCTURE OF THE HOT NEUTRON STAR WITH THE QUARK CORE

The structure of the neutron star is determined by numerically integrating the Tolman-Oppenheimer-Volkoff equation (TOV) (Shapiro & Teukolsky 1983; Glendenning 2000; Weber 1999; Adler et al.

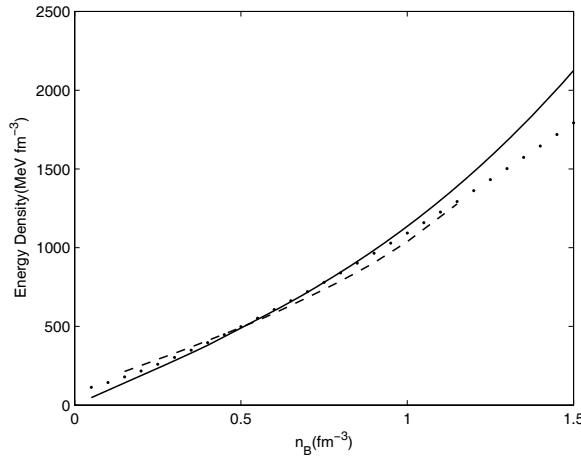


Fig. 2 Same as Fig. 1 but at $T = 20$ MeV.

1965),

$$\frac{dP}{dr} = -\frac{G[\mathcal{E}(r) + \frac{P(r)}{c^2}][m(r) + \frac{4\pi r^3 P(r)}{c^2}]}{r^2[1 - \frac{2Gm(r)}{rc^2}]}, \quad (42)$$

$$\frac{dm}{dr} = 4\pi r^2 \mathcal{E}(r), \quad (43)$$

where P is the pressure and \mathcal{E} is the total energy density. For a given equation of state in the form $P(\mathcal{E})$, the TOV equation yields the mass and radius of the star as a function of the central mass density.

In our calculations for the structure of the hot neutron star with the quark core, we use the following equations of state: (i) Below the density of 0.05 fm^{-3} , we use the equation of state calculated by Baym (Baym et al. 1971). (ii) From the density of 0.05 fm^{-3} up to the density where the mixed phase starts, we use the equation of state of the pure hadron phase calculated in Section 2.1. (iii) In the range of densities in which there is the mixed phase, we use the equation of state calculated in Section 2.3. (iv) Beyond the density of the end point of the mixed phase, we use the equation of state of the pure quark phase calculated in Section 2.2. All calculations are done for $\mathcal{B} = 90 \text{ MeV fm}^{-3}$ at two different temperatures: $T = 10$ and 20 MeV . Our results are as follows.

The gravitational mass as a function of the central mass density for the hot neutron star with the quark core at two different temperatures has been presented in Figures 3 and 4. It is seen that for both relevant temperatures, the gravitational mass increases by increasing the central mass density and finally reaches a limiting value (maximum mass). In Figures 3 and 4, our results for the case of the neutron star without the quark core have also been given for comparison. We see that by including the quark core for the neutron star, our results for the gravitational mass are substantially affected. For the neutron star with the quark core, our results for the gravitational mass at three different temperatures ($T = 0, 10$ and 20 MeV) have been compared in Figure 5. It is seen that the gravitational mass increases by increasing the temperature.

Figures 6 and 7 show the gravitational mass versus the radius for both cases of the neutron star with and without the quark core at two different temperatures. At each temperature, it is seen that there is a reasonable difference between the mass-radius relations for these two cases of the neutron star. However, for both cases, we see that the radius decreases as the mass increases. By comparing

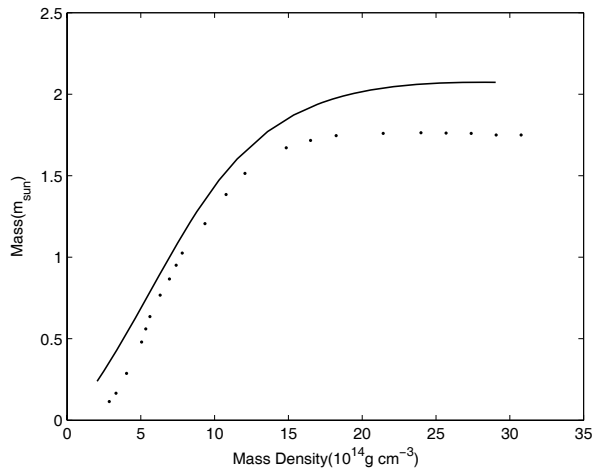


Fig. 3 Gravitational mass versus the central mass density for the neutron star with (*dotted line*) and without (*solid line*) the quark core at $T = 10$ MeV.

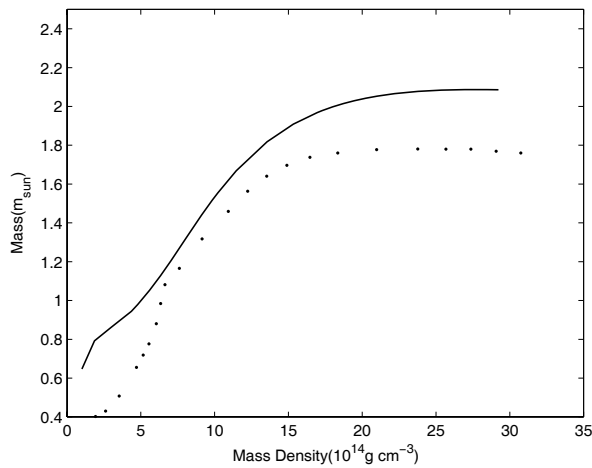


Fig. 4 Same as Fig. 3 but at $T = 20$ MeV.

Figures 6 and 7, we can see that the rate of decrease of the radius versus the mass is substantially different for different temperatures.

Our results for the maximum gravitational mass of the hot neutron star with the quark core and the corresponding values of radius and central mass density have been given in Tables 1 and 2 for two different temperatures. Our results for the case of a hot neutron star without a quark core have also been presented for comparison. For different temperatures, it is seen that the inclusion of the quark core considerably reduces the maximum mass of the hot neutron star. This is due to the fact that by including the quark core in the neutron star, the equation of state becomes softer than that without the quark core. However, we do not see any substantial changes in the radius and central mass density of these two cases for the hot neutron star.

Table 1 Maximum gravitational mass (M_{max}), and the corresponding radius (R) and central mass density (ϵ_c) of the hot neutron star without (NS) and with (NS+Q) the quark core at $T = 10$ MeV.

	$M_{\text{max}} (M_{\odot})$	R (km)	$\epsilon_c (10^{14} \text{ g cm}^{-3})$
NS	2.07	10.22	26.94
NS+Q	1.76	10.45	27.38

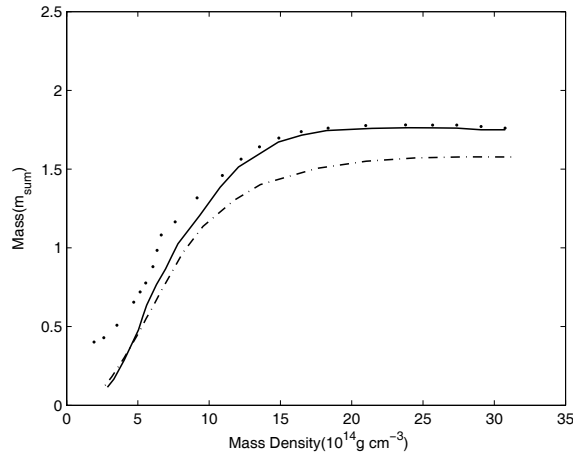


Fig. 5 Gravitational mass versus the central mass density for the neutron star with the quark core at $T = 0$ (dot-dashed line), 10 (solid line) and 20 MeV (dotted line).

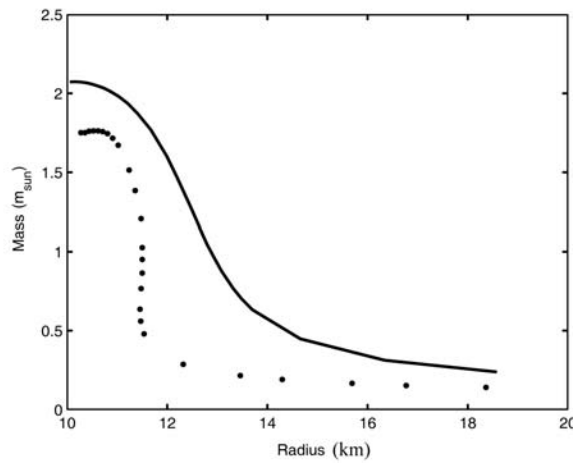
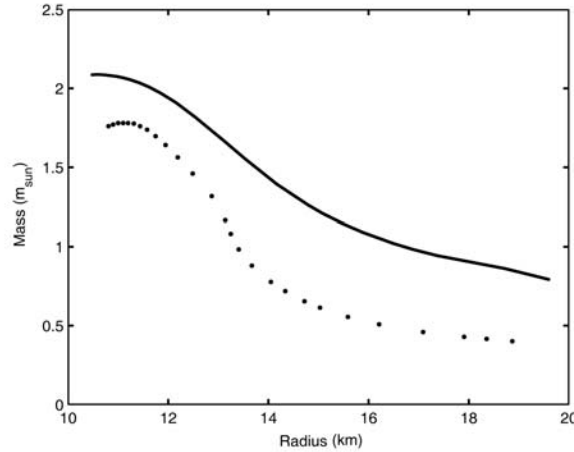


Fig. 6 Mass-radius relation for the neutron star with (dotted line) and without (solid line) the quark core at $T = 10$ MeV.

Table 2 Same as Table 1 but at $T = 20$ MeV.

	$M_{\max} (M_{\odot})$	R (km)	$\varepsilon_c (10^{14} \text{g cm}^{-3})$
NS	2.09	10.64	27.01
NS+Q	1.78	11	27.37

**Fig. 7** Same as Fig. 6 but at $T = 20$ MeV.

4 SUMMARY AND CONCLUSIONS

For the hot neutron star, from the surface toward the center, we have considered a pure hadronic matter layer, a mixed phase of quarks and hadrons in a range of densities which are determined by employing the Gibbs conditions, and pure quark matter in the core, to calculate its equation of state at a finite temperature. For calculating the equation of state of the hot hadronic matter, we have applied the LOCV method at a finite temperature. The equation of state of the hot quark matter has been computed using the MIT bag model with the bag constant $\mathcal{B} = 90 \text{ MeV fm}^{-3}$. Using this equation of state, we have solved the TOV equation by a numerical method to determine the structural properties of the hot neutron star with the quark core at $T = 10$ and 20 MeV. Then, we have compared the results of these calculations with those for the hot neutron star without the quark core. It is found that our results for the maximum gravitational mass of the neutron star with a quark core are less than those of the neutron star without the quark core.

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References

- Adler, R., Bazin, M., & Schiffer, M. 1965, *Introduction to General Relativity* (New York: McGraw-Hill)
 Baym, G., Pethick, C., & Sutherland, P. 1971, *ApJ*, 170, 299
 Bigdeli, M., Bordbar, G. H., & Rezaei, Z. 2009, *Phys. Rev. C*, 80, 034310

- Bordbar, G. H., Bigdeli, M., & Yazdizadeh, T. 2006, *International Journal of Modern Physics A*, 21, 5991
- Bordbar, G. H., & Hayati, M. 2006, *International Journal of Modern Physics A*, 21, 1555
- Bordbar, G. H., & Modarres, M. 1997, *Journal of Physics G Nuclear Physics*, 23, 1631
- Bordbar, G. H., & Modarres, M. 1998, *Phys. Rev. C*, 57, 714
- Bordbar, G. H., & Bigdeli, M. 2007a, *Phys. Rev. C*, 75, 045804
- Bordbar, G. H., & Bigdeli, M. 2007b, *Phys. Rev. C*, 76, 035803
- Bordbar, G. H., & Bigdeli, M. 2008a, *Phys. Rev. C*, 77, 015805
- Bordbar, G. H., & Bigdeli, M. 2008b, *Phys. Rev. C*, 78, 054315
- Burgio, G. F., Baldo, M., Sahu, P. K., Santra, A. B., & Schulze, H.-J. 2002, *Phys. Lett. B*, 526, 19
- Burgio, G. F., Baldo, M., Nicotra, O. E., & Schulze, H.-J. 2007, *Ap&SS*, 308, 387
- Burrows, A., & Lattimer, J. M. 1986, *ApJ*, 307, 178
- Farhi, E., & Jaffe, R. L. 1984, *Phys. Rev. D*, 30, 2379
- Fetter, A. L., & Walecka, J. D. 1971, *Quantum theory of many-particle systems*, by Fetter, Alexander L.; Walecka, John Dirk. San Francisco, McGraw-Hill [c1971]. International series in pure and applied physics
- Glendenning, N. K. 1992, *Phys. Rev. D*, 46, 1274
- Glendenning, N. K. 2000, *Compact Stars, Nuclear Physics, Particle Physics and General Relativity* (New York: Springer)
- Kutschera, M., & Niemić, J. 2000, *Phys. Rev. C*, 62, 025802
- Lagaris, I. E., & Pandharipande, V. R. 1981, *Nuclear Physics A*, 369, 470
- Modarres, M., & Bordbar, G. H. 1998, *Phys. Rev. C*, 58, 2781
- Prakash, M., Lattimer, J. M., Steiner, A. W., & Page, D. 2003, *Nuclear Physics A*, 715, 835
- Shapiro, S. L., & Teukolsky, S. A. 1983, *Black Holes, White Dwarfs, and Neutron Stars* (New York: Wiley)
- Strobel, K., & Weigel, M. K. 2001, *A&A*, 367, 582
- Weber, F. 1999, *Pulsars as Astrophysical Laboratories for Nuclear and Particle Physics* (Bristol: Institute of Physics Publishing)
- Wiringa, R. B., Fiks, V., & Fabrocini, A. 1988, *Phys. Rev. C*, 38, 1010