

Constraints on smoothness parameter and dark energy using observational $H(z)$ data *

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Abstract With large-scale homogeneity, the universe is locally inhomogeneous, clustering into stars, galaxies and larger structures. Such property is described by the smoothness parameter α which is defined as the proportion of matter in the form of intergalactic medium. If we consider the inhomogeneities over a small scale, there should be modifications of the cosmological distances compared to a homogenous model. Dyer and Roeder developed a second-order ordinary differential equation (D-R equation) that describes the angular diameter distance-redshift relation for inhomogeneous cosmological models. Furthermore, we may obtain the D-R equation for observational $H(z)$ data (OHD). The density-parameter Ω_M , the state of dark energy ω , and the smoothness-parameter α are constrained by a set of OHD in a spatially flat Λ CDM universe as well as a spatially flat XCDM universe. By using a χ^2 minimization method, we get $\alpha = 0.81^{+0.19}_{-0.20}$ and $\Omega_M = 0.32^{+0.12}_{-0.06}$ at the 1σ confidence level. If we assume a Gaussian prior of $\Omega_M = 0.26 \pm 0.1$, we get $\alpha = 0.93^{+0.07}_{-0.19}$ and $\Omega_M = 0.31^{+0.06}_{-0.05}$. For the XCDM model, α is constrained to $\alpha \geq 0.80$ but ω is weakly constrained around -1 , where ω describes the equation of state of the dark energy ($p_X = \omega\rho_X$). We conclude that OHD constrains the smoothness parameter more effectively than the data of SNe Ia and compact radio sources.

Key words: cosmology — dark energy — cosmological parameters

1 INTRODUCTION

In the past few decades, we have entered into an era of active cosmological research and have gained a better understanding of our universe. According to the cosmological principle, our universe is homogeneous and isotropic on a large scale, but deviations from the homogeneities are also observed. The corresponding researches are still exciting.

Recently, there have been mounting data from type Ia supernovae, cosmic microwave background (CMB) and large scale structure studies which suggest that the present universe is spatially flat and has accelerating expansion. Combined analyses of the above cosmological observations

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support the finding that approximately 26% of the universe is cold dark matter (CDM) while the remaining 74% is dominated by an unknown exotic component with negative pressure – the so-called dark energy – driving the current acceleration (Perlmutter et al. 1998; Perlmutter et al. 1999; Riess et al. 1998; Riess et al. 2007; Efstathiou et al. 2002; Allen et al. 2004; Astier et al. 2006; Spergel et al. 2007). The most likely candidate of this component is the cosmological constant (Carroll et al. 1992). In addition, dynamical models like quintessence (Caldwell et al. 1998), Chaplygin Gas (Kamenshchik et al. 2000), the “X-matter” model (Turner & White 1997; Chiba et al. 1997; Alcaniz & Lima 1999; Alcaniz & Lima 2001; Lima & Alcaniz 2000; Lima et al. 2003; Dabrowski 2007), Braneworld models (Csáki et al. 2000) and the Cardassian models (Freese & Lewis 2002) were proposed to explain the accelerating expansion of the universe. In the case of X-matter, the dark energy has the following property with an equation of state:

$$p_X = \omega \rho_X, \quad (1)$$

where ω is a constant independent of time or redshift. If $\omega = -1$, it is reduced to the case of the cosmological constant (the Λ CDM model).

However, these models could not perfectly explain the observations of our universe. Except for the cosmological constant problem (Weinberg 1989), the deviation from the cosmological principle, that the universe is homogenous and isotropic over a large scale, needed to be considered. It is obvious that the matter in the universe is clustered into stars, galaxies and clusters of galaxies, rather than being completely uniformly distributed everywhere in the space. It is also well known that the universe is grouped into superclusters, or perhaps filaments, great walls and voids on larger scales. Only on a scale larger than 1 Gpc does the universe appear smooth. Such problems may have effects on the distance redshift relation. Therefore, a smoothness-parameter α was introduced to describe the proportion of the mean density ρ in the form of intergalactic matter (Dyer & Roeder 1973):

$$\alpha \equiv \frac{\rho_{\text{int}}}{\rho}, \quad (2)$$

where ρ_{int} is the mean density of the universe in the form of intergalactic matter, while ρ denotes the mean density of the whole universe, so $\alpha \in [0, 1]$. In the case of $\alpha = 0$, it describes a universe where all the matter is clustered into stars, galaxies and so on, while $\alpha = 1$ is for a normal homogeneous universe. Generally, $0 < \alpha < 1$ describes the universe as being partially in the form of clustered matter and partially in the form of intergalactic matter.

The properties of angular diameter distance in a locally inhomogeneous universe have been discussed (Weinberg 1989; Zeldovich 1967; Dashveski & Slysh 1966; Kayser et al. 1997). Later, Dyer et al. established the Dyer-Roeder (D-R) equation to explain the distance-redshift relation in a universe with a fractional intergalactic medium (Dyer & Roeder 1973), as well as without an intergalactic medium (Dyer & Roeder 1972). In the literature (Santos et al. 2008), by using two different samples of SNe type Ia data, the Ω_M and α parameters are constrained by minimized χ^2 fitting, which applies the Zeldovich-Kantowski-Dyer-Roeder (ZKDR) luminosity distance redshift relation for a flat Λ CDM model. A χ^2 -analysis, by using the 115 SNe Ia data of the Astier et al. sample (Astier et al. 2006), constrains the density-parameter to be $\Omega_M = 0.26^{+0.17}_{-0.07}$ (2σ) while the α parameter is free (all the values $\alpha \in [0, 1]$ are allowed even at 1σ). The analysis based on the 182 SNe Ia data of Riess et al. (Riess et al. 2007) constrains the pair of parameters to be $\Omega_M = 0.33^{+0.09}_{-0.07}$ and $\alpha \geq 0.42$ (2σ), which provides a more stringent constraint because the sample extends to higher redshifts.

Santos et al. (2008) have proposed constraining α , Ω_M and ω by the angular diameter distances of compact radio sources with the XCDM model. However, only the ω and Ω_M parameters are well constrained, but the α parameter is totally free at the 1σ level.

As can be seen, neither SNe Ia data nor compact radio source data are capable of constraining the smoothness parameter. We could make use of other astronomical data to constrain the smoothness

parameter. It is also feasible to constrain the inhomogeneous model by making use of the observational $H(z)$ data (OHD), which can be obtained by the method to estimate the differential ages of the oldest galaxies. Yi and Zhang (Zeus Collaboration et al. 2007) present a constraint on a flat Friedmann-Robertson-Walker (FRW) universe with a matter component and a holographic dark energy component that uses OHD.

Wan et al. (2007) use OHD to constrain the Dvali-Gabadadze-Porrati (DGP) Universe. Lin et al. (2009) successfully use OHD together with other observational data to constrain the Λ CDM cosmology. The wiggling Hubble parameter $H(z)$ is also studied (in Zhang & Zhu 2008).

It can be concluded that OHD is complementary to other cosmological probes and may also present a better constraint on the smoothness parameter. In this article, the parameters Ω_M , α and ω are constrained by a total of 12 bins of OHD from Simon et al. (2005) and Ruth et al. (2008) in spatially flat Λ CDM universes as well as in the XCDM model. This paper is organized as follows: In Section 2, we review the basic origin of the Dyer-Roeder Equation and the relationship between the Hubble parameter and different cosmological models, which is performed in an inhomogeneous universe. In Section 3, we constrain the parameters Ω_M , ω and α from OHD. Discussions and prospects are presented in Section 4.

2 DYER-ROEDER EQUATION AND THE RELATIONSHIP BETWEEN ZKDR DISTANCE AND THE HUBBLE PARAMETER

We consider a stellar object which emits a beam of light propagating throughout space-time described by the metric tensor $g_{\mu\nu}$. We can identify a null surface Σ determined by the eikonal equation $g^{\mu\nu}\Sigma_{,\mu}\Sigma_{,\nu} = 0$ along which the beam of light propagates. The direction of this light is the tangent vector of the null surface, i.e. the null geodesic $k_\mu = -\Sigma_{,\mu}$. The beam of light rays can be described by $x^\mu = (v, y^i)$, where $x^0 = v$ is the affine parameter and y^i ($i = 1, 2, 3$) indicates the three different directions of the propagation of the light. The vector field is tangent to the light ray congruence, $k^\mu = \frac{dx^\mu}{dv} = -\Sigma_{,\mu}$, which determines two optical scalars: θ describing the convergence of the light and the shear parameter σ ,

$$\theta \equiv \frac{1}{2}k^\mu{}_{;\mu}, \quad \sigma \equiv k_{\mu;\nu}\tilde{m}^\mu\tilde{m}^\nu, \quad (3)$$

where $\tilde{m}^\mu = \frac{1}{\sqrt{2}}(\xi^\mu - i\eta)$ is a complex vector that is orthogonal to k^μ ($k^\mu\tilde{m}_\mu = 0$). Since $k_\mu = -\Sigma_{,\mu}$, the vorticity which is connected with the light beam is zero, therefore the congruence of light is characterized by these two optical scalars, σ and θ . These two optical scalars satisfy the Sachs propagation equations (Kristian & Sachs 1966)

$$\dot{\theta} + \theta^2 + |\sigma|^2 = -\frac{1}{2}\mathcal{R}_{\mu\nu}k^\mu k^\nu, \quad (4)$$

$$\dot{\sigma} + 2\theta\sigma = -\frac{1}{2}\mathcal{C}_{\mu\nu\tau\lambda}\tilde{m}^\mu k^\nu\tilde{m}^\tau k^\lambda, \quad (5)$$

where a dot denotes the derivative with respect to v , $\mathcal{R}_{\mu\nu}$ and \mathcal{R} are the Ricci tensor and Ricci scalar respectively, and $\mathcal{C}_{\mu\nu\tau\lambda}$ is the Weyl tensor which is zero in a conformally flat FRW space-time. One can see that if the shear σ is initially zero, the Weyl tensor could always be zero, which automatically satisfies the condition in FRW space-time (Demianski et al. 2003). Therefore, assuming that the light beam has no shear, the condition $\sigma = 0$, we may describe the convergence and divergence of this beam of light by the parameter θ (empty beam approximation).

The relative rate of the change of an infinitesimal area A on the cross section of the beam can be described by the optical scalar θ (and the distortion by σ), the only parameter that characterizes

the congruence of light, which relates to A by

$$\theta = \frac{1}{2} \frac{\dot{A}}{A}. \quad (6)$$

Substituting Equation (4) into the above expression, one can reduce the optical scalar equation to (Sachs 1961)

$$\ddot{\sqrt{A}} + \frac{1}{2} \mathcal{R}_{\mu\nu} k^\mu k^\nu \sqrt{A} = 0. \quad (7)$$

The Einstein field equation is

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} - \lambda g_{\mu\nu} = -\frac{8\pi G}{c^2} T_{\mu\nu}. \quad (8)$$

Multiply each side by $k^\mu k^\nu$, then the two $g_{\mu\nu}$ terms vanish while leaving the following form:

$$\mathcal{R}_{\mu\nu} k^\mu k^\nu = -\frac{8\pi G}{c^2} T_{\mu\nu} k^\mu k^\nu. \quad (9)$$

The universe, though locally inhomogeneously distributed, is homogeneous and isotropic on the largest scale on average, so we choose the Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a^2(t) d\sigma^2, \quad (10)$$

where $d\sigma^2$ describes the spatial part of the metric and $a(t)$ is the scale factor of the universe. We set $a(t) = 1$ at the present time and choose a proper affine parameter v so that (Schrodinger 1956)

$$\frac{dt}{d\tau} = \frac{a_0}{H_0 a}, \quad (11)$$

where a_0 and H_0 are the present values of a and the Hubble constant, respectively. Then we have

$$k^0 = \frac{dx^0}{d\tau} = \frac{d(ct)}{d\tau} = \frac{ca_0}{H_0 a}. \quad (12)$$

We consider a pressureless matter dominant universe in which the energy-momentum tensor only has a nonzero 0-0 part (with comoving coordinates), i.e. $T_{00} = \rho$ and $T_{ik} = 0$. In addition, if $a/a_0 = (1+z)^{-1}$ and $\rho/\rho_0 = (1+z)^3$, we get

$$T_{\mu\nu} k^\mu k^\nu = \left(\frac{ca_0}{H_0 a} \right)^2 \alpha \rho = \frac{c^2}{H_0^2} (1+z)^5 \alpha \rho_0. \quad (13)$$

Substituting it into Equation (9) and then substituting the resulting equation into Equation (7), we can finally obtain

$$\ddot{\sqrt{A}} + \frac{2}{3} \alpha \Omega_M (1+z)^5 \sqrt{A} = 0. \quad (14)$$

Here, we use the density-parameter Ω_M instead of ρ_0 . Due to a relationship between the angular diameter distance D_A and A , $D_A = \sqrt{A}$ (Schneider & Weiss 1988a; Schneider & Weiss 1988b; Bartelmann & Schneider 1991; Watanabe et al. 1992), the Equation (14) becomes the Dyer-Roeder Equation (Dyer & Roeder 1973)

$$\ddot{D}_A + \frac{2}{3} \alpha \Omega_M (1+z)^5 D_A = 0, \quad (15)$$

where the dots denote the derivatives with respect to the affine parameter v .

It is necessary to mention that the smoothness parameter must be different at various epochs of the universe due to the theory of formation of the large scale structure (Santos & Lima 2008; Efstathiou et al. 2002). For very high redshift, the matter in the universe must be more smoothly distributed compared to that of the present. From this point of view, we would have to identify the smoothness parameter α as a function of z , $\alpha(z)$ in Equation (15), especially when discussing the properties of the angular diameter distance at high redshift. However, because the samples of compact radio sources, SNe Ia and OHD, are mostly located at low redshift ($z < 2$ for the Hubble parameter), we set α to be a constant in the following discussion. We will also not consider the variations of α with respect to z because the data are neither adequate nor precise enough. The redshift dependence of α was discussed by Santos & Lima (2008).

Considering Equation (15) again, we may change the variable by substituting redshift z for the affine parameter v and obtain,

$$\left(\frac{dz}{dv}\right)^2 \frac{d^2 D_A}{dz^2} + \frac{d^2 z}{dv^2} \frac{dD_A}{dz} + \frac{2}{3} \alpha \Omega_M (1+z)^5 D_A = 0. \quad (16)$$

Note that the universe discussed is spatially flat, i.e. $\Omega_k = 0$, so that $\Omega_\Lambda = 1 - \Omega_M$. Finally, after the substitution of a variable (Demianski et al. 2003), we get a second-order ordinary differential equation in which the angular diameter distance D_A is a function of redshift z , and D_A is in the unit of c/H_0

$$\frac{d^2 D_A}{dz^2} + \mathcal{P} \frac{dD_A}{dz} + \mathcal{Q} D_A = 0, \quad (17)$$

where the initial conditions

$$\begin{cases} D_A(0) = 0, \\ \left. \frac{dD_A}{dz} \right|_{z=0} = 1, \end{cases} \quad (18)$$

are satisfied.

In addition, the functions \mathcal{P} and \mathcal{Q} read

$$\begin{aligned} \mathcal{P} &= \frac{\frac{7}{2} \Omega_M (1+z)^3 + \frac{3\omega+7}{2} (1-\Omega_M)(1+z)^{3\omega+3}}{\Omega_M (1+z)^4 + (1-\Omega_M)(1+z)^{3\omega+4}}, \\ \mathcal{Q} &= \frac{\frac{3}{2} \alpha \Omega_M + \frac{3\omega+3}{2} (1-\Omega_M)(1+z)^{3\omega}}{\Omega_M (1+z)^2 + (1-\Omega_M)(1+z)^{3\omega+2}}. \end{aligned} \quad (19)$$

The numerical results of D_A and dD_A/dz (hereafter $D'_A(z)$) are shown in Figure 1 with iterative calculations by the fourth-order Runge-Kutta scheme (see Sect. 3.1). From the well known Etherington principle, the relation between the luminosity distance and angular diameter distance (Etherington 1933),

$$D_L = (1+z)^2 D_A, \quad (20)$$

we can get the luminosity distance as a function of z .

At present, by the aid of the method based on the differential age of the oldest galaxies, the Hubble parameter can be determined as a function of redshift. It reads

$$H(z) = -\frac{dz}{dt} \frac{1}{1+z}, \quad (21)$$

which can be directly measured by the determination of dz/dt .

Since the comoving radial distance $r(z)$ (in units of c/H_0) in flat geometry can be expressed as

$$r(z) = \int_0^z \frac{dz}{E(z)}, \quad (22)$$

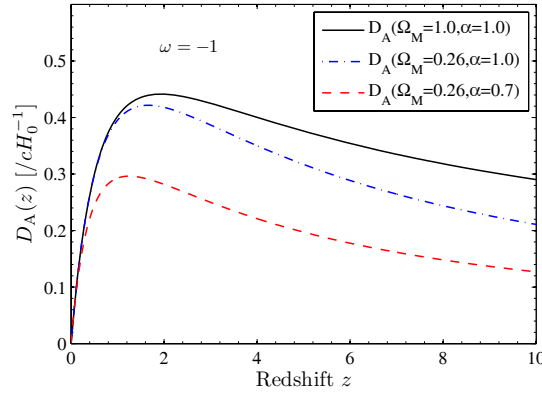


Fig. 1 Angular diameter distance $D_A(z)$ as a function of redshift z for a flat Λ CDM model. Several selected values of Ω_M and α are shown. $D_A(z)$ is in units of c/H_0 (*color online*).

where $E(z)$ is the expansion rate of the universe, which relates the Hubble parameter to the Hubble constant H_0 in the equation

$$H(z) = H_0 E(z), \quad (23)$$

the angular diameter distance can be written as,

$$D_A = \frac{r(z)}{1+z}. \quad (24)$$

Differentiating Equation (22) with respect to redshift z and combining with Equation (24), we can get the expansion rate of the universe expressed by D_A and $D'_A(z)$ at any redshift z

$$E(z) = \frac{1}{(1+z)D'_A(z) + D_A}. \quad (25)$$

3 SAMPLES AND RESULTS

3.1 Observational Data of $H(z)$

In order to constrain the smoothness parameter and other cosmological parameters with OHD, we need to integrate Equation (17) to obtain $D_A(z)$ and $D'_A(z)$ as a function of z (Fig. 1), then from Equation (23) and Equation (25) derive the Hubble parameter as a function of redshift z .

Although it is not possible to obtain the analytical solution of Equation (17) (Kantowski 1998; Kantowski et al. 2000; Kantowski & Thomas 2001), one can get an approximate expression of the equation which is accurate enough to use in practice (Demianski et al. 2003). It is convenient for controlling the precision that we integrate Equation (17), iteratively applying the 4th-order Runge-Kutta scheme, and then calculate numerical results of $D_A(z)$ and $D'_A(z)$. Furthermore, the Hubble parameter $H(\Omega_M, \omega, \alpha, h; z)$ can be obtained with any cosmological model at an arbitrary z .

Note that OHD, consisting of two parts, is obtained from two different sources. For one part, from the Simon et al. sample (Simon et al. 2005), we have a sample of nine bins and $z \in [0, 1.75]$, while for the other, from the Ruth et al. sample (Daly et al. 2008), we choose the data separated into three bins. In Table 1 we listed all the data and errors mentioned above with their sources marked, and we plotted them in Figures 2 and 3 respectively. We can compare the theoretical curves of $H(z)$ and the data in different models (see Sects. 3.2 and 3.3).

Table 1 Observational $H(z)$ Data (OHD)

Redshift z	$H(z)$	1σ interval	Data
0.05	75.4	± 2.3	●
0.09	69	± 12	★
0.17	83	± 8.3	★
0.27	70	± 14	★
0.40	87	± 17.4	★
0.505	96.9	± 6.9	●
0.88	117	± 23.4	★
0.905	116.9	± 11.5	●
1.30	168	± 13.4	★
1.43	177	± 14.2	★
1.53	140	± 14	★
1.75	202	± 40.4	★

The data marked with stars are from the Simon et al. sample, and the data marked with ● symbols are from the Ruth et al. sample.

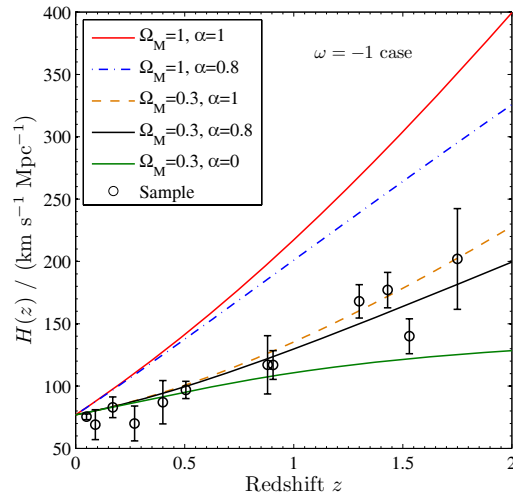


Fig. 2 Hubble parameter $H(z)$ as a function of redshift z for a flat Λ CDM model with selected values of Ω_M and α . The data sets in Table 1 are also shown (*color online*).

3.2 Constraining α and Ω_M for the Λ CDM Model

Firstly, we can find the relationship between $H(z)$ and α , Ω_M . In Figure 2, we plot the theoretical $H(z)$ at redshift $z \in [0, 2]$ according to Section 2 for some typically selected α and Ω_M , where a flat Λ CDM model is assumed. The figure includes the mean values of OHD in redshift bins and their error bars. One can see from Figure 2 that the curve of $H(z)$ strongly depends on Ω_M , i.e., the larger the value of Ω_M , the faster the growth of $H(z)$. In contrast, the smoothness-parameter α mainly has effects on the properties of the Hubble parameter at high redshift. For a certain Ω_M , theoretical $H(z)$ terms with different α values seem to appear similar at lower redshift, but they start to differ at high redshift.

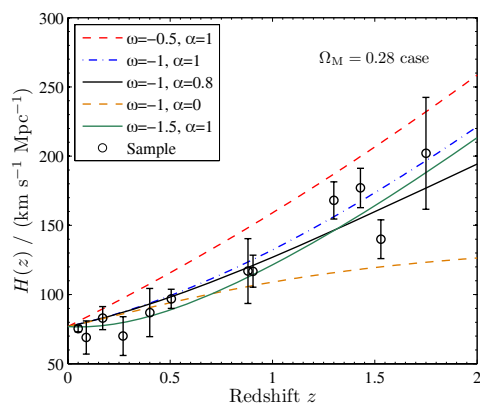


Fig. 3 Hubble parameter $H(z)$ as a function of redshift z for a flat XCDM model with selected values of ω and α . The data sets in Table 1 are also shown (*color online*).

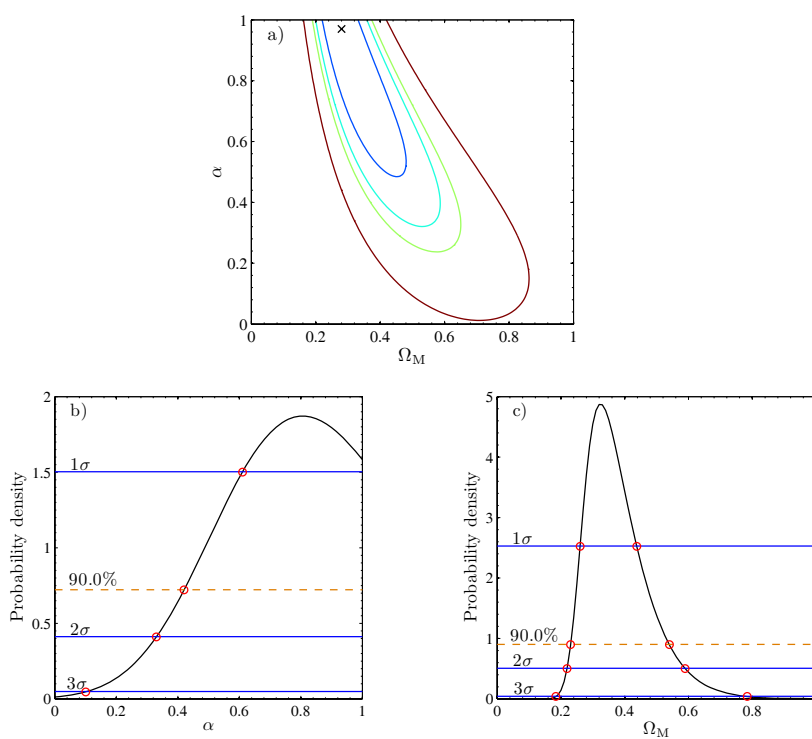


Fig. 4 (a) Confidence regions at 68.3%, 90.0%, 95.4%, and 99.7% levels from inner to outer respectively on the (Ω_M, α) plane for a flat Λ CDM model (without considering the prior on Ω_M). The “x” in the center of the confidence regions indicates the best-fit values (0.28, 0.97). (b) The one-dimensional probability distribution function (PDF) for the α parameter. (c) PDF for the Ω_M parameter (*color online*).

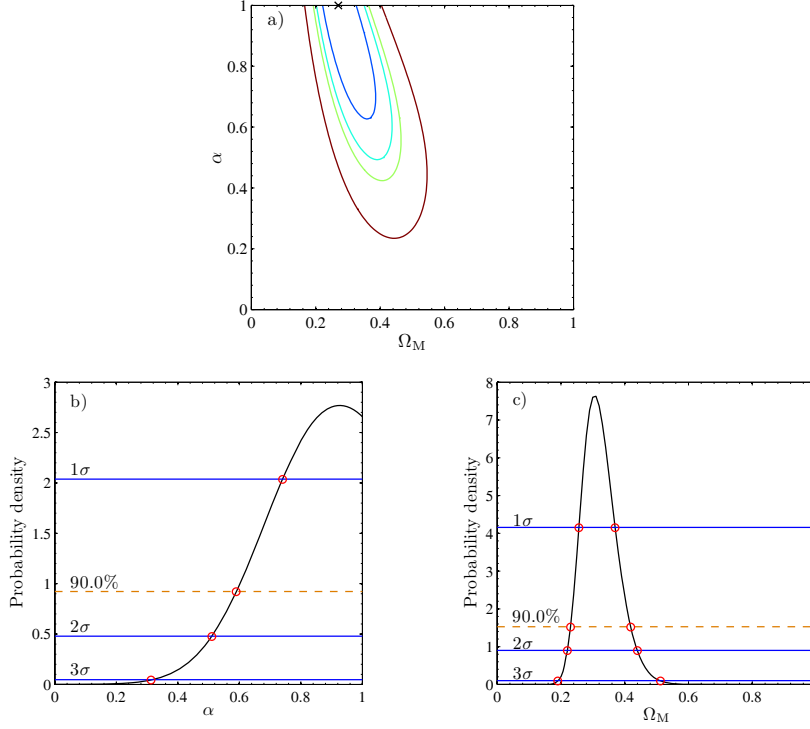


Fig. 5 (a) Confidence regions at 68.3%, 90.0%, 95.4%, and 99.7% levels from inner to outer respectively on the (Ω_M, α) plane for a flat Λ CDM model (with a prior on Ω_M considered). The “ \times ” in the center of the confidence regions indicates the best fit values (0.27, 1.0). (b) PDF for the α parameter. (c) PDF for the Ω_M parameter (*color online*).

In order to constrain α and Ω_M , we use χ^2 minimization

$$\chi^2(H_0, \alpha, \Omega_M) = \sum_{i=1}^{12} \left[\frac{H(H_0, \alpha, \Omega_M; z_i) - H_{\text{obs}}(z_i)}{\sigma(z_i)} \right]^2, \quad (26)$$

where $H(H_0, \alpha, \Omega_M; z_i)$ is the theoretical expectation of the Hubble parameter which is determined by Equations (17), (25) and (23), and $H_{\text{obs}}(z_i)$ is the observational value of the Hubble parameter with errors $\sigma(z_i)$ in the sample.

In the analysis, we marginalize the Hubble constant H_0 by integrating over it, and assume a Gaussian prior according to the best fitting value obtained from Bonamente et al. (2006), i.e., $H_0 = 76.9^{+3.9}_{-3.4}$ km s $^{-1}$ Mpc $^{-1}$. On the basis of the cosmic concordance from observations, we can optionally choose to add a Gaussian prior on Ω_M , $\Omega_M = 0.26 \pm 0.1$. We investigate the minimization both considering this prior (Fig. 5(a)) and without considering it (Fig. 4(a)). In Figures 4(a) and 5(a), we plot the regions of confidence on the Ω_M - α plane. The contours of the confidence levels of 68.3%, 90.0%, 95.4% and 99.7% are determined by two-parameter levels 2.30, 4.61, 6.17 and 11.8, respectively.

We also plot the one-dimensional probability distribution function (PDF) of parameters Ω_M and α . In Figure 4(b) and (c), PDFs were plotted without considering the Gaussian prior, while in Figure 5(b) and (c) we considered this prior. One can see that in the 90% confidence region,

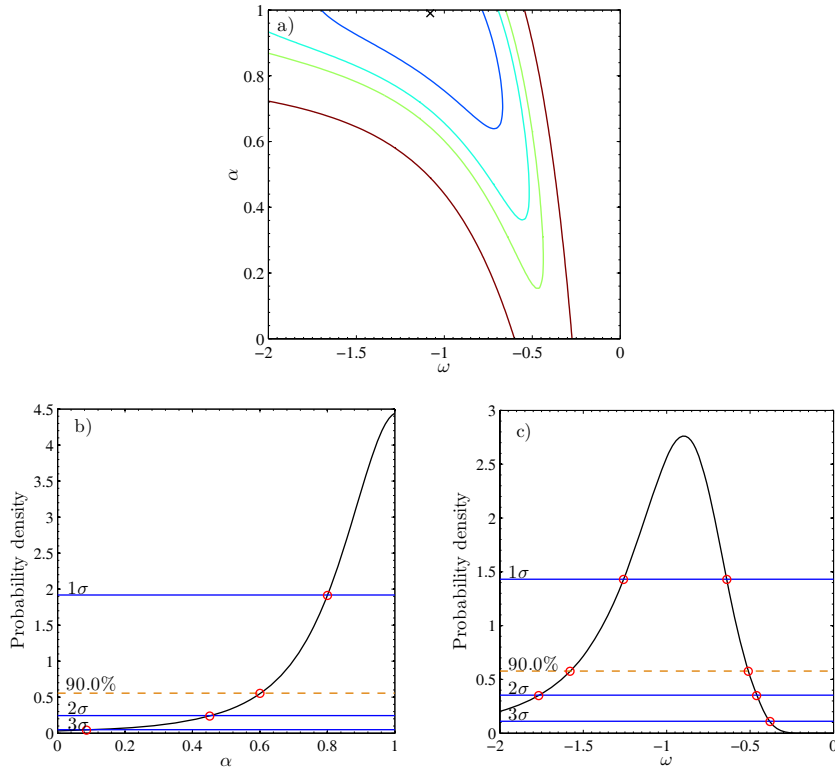


Fig. 6 (a) Confidence regions at 68.3%, 90.0%, 95.4%, and 99.7% levels from inner to outer respectively on the (ω, α) plane for a flat XCDM model. The “x” in the center of the confidence regions indicates the best fit values $(-1.08, 0.99)$. (b) PDF for the α parameter. (c) PDF for the ω parameter (*color online*).

$0.59 \leq \alpha \leq 1.0$ and $0.23 \leq \Omega_M \leq 0.41$ if we consider the prior, but without any prior, a 90% confidence lies in the region of $0.42 \leq \alpha \leq 1.0$ and $0.23 \leq \Omega_M \leq 0.54$. The symbols “x” indicate the model with the best fitting values that occur at $\alpha = 1, \Omega_M = 0.27$ and $\alpha = 0.97, \Omega_M = 0.28$, respectively. It is clear that whether or not we consider the Gaussian prior, the best fitting models are nearly the same with α slightly lower than one, corresponding to a universe with a uniform distribution of cold dark matter, and the value of Ω_M favors other observations.

3.3 Constraining α and ω for the XCDM Model

We constrain α and ω for the XCDM model. In this case, we set the density-parameter Ω_M to be 0.28 from the best fitting results in Section 3.2. In Figure 3, we plot theoretical $H(z)$ with some typically selected α and ω values, from which one can see how these two parameters modify the theoretical curve. Unfortunately, we find that the curves do not strongly depend on the parameters as much as in Section 3.2. We constrain these two parameters by χ^2 minimization

$$\chi^2(H_0, \alpha, \omega) = \sum_{i=1}^{12} \left[\frac{H(H_0, \alpha, \omega; z_i) - H_{\text{obs}}(z_i)}{\sigma(z_i)} \right]^2, \quad (27)$$

where $H(H_0, \alpha, \omega; z_i)$ is the theoretical value of the Hubble parameter; $H_{\text{obs}}(z_i)$ and $\sigma(z_i)$ are their observational values and errors, respectively. Again, marginalizing the parameter H_0 , we get the regions of confidence on the $\omega - \alpha$ plane. Then integrating over ω and α in the two-dimensional probability function $p(\alpha, \omega)$, we obtain PDFs of α and ω . The numerical results are plotted in Figure 6.

From Figure 6(a), we find that α is only mildly constrained. The best fitting point occurs at $\alpha = 0.99$ and $\omega = -1.08$, which indicates that the state of equation of XCDM is approximately that of the cosmological constant. All the allowable values of α in $[0, 1]$ are permitted at the 3σ confidence level.

4 CONCLUSIONS AND DISCUSSION

In this article, we study how the inhomogeneous distribution of cold dark matter affects the Hubble parameter at different redshift. By using OHD from Simon et al., together with Ruth et al. in a flat Λ CDM model, the smoothness-parameter and density-parameter are constrained at different confidence intervals. By marginalizing the Hubble constant H_0 , we found that the best fitting values are $\alpha = 0.97$ and $\Omega_M = 0.28$ for the Λ CDM model. However, if we assume a Gaussian prior of $\Omega_M = 0.26 \pm 0.1$, we find that the best fitting values are $\alpha = 1$ and $\Omega_M = 0.27$. With the XCDM model, setting $\Omega_M = 0.28$, we get the best fitting values of $\alpha = 0.99$ and $\omega = -1.08$. Comparing the constraints on the smoothness-parameter with samples of compact radio sources, where all the values of α from 0 to 1 are allowed at the 68.3% statistical confidence level (Santos & Lima 2008), the constraint on luminosity distance from SNe Ia gave slightly better results, which are $0.42 \leq \alpha \leq 1.0$ and $0.25 \leq \Omega_M \leq 0.44$ at the 90% confidence level (Santos et al. 2008). In our work, from Figures 4 and 5, the empty beam ($\alpha = 0$) case is even excluded at the 3σ confidence level. This result is better than the two previous works (Santos et al. 2008; Santos & Lima 2008) which constrain the α parameter. We can see that OHD constrains the smoothness parameter more effectively than both the SNe Ia data and the data about angular diameters of compact radio sources.

At the same time, as one can see from Figures 2 and 3, the errors of the data are too large to constrain α into a relatively small interval, especially at high redshift, or to give α a moderately accurate value. The statistical effects may not be neglected in this analysis. In the near future, we expect better constraints on α from more accurate data and/or more data at high redshift. In addition, OHD, as well as angular diameter distances of compact radio sources and luminosity distances of SNe Ia, may also enable us to study $\alpha(z)$ as a function of z .

Throughout our work, we discuss how the smoothness-parameter is independent of space-time, i.e., $\rho_{\text{int}} + \rho_{\text{clus}}$ is constant everywhere, and α only describes their ratio. In a real universe, the structures of walls and voids could not be described by such model, therefore these complex structures, which might be characterized by more parameters and/or more complicated models, may need ever more precise data. Hunt and Sarkar have discussed the case where we are located in a 200–300 Mpc void with especially low density, which is expanding at a rate 20%–30% higher than the average rate (Hunt & Sarkar 2010). In future work, we should consider more complex models that better describe our universe, together with better physical theories, in order to find a more accurate answer.

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