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Inhomogeneous cosmological models in the presence of massless scalar fields

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Abstract Non-static inhomogeneous cosmological models are obtained in general relativity for the case of a plane symmetric massless scalar field with cosmological constant Λ, when the source of the gravitational field is a viscous fluid. Some physical and geometrical behaviors of the solutions are also discussed.

Key words: cosmology: cosmological parameters

1 INTRODUCTION

General relativity proposed by Einstein (1916) has served as the basis for studies of cosmological models of the universe. As we know, present cosmology heavily depends on particle physics, where the role of scalar fields is very important. So, researchers are interested in studying relativistic field equations with scalar fields as a source of dark energy. They are also interested in studying the nature of scalar fields (with or without a mass parameter) interacting with a viscous fluid and cosmological constant Λ to draw an analogy between physics of the cosmos and experimental results. We know that plane symmetric solutions have interesting applications in cosmology, astrophysics and special relativistic hydrodynamics and the viscosity mechanism helps to obtain more realistic models. The viscosity mechanism has again attracted the attention of researchers because it can account for the high entropy of the present universe (Weinberg 1971, 1972). High entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation suggests that one should analyze dissipative effects in cosmology. Moreover, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era, the decomposition of matter and radiation during the recombination era (Kolb & Turner 1990), models where massive superstrings decay into massless ones (Myung & Cho 1986), gravitational string production (Turk 1988; Barrow 1988) and particle creation effects in the grand unification era. Murphy (1973) showed that introduction of bulk viscosity can avoid the Big Bang singularity. Hence, one should consider the presence of material distribution other than the perfect fluid to get realistic models (see Gron 1990 for a review of cosmological models with bulk viscosity).

It is known that Pradhan et al. (1997) have obtained a class of non-static plane symmetric cosmological models with bulk viscosity. Recently, Sahu & Mahapatra (2009) have obtained plane symmetric perfect fluid models in the presence of the cosmological constant in general relativity. However, to our knowledge, none of the authors have studied the theory of general relativity for a plane symmetric space-time for the case of a massless scalar field with Λ when the source of the gravitational field is a viscous fluid. Hence, in the present paper, we have considered this problem of studying and constructing cosmological models of the universe.

2 FIELD EQUATIONS

The space-time is described by the metric of the form

$$ds^2 = D^2dt^2 - A^2dx^2 - B^2\left(dy^2 + dz^2\right), \tag{1}$$

where A, B and D are functions of "x" and "t".

Einstein's field equations corresponding to a massless scalar field with a viscous fluid and Λ are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda_{ij} = -8\pi\left(T^{\nu}_{ij} + T^{m}_{ij}\right), \tag{2}$$

where

$$T^{\nu}_{ij} = (\rho + \overline{p})\, u_i u_j - \overline{p} g_{ij}\,, \tag{3}$$

is the energy-momentum tensor corresponding to a viscous fluid with

$$\overline{p} = p - \eta u^{i}_{;i} = p - 3\eta H, \tag{4}$$

and

$$T^{m}_{ij} = v_i v_j - \frac{1}{2}g_{ij}\nu_k\nu^k \tag{5}$$

is the energy-momentum tensor corresponding to a massless scalar field ν satisfying the Klein–Gordon equation

$$g_{ij}\nu_{;ij} = 0, \tag{6}$$

where ρ, p, $\overline{p}$, η, u_i and H are, respectively, the energy density, isotropic pressure, effective pressure, bulk viscosity coefficient, four velocity vector of the fluid and Hubble parameter. In general, η is a function of time and (;) denotes covariant differentiation. Since the bulk viscous pressure represents only a small correction to the thermal dynamical pressure, the inclusion of a viscous term in the energy momentum tensor is a reasonable assumption which does not fundamentally change the dynamics of the cosmic evolution. Using a comoving coordinate system, the set of field equations (2) for metric (1) reduces to the following forms:

$$\begin{aligned}&\frac{2}{BD^2}\left(B_{44} - \frac{DB_1D_1}{A^2} - \frac{B_4D_4}{D}\right) - \frac{1}{B^2}\left(\frac{B_1^2}{A^2} - \frac{B_4^2}{D^2}\right) - \Lambda \\ &= -8\pi\cdot\left[\bar{p} + \frac{1}{2}\left(\frac{\nu_1^2}{A^2} + \frac{\nu_4^2}{D^2}\right)\right],\end{aligned} \tag{7}$$

$$\frac{2}{B}\left(B_{14} - \frac{B_1A_4}{A} - \frac{D_1B_4}{D}\right) = -8\pi\nu_1\nu_4, \tag{8}$$

$$\begin{aligned}&\frac{1}{BD^2}\left(B_{44} - \frac{DB_1D_1}{A^2} - \frac{B_4D_4}{D}\right) - \frac{1}{A^2B}\left(B_{11} - \frac{A_1B_1}{A} - \frac{AA_4B_4}{D^2}\right) \\ &+ \frac{1}{A^2D^2}\left(AA_{44} - \frac{AA_4D_4}{D} - DD_{11} + \frac{DA_1D_1}{A}\right) - \Lambda \\ &= -8\pi\left[\overline{p} + \frac{1}{2}\left(\frac{-\nu_1^2}{A^2} + \frac{\nu_4^2}{D^2}\right)\right],\end{aligned} \tag{9}$$

$$\begin{aligned}&\frac{2}{A^2B}\left(B_{11} - \frac{A_1B_1}{A} - \frac{AA_4B_4}{D^2}\right) + \frac{1}{B^2}\left(\frac{B_1^2}{A^2} - \frac{B_4^2}{D^2}\right) + \Lambda \\ &= -8\pi\left[\rho + \frac{1}{2}\left(\frac{\nu_1^2}{A^2} + \frac{\nu_4^2}{D^2}\right)\right].\end{aligned} \tag{10}$$

The Klein-Gordon Equation (6) for metric (1) yields

$$\frac{\nu_{44}}{D^2}+\left(\frac{A_4}{A}+\frac{2B_4}{B}-\frac{D_4}{D}\right)\frac{\nu_4}{D^2}+\left(\frac{A_1}{A}-\frac{2B_1}{B}-\frac{D_1}{D}\right)\frac{\nu_1}{A^2}-\frac{\nu_{11}}{A^2}=0. \tag{11}$$

The suffixes 1 and 4 after a field variable indicate partial differentiation with respect to x and t respectively. As the field equation system is underdetermined, it needs extra conditions to obtain solutions. Since η does not appear in explicit form, for specification of η,

$$p=\gamma\rho, \qquad 0\le\gamma\le 1\,, \tag{12}$$

is considered.

3 SOLUTIONS OF THE FIELD EQUATIONS

As the field equations are highly non-linear, the following physically meaningful explicit solutions are considered (Patel & Dadhich 1993), i.e.

$$A=t^{\alpha}(1+x^2)^a, \quad B=t^{\beta}(1+x^2)^b \quad \text{and} \quad D=(1+x^2)^d, \tag{13}$$

where α, β, a, b and d are real constants.

To avoid mathematical complexities, ν is only considered to be a function of "t." Using values of A, B and D from Equation (13) in Equation (11), we obtain

$$\nu_{44}+\frac{(\alpha+2\beta)\nu_4}{t}=0. \tag{14}$$

After integration, Equation (14) yields

$$\nu=\frac{k_1 t^{-(\alpha+2\beta)+1}}{-(\alpha+2\beta)+1}+k_2, \tag{15}$$

where $k_1\neq 0$ and $k_2=0$ are constants of integration. In view of Equations (13) and (15), Equations (7)–(10) yield:

$$\left[\frac{(3\beta^2-2\beta)}{t^2(1+x^2)^{2d}}-\frac{4b(2d+b)x^2}{t^{2\alpha}(1+x^2)^{2a+2}}\right]+\frac{4\pi k_1^2}{t^{2(\alpha+2\beta)}(1+x^2)^{2d}}=\Lambda-8\pi\overline{p}, \tag{16}$$

$$\beta(d-b)+\alpha b=0, \tag{17}$$

$$\left\{\frac{4b[(3b-2a-1)x^2+1]}{t^{2\alpha}(1+x^2)^{2a+2}}-\frac{\beta(2\alpha+\beta)}{t^2(1+x^2)^{2d}}\right\}+\frac{4\pi k_1^2}{t^{2(\alpha+2\beta)}(1+x^2)^{2d}}=-\Lambda-8\pi\rho. \tag{18}$$

Now we obtain two types of solutions, i.e. for $\Lambda=0$ and $\Lambda\neq 0$.

Case-I: When $\Lambda=0$. Here we find two sub-cases corresponding to Equation (17).
Sub-case-i: When, for example

$$b=0=d \quad \text{and} \quad \alpha=\beta=r. \tag{19}$$

Using the values from Equation (19), Equations (15), (16) and (18) reduce to

$$\nu=\frac{k_1}{1-3r}\cdot\frac{1}{t^{3r-1}} \tag{20}$$

$$\frac{1}{8\pi}\left[\frac{(2r-3r^2)}{t^2}-\frac{4\pi k_1^2}{t^{6r}}\right]=\overline{p}, \tag{21}$$

and

$$\frac{1}{8\pi}\left(\frac{3r^2}{t^2}-\frac{4\pi k_1^2}{t^{6r}}\right)=\rho. \tag{22}$$

Using Equation (22) in Equation (12), we obtain

$$p=\gamma\left(\frac{3r^2}{8\pi t^2}-\frac{k_1^2}{2t^{6r}}\right). \tag{23}$$

In view of Equations (13), (21) and (23), Equation (4) yields

$$\eta=\frac{t}{3r}\left[\frac{3r^2(\gamma+1)-2r}{8\pi t^2}+\frac{k_1^2(1-\gamma)}{2t^{6r}}\right], \tag{24}$$

and

$$H=\frac{r}{t}. \tag{25}$$

Thus geometry of the space time (1) can be written as

$$ds^2=dt^2-t^{2r}(1+x^2)^{2a}dx^2-t^{2r}(dy^2+dz^2). \tag{26}$$

It is interesting to see that for $a=0$, model (26) reduces to an Einstein–deSitter universe.

Sub-case-ii: When

$$b=0=d \quad \text{and} \quad \alpha=-2\beta+1. \tag{27}$$

After substitution of the value of "α" from Equation (27) in Equation (14), we obtain

$$\nu_{44}+\frac{\nu_4}{t}=0. \tag{28}$$

On integration, Equation (28) reduces to

$$\nu=k_3\ln t+k_4, \tag{29}$$

where $k_3\neq 0$ and $k_4=0$ are constants of integration. Using Equation (27) in Equations (16) and (18), we get

$$\overline{p}=\rho=\frac{\beta(2-3\beta)}{8\pi t^2}-\frac{k_1^2}{2t^2}. \tag{30}$$

Using Equation (30) in Equation (12), we obtain

$$p=\gamma\left[\frac{(2-3\beta)\beta}{8\pi t^2}-\frac{k_1^2}{2t^2}\right]. \tag{31}$$

Now using Equations (13), (30) and (31) in Equation (4) yields

$$\eta=(\gamma-1)\left[\frac{(2\beta-3\beta^2)}{8\pi t}-\frac{k_1^2}{2t}\right]. \tag{32}$$

Therefore, the model of the universe described by the space time (1) is

$$ds^2=dt^2-t^{-4\beta+2}(1+x^2)^{2a}dx^2-t^{2\beta}(dy^2+dz^2). \tag{33}$$

As in Sub-case-i, here also for $a=0$ and $\beta=1/3$, model (33) reduces to an Einstein–deSitter Universe.

Case-II: For $\Lambda \neq 0$, it is difficult to determine solutions. So we only consider the vacuum case, i.e.

$$p = \rho = 0, \tag{34}$$

where $\gamma = 0$ is taken in Equation (12). Applying Equation (34) in Equation (18), we get

$$\Lambda = \frac{\beta(2\alpha+\beta)}{t^2(1+x^2)^{2d}} - \frac{4b[(3b-2a-1)x^2+1]}{t^{2\alpha}(1+x^2)^{2a+2}} - \frac{4\pi k_1^2}{t^{2(\alpha+2\beta)}(1+x^2)^{2d}}. \tag{35}$$

Using Equation (35), Equation (16) reduces to

$$\overline{p} = \frac{1}{8\pi}\left\{\frac{2\beta(\alpha-\beta+1)}{t^2(1+x^2)^{2d}} + \frac{4b[(2a-2b+2d+1)x^2-1]}{t^{2\alpha}(1+x^2)^{2a+2}} - \frac{8\pi k_1^2}{t^{2(\alpha+2\beta)}(1+x^2)^{2d}}\right\}. \tag{36}$$

As in Case-I, here are also two sub-cases, i.e.

Sub-case-i: When, for example

$$b = 0 = d \quad \text{and} \quad \alpha = \beta = r, \tag{37}$$

Equations (35), (36) and (4) reduce to

$$\Lambda = \frac{3r^2}{t^2} - \frac{4\pi k_1^2}{t^{6r}}, \quad \overline{p} = \frac{1}{8\pi}\left(\frac{2r}{t^2} - \frac{8\pi k_1^2}{t^{6r}}\right), \tag{38}$$

and

$$\eta = \left(\frac{-1}{12\pi t} + \frac{k_1^2}{3rt^{6r-1}}\right), \tag{39}$$

respectively, where ν and H are the same as found in Sub-case-i of Case-I. Thus the vacuum model of metric (1) is given by Equation (26).

Sub-case-ii: When

$$b = 0 = d \quad \text{and} \quad \alpha = -2\beta + 1. \tag{40}$$

Equations (35), (36) and (4) reduce to

$$\Lambda = \frac{\beta(2-3\beta) - 4\pi k_1^2}{t^2}, \quad \overline{p} = \frac{\beta(2-3\beta) - 4\pi k_1^2}{4\pi t^2}, \tag{41}$$

and

$$\eta = \left[\frac{\beta(3\beta-2) + 4\pi k_1^2}{4\pi t}\right], \tag{42}$$

respectively.

In this case also, ν and H are the same as found in Sub-case-ii of Case-I and the vacuum model of the metric (1) can be written by Equation (33).

4 SOME PHYSICAL AND GEOMETRIC PROPERTIES OF THE MODELS

The physical parameters, such as $\overline{p}$, p, ρ, Λ and η, which are involved in models (26) and (33), are given in the preceding section.

4.1 Energy Conditions for a Viscous Fluid

In Case-I of model (26), the strong energy condition

$$\rho + 3\overline{p} = \frac{3r(1-r)}{4\pi t^2} - \frac{2k_1^2}{t^{6r}} \geq 0 \tag{43}$$

is satisfied when $r \neq 0, 1$. Weak and dominant energy conditions given by

$$\rho = \frac{3r^2}{8\pi t^2} - \frac{k_1^2}{t^{6r}} \geq 0, \quad \rho + \overline{p} = \frac{r}{4\pi t^2} - \frac{k_1^2}{t^{6r}} \geq 0 \tag{44}$$

are satisfied when $r \neq 0$ and $r \geq 1/3$. However in Case-II, strong and dominant energy conditions are satisfied for $r \neq 0$.

Similarly in Case-I of model (33), we have

$$\rho + 3\overline{p} = \frac{(2-3\beta)\beta}{2\pi t^2} - \frac{k_1^2}{2t^2} \geq 0, \quad \rho = \frac{(2-3\beta)\beta}{8\pi t^2} - \frac{k_1^2}{2t^2} \geq 0 \tag{45}$$

and

$$\rho + \overline{p} = \frac{(2-3\beta)\beta}{4\pi t^2} - \frac{k_1^2}{2t^2} \geq 0, \tag{46}$$

which are only possible when $\beta \neq 0$ or 2/3. These conditions also hold in Case-II.

In model (26) of Case-I, if $t \to 0$ then ρ and p are both undetermined and ρ, $p \to 0$ as $t \to \infty$. However, in model (33), $\overline{p} = \rho \to \infty$ as $t \to 0$ and $\overline{p} = \rho \to 0$ as $t \to \infty$. Thus, these results show the presence of the Big Bang singularity at the initial epoch.

4.2 Bulk Viscous Coefficient η

In both models of Case-I, it is found that $\eta \to 0$ when $t \to \infty$ provided $r \neq 0$ in model (26) and $\eta \to \infty$ when $t \to 0$ provided $\gamma \neq 1$ in model (33). However in models of Case-II, $\eta \to 0$ when $t \to \infty$ and $\eta \to \infty$ when $t \to 0$. Thus, the above results indicate the presence of the Big Bang singularity.

4.3 Expansion Scalar θ

The expansion scalar for models in both Case-I and Case-II are found to be $\theta = \frac{3r}{t}$ and $\theta = \frac{1}{t}$, respectively. It is evident from the above that $\theta \to 0$ as $t \to \infty$ and $\theta \to \infty$ as $t \to 0$. Hence models start expanding with a Big Bang at $t = 0$. Also, expansion in both models decreases as time increases. However, expansion stops at infinite future or for the case $r = 0$.

4.4 Hubble Parameter H

The Hubble parameter H values in both models of Case-I and Case-II are found to be $H = \frac{r}{t}$ and $H = \frac{1}{3t}$, respectively. Thus, it is observed that H is a function of t and we conclude that models are not steady-state.

4.5 Shear Scalar σ

The anisotropy (Raychoudhuri 1955) defined by

$$\sigma^2 = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{11,4}}{g_{11}}\right)^2\right] \tag{47}$$

for models (26) and (33) of Case-I and Case-II are found to be $\sigma = 0$ and $\sigma = \sqrt{\frac{2}{3}}\left(\frac{1-3\beta}{t}\right)$, respectively.

The model (26) clearly approaches a state of isotropy as $\sigma = 0$ and $\lim_{t\to\infty}\frac{\sigma}{\theta} = 0$, but the model (33) does not approach a state of isotropy as $\sigma \neq 0$ and $\lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$. Thus in the case of the second model, the anisotropy exists throughout the evolution. Since $\lim_{t\to 0}\sigma^2 = \infty$ and $\lim_{t\to 0}\sigma^2 = 0$ (subject to restriction $\beta \neq \frac{1}{3}$), the shape of the universe only changes uniformly in the Y and Z directions. However, the rate of change of shape of the universe becomes slower with the increase of time. It is clear that there exists a real physical singularity in the model at $t = 0$ and the present upper limit for $\frac{\sigma}{\theta} = 10^{-3}$, obtained from indirect arguments, is related to the isotropy of primordial black body radiation (Collins et al. 1980). In Case-I, we see that $\frac{\sigma}{\theta} = 0$, which satisfies the inequality, but in Case-II, we observe that $\frac{\sigma}{\theta} < 10^{-3}$, provided $\beta > \frac{1}{3} - \frac{1}{2}\times 10^{-6}$. Hence models found here can be applied to all stages of the evolution of the universe.

4.6 Einstein Space

A space-time is said to be an Einstein space if it holds the property $R_{ij} = \frac{R}{4}g_{ij}$. Here, the space-time in this paper does not satisfy the above property and hence is not an Einstein space.

4.7 Massless Scalar Field ν

The massless scalar field ν for models (26) and (33) in both Case-I and Case-II are found respectively to be $\nu = \frac{k_1}{1-3r}\cdot\frac{1}{t^{3r-1}}$ and $\nu = k_3 \ln t$.

From above it is evident that ν is a function of cosmic time. In model (26), it is seen that ν is not defined at $r = \frac{1}{3}$. However, for $r < \frac{1}{3}$, ν is an increasing function of time and for $r > \frac{1}{3}$, ν is a decreasing function of time. Also, we see that as $t \to 0$, $\nu \to \infty$ and as $t \to \infty$, $\nu \to a$ which is constant, but in model (33), $|\nu| \to \infty$ as $t \to 0$ and $\nu \to \infty$ as $t \to \infty$. These results show the presence of the Big Bang singularity.

The energy density associated with ν is given by (Anderson 1967) $\varepsilon = \frac{1}{2}(\nu_4^2 + m^2\nu^2)$, with $m = 0$. Thus from Equations (15) and (29), we have $\varepsilon = \frac{k_1^2}{2t^{2k}}$ and $\varepsilon = \frac{k_3^2}{2t^2}$. From the above results, it is observed that energy density ε of massless scalar field ν decreases with time (in both models) at a faster rate than ν. For a physically acceptable mesonic field, we have $\varepsilon > 0$, which leads to the situation where k_1 and k_3 are both $+ve$ or both $-ve$, which are real constants.

4.8 The Parameter γ

For realistic situations, it is required that $\rho \geq p \geq 0$, which yields restrictions on the parameter γ, i.e. $0 \leq \gamma \leq 1$. Thus in Sub-case-i of Case-I, it is found that when $\gamma = 0, 1$ and 1/3 the model (26) reduces to a dust universe, a stiff fluid universe and a radiating universe, respectively. Similarly, in Sub-case-ii of Case-I, we see that when $\gamma = 0$ and 1/3 the model (33) reduces to, respectively, a dust dominated universe and a radiating universe. When $\gamma = 1$ then $p = \rho$ and $\eta = 0$. However in Case-II, we obtain the vacuum models.

4.9 Acceleration and Vorticity Tensor

The velocity field from geodesic motion is given by acceleration $\dot{u}_\mu$. In both models, there is no acceleration satisfying $\dot{u}_\mu = 0$. Thus, the mesonic viscous fluid flow is geodesic in nature. Further, vorticity tensor ω_{ij} becomes zero in each case of both models. Hence the rotation ω turns out to be zero for each case and for the models which are not rotating.

4.10 Spatial Volume

The spatial volumes $V = (-g)^{1/2}$ for models (26) and (33) in both Case-I and Case-II are found to be $V = t^{3r}(1+x^2)^a$ and $V = t(1+x^2)^a$, respectively. In both cases, it is seen that $V \to 0$ as $t \to 0$ and $V \to \pm\infty$ as $t \to \pm\infty$. Thus, the result shows expansion of the universe with time. Also, these model universes start expanding with zero volume and blow up at infinite past and at infinite future.

4.11 Dark Energy

The real nature of dark energy is a matter of speculation. It is known to be very homogeneous, not very dense and is not known to interact through any of the fundamental forces other than gravity. Before dark energy, there was the notion of the cosmological constant represented by the Greek symbol Lambda. It was a feature of the original equations of Einstein's general relativity and caused the universe to be static. Later evidence supported the fact that the universe was indeed expanding and the cosmological constant was believed to be zero. Evidence in the late 1990s has begun to support the idea that the universe is not only expanding, but that the expansion rate is actually accelerating due to the presence of dark energy.

Dark energy is an additional energy that penetrates all of space and tends to increase the rate of expansion of the universe (Peebles & Ratra 2003). There are two proposed forms for dark energy, i.e. the cosmological constant Λ (lambda) and scalar fields such as quintessence (a scalar field with gravitational interactions called quintessence). The cosmological constant is a *constant* energy density filling space homogeneously (Carroll 2001). Quintessence is a *dynamic* quantity whose energy density can vary in time and space. Contributions from scalar fields, which are constant in space, are usually also included in the cosmological constant. The cosmological constant is physically equivalent to vacuum energy and scalar fields, which do change in space, and can be difficult to distinguish from a cosmological constant because the change may be extremely slow.

The latest 2005 Supernova Legacy Survey reveals that the average behavior (i.e. equation of state) of dark energy behaves like Einstein's cosmological constant to a precision of 10% (Astier et al. 2006). The existence of dark energy, in whatever form, is needed to reconcile the measured geometry of space with the total amount of matter in the universe. Measurements of the cosmic microwave background (CMB) account for anisotropies and the most recent WMAP satellite indicates that the universe is very close to being flat. For the shape of the universe to be flat, the mass/energy density of the universe must be equal to a certain critical density. The total amount of matter in the universe (including baryons and dark matter), as measured by the CMB, accounts for only about 30% of the critical density. This implies the existence of an additional form of energy to account for the remaining 70% (Spergel et al. 2006). The most recent WMAP observations are consistent with a universe made up of 74% dark energy, 22% dark matter, and 4% ordinary matter (Hinshaw 2008).

5 CONCLUSIONS

Two distinct classes of solutions to the problem of a plane symmetric non-static massless scalar field in general relativity with a viscous fluid and a cosmological constant have been obtained. In Case-I, it is found that the cosmological constant is zero and the massless scalar field is time-dependent, whereas in Case-II, both of them are time-dependent. The Hubble's parameter is found to be time-dependent in both the models. Also, it is observed that the cosmological constant and massless scalar field are not mutually dependent, but both are divergent at $t = 0$ and convergent at $t = \infty$.

Earlier, it was an assumption that the effect of bulk viscosity is to introduce a change in the perfect fluid models and the bulk viscosity exhibits essential influence on the character of the solution. However, it is observed from the models obtained in Section 3 that Murphy's conclusion (Murphy 1973) about the absence of the Big Bang singularity in the models with bulk viscous fluid at infinite past or at the initial epoch is, in general, not true.

It is seen that models are found to expand in nature and that the model universes start expanding with zero volume and blow up at infinite past and at infinite future. Also, the models start expanding with a Big Bang at the initial epoch and the expansion stops at the infinite future. Further, it is observed that the first model is isotropic and the second model is anisotropic, but both models are non-rotating and mesonic fluid flow is geodesic. Again, in the case of the second model, the shape of the universe changes uniformly in the Y and Z directions, but the rate of change becomes slow as time increases. Moreover, the space-time considered here is not an Einstein space and models found can be applied to all the stages of the evolution of the universe.

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References

Anderson, J. L. 1967, Principle of Relativity Physics (New York: Academic Press)
Astier, P., et al. 2006, A&A, 447, 31
Barrow, J. D. 1988, Nucl. Phys. B, 310, 743
Carroll, S. 2001, Living Reviews in Relativity, 4, 1
Collins, C. B., Glass, E. N., & Wilkinson, D. A. 1980, General Relativity and Gravitation, 12, 805
Einstein, A. 1916, Annalen der Physik, 49, 769
Gron, O. 1990, Astrophysics and Space Science, 173, 191
Hinshaw, G. F. 2008, WMAP Cosmological Parameters Model: NASA
Kolb, E. W., & Turner, M. S. 1990, Frontiers in Physics, 69
Murphy, G. L. 1973, Phys. Rev. D, 8, 4231
Myung, S., & Cho., B. M. 1986, Mod. Phys. Lett. A, 1, 37
Patel, L. K., & Dadhich, N. 1993, J. Math. Phys., 34, 1927
Peebles, P. J. E., & Ratra, B. 2003, Reviews of Modern Phys., 75, 559
Pradhan, A., Saraykar, R. V., & Beesham, A. 1997, Astro Lett. Comm., 35, 383
Raychoudhuri, A. K. 1955, Physical Rev., 98, 1123
Sahu, R. C., & Mahapatra, L. K. 2009, Bull. Cal. Math. Soc., 101, 497
Spergel, D. N., et al. 2006 (WMAP Collaboration), Wilkinson Microwave Anisotropy Probe (WMAP) three years results: Implications for Cosmology
Turok, N. 1988, Phys. Rev. Lett., 60, 549
Weinberg, S. 1971, ApJ, 168, 175
Weinberg, S. 1972, in Gravitation and Cosmology, Principles and Applications of the General Theory of Relativity, ed. S. Weinberg (Wiley-VCH), 688