Testing the distance-duality relation with a combination of cosmological distance observations *

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Abstract We propose an accurate test of the distance-duality (DD) relation, $\eta = D_{\mathrm{L}}(z)(1 + z)^{-2}/D_{\mathrm{A}}(z) = 1$ (where D_{L} and D_{A} are the luminosity distances and angular diameter distances, respectively), with a combination of cosmological observational data of Type Ia Supernovae (SNe Ia) from the Union2 set and the galaxy cluster sample under an assumption of the spherical model. In order to avoid bias brought on by redshift non-coincidence between observational data and to consider redshift error bars of both clusters and SNe Ia in the analysis, we carefully choose the SNe Ia points which have the minimum acceptable redshift difference of the galaxy cluster sample ($|\Delta z|_{\mathrm{min}} = \sigma_{z,\mathrm{SN}} + \sigma_{z,\mathrm{cluster}}$). By assuming η to be a constant and defined as functions of the redshift parameterized by six different expressions, we find that there exists no observable evidence for variations in the DD relation based on the collected data, since related statistical tests are well satisfied within the 1σ confidence level for most cases. Further, considering different values of Δz as constraints, we also find that the choice of Δz may play an important role in this model-independent test of the DD relation for the spherical sample of galaxy clusters.

Key words: distance scale — galaxies: clusters: general — supernovae: general

1 INTRODUCTION

The distance-duality (DD) relation, also known as the Etherington's reciprocity relation (Etherington 1933), is of fundamental importance in cosmology, which relates the luminosity distance (LD, D_{L}) with the angular diameter distance (ADD, D_{A}) by means of the following expression,

$$\eta = \frac{D_{\mathrm{L}}}{D_{\mathrm{A}}}(1 + z)^{-2} = 1. \tag{1}$$

We notice that the DD relation is completely general, valid for all cosmological models based on Riemannian geometry, being dependent neither on Einstein field equations nor on the nature of matter-energy content. It only requires that source and observer are connected by null geodesics in a

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Riemannian spacetime and that the number of photons is conserved. This equation plays an essential role in modern cosmology (Csáki et al. 2002), ranging from gravitational lensing studies (Schneider et al. 1992; Fu et al. 2008) to analysis of galaxy cluster observations (Lima et al. 2003; Cunha et al. 2007), as well as to the plethora of cosmic consequences involving primary and secondary temperature anisotropies of the cosmic microwave background (CMB) observations (Komatsu et al. 2011).

Up to now, diverse astrophysical mechanisms such as gravitational lensing and dust extinction have been proved to be capable of causing obvious deviation from the distance duality and testing this equality with high accuracy can also provide a powerful probe of exotic physics (Bassett & Kunz 2004a,b; Corasaniti 2006). Therefore, it is rewarding to explore the DD relation to test the validity of photon conservation and related phenomena.

On the side of the observational data, if one is able to find cosmological sources whose intrinsic luminosities are known (standard candles) as well as their intrinsic sizes (standard rulers), one can determine both D_L and D_A and after measuring the common redshifts, to directly test the above Etherington's result. The possibility of using the Sunyaev-Zeldovich effect (SZE) together with X-ray emission of galaxy clusters to measure angular distances was suggested soon after the SZE was found (Silk & White 1978). Used jointly, this provides an independent method to determine distance scales and thus to measure the value of the Hubble constant (Silk & White 1978; Birkinshaw et al. 1991; Inagaki et al. 1995; Nozawa et al. 2006).

By using an isothermal spherical model for which the hydrostatic equilibrium model and spherical symmetry are assumed, Reese et al. (2002) selected 18 galaxy cluster samples and Bonamente et al. (2006) obtained 38 ADD galaxy clusters samples. De Filippis et al. (2005) have corrected the samples by using an isothermal elliptical model to get 25 ADDs of galaxy clusters. Uzan et al. (2004) considered 18 ADD samples (Reese et al. 2002) to test the DD relation by assuming the ΛCDM model via the technique $D_A^{\text{cluster}}(z) = D_A^{\Lambda\text{CDM}}(z)\eta^2(z)$. They showed that no violation of the DD relation is only marginally consistent with the accepted model. Cases of using the DD relation for astrophysical research can be found in many works, e.g., Bassett & Kunz (2004b); More et al. (2009); Avgoustidis et al. (2010); Holanda et al. (2011a); Cao & Zhu (2011).

In order to test the DD relation in a model-independent way, one should use measurements of D_L from cosmological observations directly. The first direct evidence for cosmic acceleration came from Type Ia Supernovae (SNe Ia) (Riess et al. 1998; Perlmutter et al. 1999), which have provided the strongest constraints on the cosmological parameters (Riess et al. 2004, 2007; Astier et al. 2006; Wood-Vasey et al. 2007; Kowalski et al. 2008). de Bernardis et al. (2006) divided the weighted average of galaxy clusters (Bonamente et al. 2006) and SNe Ia (Riess et al. 2004) in redshift bins and concluded that the validity of $\eta = 1$ is consistent at the 68.3% (1σ) confidence level (CL).

Recently, Holanda et al. (2010) tested the DD relation with ADD samples and the Constitution set of SNe Ia data (Hicken et al. 2009). In order to avoid the corresponding bias of redshift differences, a selection criteron ($\Delta z = |z_{\text{SN}} - z_{\text{cluter}}| \leq 0.005$) for a given pair of data is used. By using two parameterizations of the η parameter, they found that the DD relation is marginally compatible, within the 2σ CL, with the elliptical model sample (De Filippis et al. 2005) and a strong violation occurs ($> 3\sigma$) of the DD relation with the spherical model sample (Bonamente et al. 2006). However, Li et al. (2011) found that, by removing more data points of galaxy cluster samples according to the selection criteron, the DD relation can be accommodated at 1σ CL for the elliptical model and at 3σ CL for the spherical model. Nair et al. (2011) discussed the validity of the DD relation with observational data and significantly ruled out some of the parameterizations.

It is obvious that the difference of redshifts between galaxy clusters and SNe Ia may cause obvious deviation in testing the DD relation. In principle, the only strict criterion to form a given pair is that galaxy clusters and SNe Ia should be at the same redshift. Liang et al. (2011) found

that the DD relation is satisfied at 1σ CL with the corrected D_{L} located at the same redshift as the corresponding 38 spherical galaxy cluster samples, which are obtained by interpolating from the nearby SNe Ia of the Union2 set. It should be noted that the redshifts of observations are not determined with infinite accuracy and there is no point in decreasing Δz below the total 1σ error of observational redshifts $\sigma_{z,\mathrm{tot}} = \sigma_{z,\mathrm{SN}} + \sigma_{z,\mathrm{cluster}}$. Therefore, the finite errors of both clusters and SNe Ia should be taken into account in the analysis.

In this paper, we consider redshift error bars of both clusters and SNe Ia in the analysis in order to avoid bias from redshift differences between observational data to test the DD relation. In practice, $\sigma_{z,\mathrm{tot}}$ is not smaller than 0.002, therefore it is not appropriate to use a smaller window constraint. For the total 38 data pairs with the spherical sample of galaxy clusters and the Union2 set, we find that differences of redshifts between all the 38 data pairs are very small ($\Delta z \leq 0.005$) and there are 33 pairs that meet the minimum selection criteron $|\Delta z|_{\mathrm{min}} = \sigma_{z,\mathrm{tot}}$. Thus we choose the SNe Ia points which have the minimum acceptable redshift difference of the galaxy cluster sample $\Delta z \leq 0.002$. This criteron serves as a much more stringent one compared with $\Delta z \leq 0.005$ (Holanda et al. 2010; Li et al. 2011), therefore the accuracy and reliability of our test should be improved. We also find that the choice of Δz may play an important role in this model-independent test.

This paper is organized as follows. In Section 2, we introduce seven parametrizations for the DD relations applied in this work. In Section 3, we present a combined data set given by the latest released Union2 SNe Ia data as well as the 38 galaxy cluster samples under an assumption of obeying the spherical model. In Section 4, we briefly describe the analysis method and show results for constraining parameters of the DD relation. Finally, we summarize conclusions in Section 5.

2 DD RELATION PARAMETRIZATIONS

Regarding the parametrization of the DD relation, a model independent test has been extensively discussed in the above quoted papers (de Bernardis et al. 2006; Holanda et al. 2010, 2011a,b; Li et al. 2011; Nair et al. 2011; Liang et al. 2011; Meng et al. 2011; Fu 2011) considered η a constant with no relation to the redshift

   I. $\eta = \eta_0$,

where η_0 is a constant to be constrained by observational data. In general, η can be treated as parameterized functions of the redshift, $\eta(z)$, which are clearly inspired from similar expressions for $w(z)$, the equation of state in time-varying dark energy models (see, for instance, Chevallier & Polarski (2001); Linder (2003); Cunha et al. (2007); Silva et al. (2007)). Recently, Holanda et al. (2010, 2011a) used two one-parameter expressions, namely,

   II. $\eta(z) = 1 + \eta_a z$,
   III. $\eta(z) = 1 + \eta_a z/(1 + z)$.

In this work, we also use other general parametric representations for a possible redshift dependence of the distance duality expression including three two-parameter parameterizations (Li et al. 2011; Nair et al. 2011; Liang et al. 2011; Meng et al. 2011):

   IV. $\eta(z) = 1 + \eta_a \ln(1 + z)$,
   V. $\eta(z) = \eta_0 + \eta_a z$,
   VI. $\eta(z) = \eta_0 + \eta_a z/(1 + z)$,
   VII. $\eta(z) = \eta_0 + \eta_a \ln(1 + z)$.

3 GALAXY CLUSTERS AND SUPERNOVAE IA SAMPLES

In this work, we consider the sample of ADD from galaxy clusters obtained by combining their SZE and X-ray surface brightness observation samples (Bonamente et al. 2006). Under an assumption of

spherical model, the cluster plasma and dark matter distributions were analyzed assuming a hydro-static equilibrium model and spherical symmetry, thereby accounting for radial variations in density, temperature and abundance. Recently, the Supernova Cosmology Project (SCP) collaboration has released its Union2 compilation which consists of 557 SNe Ia (Amanullah et al. 2010), which is the largest published and spectroscopically confirmed SNe Ia sample to date.

For a given $D_{\mathrm{A}}^{\mathrm{cluster}}$ data point, theoretically, we should select an associated SNe Ia data point $D_{\mathrm{L}}^{\mathrm{SN}}$ at the same redshift to obtain an η_{obs}. In order to avoid any bias of redshift differences between SNe Ia and galaxy clusters and to consider redshift error bars of both clusters and SNe Ia in the analysis, we should determine the value of $\sigma_{z,\mathrm{tot}} = \sigma_{z,\mathrm{SN}} + \sigma_{z,\mathrm{cluster}}$ for the combination of observational data pairs. For the observations of SNe Ia, the peculiar velocity's uncertainty is set at $400\ \mathrm{km\ s}^{-1}$ (Wood-Vasey et al. 2007) (or $300\ \mathrm{km\ s}^{-1}$, Kowalski et al. (2008)) and the redshift uncertainty is $\sigma_{z,\mathrm{SN}} = 0.001$ (Hicken et al. 2009). For the observations of galaxy clusters, the rms one-dimensional cluster peculiar velocity's uncertainty is set at $256^{+106}_{-75}\ \mathrm{km\ s}^{-1}$, which corresponds to the three-dimensional rms velocity $459^{+184}_{-130}\ \mathrm{km\ s}^{-1}$ (Watkins 1997) (or $341\pm93\ \mathrm{km\ s}^{-1}$ for the rms one-dimensional cluster peculiar velocity, which corresponds to the three-dimensional rms velocity $591 \pm 161\ \mathrm{km\ s}^{-1}$, Dale et al. (1999)) and the redshift uncertainty is $\sigma_{z,\mathrm{cluster}} = 0.001$. Therefore, $\Delta z = \sigma_{z,\mathrm{tot}} = 0.002$ is considered in our work. Obviously, this strict choice with $\Delta z = 0.002$ may hopefully ease the systematic errors brought by redshift inconsistence between SNe Ia and galaxy clusters. Therefore, we obtain a sub-sample of SNe Ia from the Union2 data set whose redshifts co-incide with the ones appearing in the galaxy cluster sample under this criterion. We then bin the SNe data in the redshift bins of the corresponding spherical galaxy cluster sample to obtain 33 data pairs in our test. Assuming that μ_i represents the ith appropriate SNe Ia distance modulus data (within the $|\Delta z| < 0.002$ redshift range) with σ_{μ_i} denoting its reported observational uncertainty, in light of the standard data reduction framework by Bevington & Robinson (2003, Chap. 4), we obtain

$$\begin{aligned} \bar{\mu} &= \frac{\sum\left(\mu_i/\sigma_{\mu_i}^2\right)}{\sum 1/\sigma_{\mu_i}^2}, \\ \sigma_{\bar{\mu}}^2 &= \frac{1}{\sum 1/\sigma_{\mu_i}^2}, \end{aligned} \tag{2}$$

where $\bar{\mu}$ stands for the weighted mean distance modulus at the corresponding galaxy cluster redshift and $\sigma_{\bar{\mu}}$ serves as its uncertainty.

It must be emphasized that, if a redshift-dependent expression for the DD relation is considered, the SZE+X-ray surface brightness observation technique gives $D_{\mathrm{A}}^{\mathrm{cluster}}(z) = D_{\mathrm{A}}(z)\eta^2(z)$ (Sunyaev & Zeldovich 1972; Cavaliere & Fusco-Femiano 1978). So, we must replace $D_{\mathrm{A}}(z)$ with $D_{\mathrm{A}}^{\mathrm{cluster}}(z)\eta^{-2}$ when we try to consistently test the reciprocity relation with the SZE+X-ray observations from galaxy clusters. Thus, the observed $\eta_{\mathrm{obs}}(z)$ is determined by the following expression

$$\eta_{\mathrm{obs}}(z) = D_{\mathrm{A}}^{\mathrm{cluster}}(z)(1 + z)^2/D_{\mathrm{L}}(z). \tag{3}$$

It should be noted that the data points of the compiled Union2 SNe Ia are given in terms of the distance modulus, which could reduce to

$$D_{\mathrm{L}}(z) = 10^{\mu(z)/5-5}. \tag{4}$$

Accordingly, the uncertainty of the luminosity distance could be expressed in terms of the distance modulus uncertainty $\sigma_{D_{\mathrm{L}}(z)} = \ln 10/5 \times 10^{(\mu/5-5)}\sigma_{\mu(z)}$.

4 ANALYSIS AND RESULTS

In this section, we estimate the η_0 and η_a parameters in seven parametrizations listed in Section 2. To estimate the model parameters of a given parameterized form, we use the minimum χ^2 estimator

following the standard route

$$\chi^2(z; \mathbf{p}) = \sum_z \frac{[\eta_{\text{th}}(z; \mathbf{p}) - \eta_{\text{obs}}(z)]^2}{\sigma^2_{\eta_{\text{obs}}}}, \qquad (5)$$

where η_{th} represents the theoretical value of the η parameter with the parameter set $\mathbf{p}$ and η_{obs} is associated with the observational technique of estimating the error of $\sigma_{\eta_{\text{obs}}}$, which comes from the statistical contributions and systematic uncertainties of the galaxy clusters and SNe Ia, as well as the redshifts

$$\sigma_{\eta_{\text{obs}}} = \sigma_{D_A}^{\text{cluster}}(1+z)^2/D_{\text{L}} - D_{\text{A}}^{\text{cluster}}\sigma_{D_{\text{L}}}(1+z)^2/D_{\text{L}}^2 + 2D_{\text{A}}^{\text{cluster}}\sigma_z(1+z)/D_{\text{L}}. \qquad (6)$$

For the one-parameter models, one should expect the likelihood of η_0 or η_a to peak at $\eta_0 = 1$ or $\eta_a = 0$ ($\Delta\chi^2$ is minimized at $\eta_0 = 1$ or $\eta_a = 0$), in order to satisfy the DD relation. As for the two-parameter models, one should expect $\eta_0 = 1$ and $\eta_a = 0$ to be the best-fit parameters in the confidence contours, if it is consistent with photon conservation and there is no visible violation of the DD relation.

In Figure 1 (left), we plot the likelihood distribution function in the $\eta_0 - \Delta\chi^2$ plane and obtain $\eta_0 = 0.97^{+0.05}_{-0.06}$ at 1σ, which is in good qualitative agreement with previous analyses ($\eta_0 = 1.01^{+0.07}_{-0.07}$) (de Bernardis et al. 2006). In Figure 1 (right), we show the likelihood distribution function from three one-parameter forms of the redshift: II. $\eta(z) = 1 + \eta_a z$; III. $\eta(z) = 1 + \eta_a z/(1 + z)$; and IV. $\eta(z) = 1 + \eta_a \ln(1 + z)$. The best-fit values at 1σ CL are $\eta_a = -0.01^{+0.15}_{-0.16}$ for model I, $\eta_a = -0.01^{+0.21}_{-0.24}$ for model II and $\eta_a = -0.01^{+0.22}_{-0.19}$ for model III, which are different from those obtained in Holanda et al. (2010), where the DD relation is ruled out at 3σ CL and those obtained in Li et al. (2011), where the DD relation is accommodated at 3σ CL for the spherical model. Fitting results from one-parameter forms with the ADDs of galaxy clusters and the luminosity distances of the Union2 set with $\Delta z = \sigma_{z,\text{tot}} = 0.002$ are summarized in Table 1.

Table 1 Summary of the Results for Seven Parameterizations at the 1σ Confidence Level with $\Delta z = 0.002$

| Parameterization ($\Delta z = 0.002$)	η_0	η_a
I. η_0	$0.97^{+0.05}_{-0.06}$	0
II. $1 + \eta_a z$	1	$-0.01^{+0.15}_{-0.16}$
III. $1 + \eta_a \frac{z}{1+z}$	1	$-0.01^{+0.21}_{-0.24}$
IV. $1 + \eta_a \ln(1+z)$	1	$-0.01^{+0.22}_{-0.19}$
V. $\eta_0 + \eta_a z$	$0.84^{+0.17}_{-0.17}$	$0.43^{+0.49}_{-0.49}$
VI. $\eta_0 + \eta_a \frac{z}{1+z}$	$0.74^{+0.23}_{-0.22}$	$1.02^{+0.84}_{-0.85}$
VII. $\eta_0 + \eta_a \ln(1+z)$	$0.82^{+0.20}_{-0.19}$	$0.57^{+0.68}_{-0.67}$

The above analyses are based on the assumption that the redshift-independent model parameter is a constant $\eta_0 = 1$. Now we take it as a varying parameter to examine the DD relation by assuming more general expressions: V. $\eta(z) = \eta_0 + \eta_a z$; VI. $\eta(z) = \eta_0 + \eta_a z/(1 + z)$; VII. $\eta(z) = \eta_0 + \eta_a \ln(1 + z)$. Fitting results from two-parameter forms with the ADDs of galaxy clusters and the luminosity distances of the Union2 set with $\Delta z = \sigma_{z,\text{tot}} = 0.002$ are shown in Figure 2 and summarized in Table 1. Our results suggest that there is no violation of the DD relation for two-parameter cases at 1σ CL for models V and VII and at 2σ CL for model VI, which are more stringent than those obtained in Li et al. (2011), where the DD relations are consistent at 2σ CL for the spherical sample of galaxy clusters.
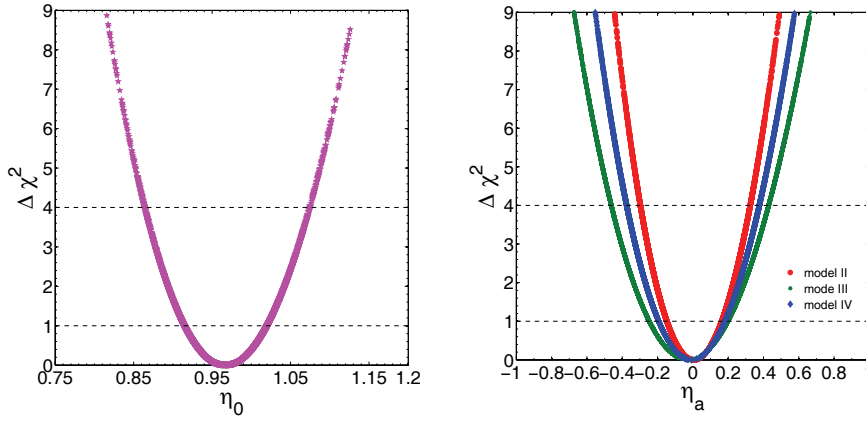
Fig. 1 Likelihood distribution function with the 33 ADDs of galaxy clusters and the luminosity distances of the Union2 set for one-parameter forms in the $\eta_0 - \Delta\chi^2$ planes (for model I) and the $\eta_a - \Delta\chi^2$ planes (for models II–IV) with $\Delta z = 0.002$.
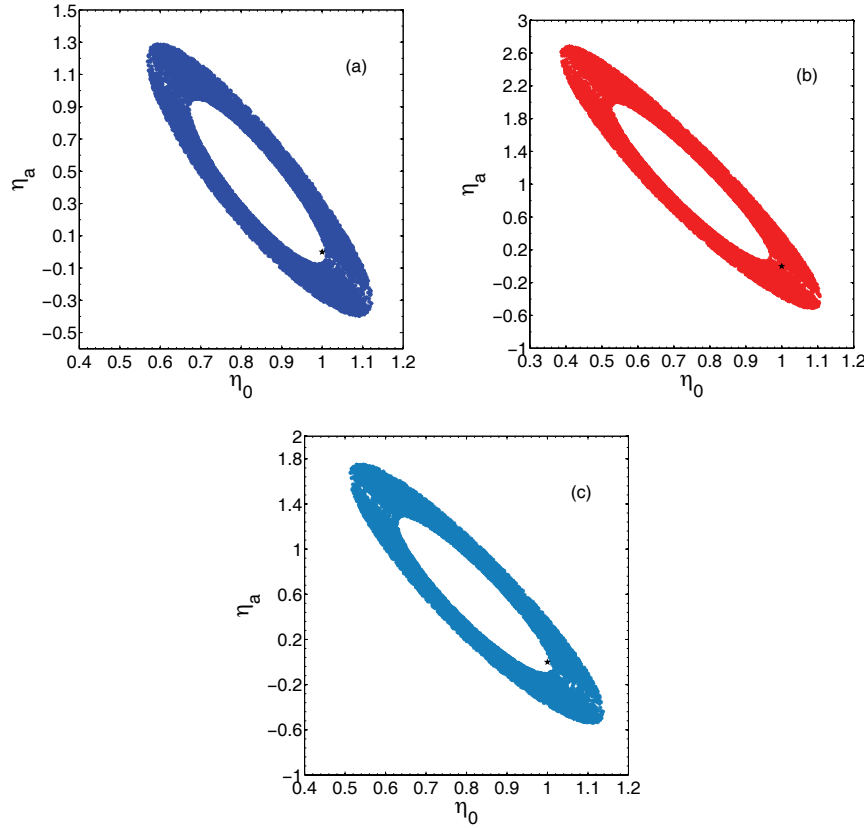
Fig. 2 Likelihood contours with the 33 ADDs of galaxy clusters and the luminosity distances of the Union2 set at 1σ and 2σ CL for two-parameter forms in the $\eta_0 - \eta_a$ plane with $\Delta z = 0.002$. (a) for $\eta(z) = \eta_0 + \eta_1 z$; (b) for $\eta(z) = \eta_0 + \eta_a \frac{z}{1+z}$; (c) for $\eta(z) = \eta_0 + \eta_a \ln(1 + z)$. The filled stars represent the cases with no violation of the DD relation ($\eta_0 = 1$ and $\eta_a = 0$).
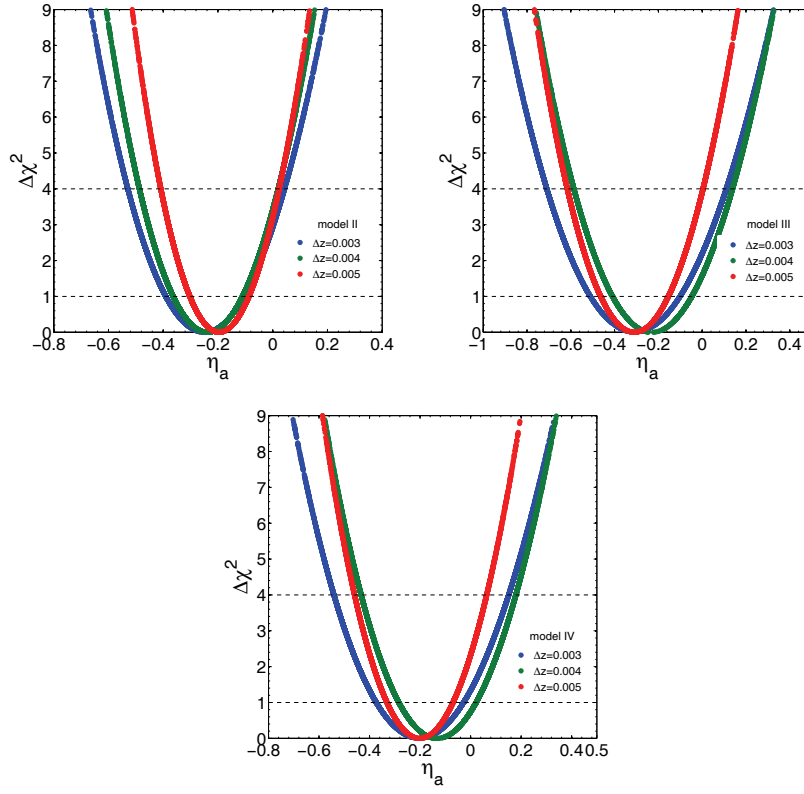
Fig. 3 Likelihood distribution function in the $\eta_a - \Delta\chi^2$ plane for the three one-parameter forms of the redshift (model II, III, IV) with varying $\Delta z = 0.003, 0.004, 0.005$.
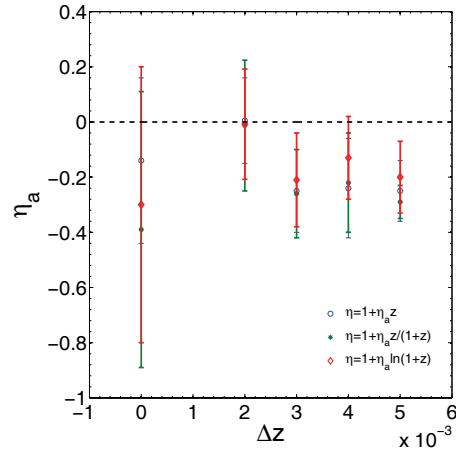
Fig. 4 1σ error bar of η_a as a function of $\Delta z = 0 - 0.005$ for the three one-parameter forms of the redshift (models II, III, IV).

From Figures 1–2 and Table 1, we can find that the DD relation can be accommodated at 1σ CL for the Bonamente et al. sample, except for model VI. Our results differ from those obtained by Holanda et al. (2010), where the results from the Bonamente et al. sample give a clear violation of the DD relation. However, these results are more stringent than those obtained by Li et al. (2011), where the DD relation is accommodated at $2\sigma - 3\sigma$ CL for the spherical sample of galaxy clusters.

After identifying the constraints on η obtained with the minimum acceptable $\Delta z = 0.002$, we may consider different values of Δz for examining the role the choice of Δz played in constraints. For the selection criteria of $\Delta z = 0.003, 0.004$ and 0.005, there are 35, 37 and 38 data pairs, respectively.

In Figure 3, we show the corresponding constraints on η_a for the three one-parameter forms of the redshift: II. $\eta(z) = 1 + \eta_a z$; III. $\eta(z) = 1 + \eta_a z/(1 + z)$; and IV. $\eta(z) = 1 + \eta_a \ln(1+z)$. Finally, we plot the 1σ error bar of η_a as a function of $\Delta z = 0.002 - 0.005$ in Figure 4. For comparison, we also show the results of η_a with 14 data pairs at $\Delta z = 0$ in Figure 4.

From Figures 3 and 4, we can find that the choice of Δz may play an important role in this model-independent test and the results for $\Delta z = 0.005$ in our test show the DD relation is ruled out at 2σ CL, which are close to those of Holanda et al. (2010) where the DD relation is ruled out at 3σ CL and consistent with those obtained in Li et al. (2011), where the DD relations are consistent at 3σ CL for the spherical sample of galaxy clusters.

5 CONCLUSIONS

In this paper, we have discussed a new model-independent cosmological test for the distance-duality relation, $\eta(z) = D_{\mathrm{L}}(1+z)^{-2}/D_{\mathrm{A}}$. We consider the angular diameter distances from galaxy clusters obtained by using SZE and X-ray surface brightness together with the luminosity distances given a sub-sample of SNe Ia taken from the Union2 data. The key aspect is that SNe Ia are carefully chosen to have the minimum acceptable redshift difference of the galaxy cluster ($\Delta z = \sigma_{z,\mathrm{tot}} = \sigma_{z,\mathrm{SN}} + \sigma_{z,\mathrm{cluster}}$). For the sake of generality, the η parameter is also parameterized in seven different forms, namely, four one-parameter models: (I) $\eta = \eta_0$, (II) $\eta = 1 + \eta_a z$, (III) $\eta = 1 + \eta_a z/(1 + z)$, (IV) $\eta = 1 + \eta_a \ln(1+z)$ and three two-parameter models: (V) $\eta = \eta_0 + \eta_a z$, (VI)$\eta = \eta_0 + \eta_a z/(1+z)$, (VII) $\eta = \eta_0 + \eta_a \ln(1 + z)$.

By assuming η to be a constant, we obtain $\eta_0 = 0.97^{+0.05}_{-0.06}$ at 1σ. For the redshift-dependent one-parameter forms of models II, III and IV, we obtain $\eta_a = -0.01^{+0.15}_{-0.16}$, $\eta_a = -0.01^{+0.21}_{-0.24}$ and $\eta_a = -0.01^{+0.22}_{-0.19}$, respectively, which are well consistent with no violation of the DD relation. We furthermore put forwards three kinds of two-parameter parametrizations corresponding to models II, III and IV, respectively. The standard values without any violation of the reciprocity relation ($\eta_0 = 1$ and $\eta_a = 0$) are still included at 68.3% CL (1σ) for models V and VII and at 95.8% CL (2σ) for model VI. It is shown that there is no observational evidence for variations of the DD relation for the Bonamente et al. sample, since it is marginally satisfied within 1σ CL for most cases, which is different from those obtained by Holanda et al. (2010), where the results from the Bonamente et al. sample give a clear violation of the DD relation and are more stringent than those obtained by Li et al. (2011). By further considering different values of the redshift difference Δz, we find that the choice of Δz may play an important role in this model-independent cosmological test of the DD relation and the results for $\Delta z = 0.005$ in our test show the DD relation is ruled out at 2σ CL, which are close to those of Holanda et al. (2010) where the DD relations are ruled out at 3σ CL and consistent with those obtained in Li et al. (2011), where the DD relations are consistent at 3σ CL for the spherical sample of galaxy clusters.

It is still interesting to see whether those conclusions may be changed with a larger sample of SNe Ia and galactic cluster data in the future, which reinforces the interest in the observational search for more samples of galaxy clusters with smaller statistical and systematic uncertainties, as well as

the determination of their angular diameters through the combination of SZE and X-ray surface brightness.

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