

Perturbations of the Yang-Mills field in the universe *

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Abstract It has been suggested that the Yang-Mills (YM) field can be a kind of candidate for the inflationary field at high energy scales or dark energy at very low energy scales, which can naturally give the equation of state $-1 < \omega < 0$ or $\omega < -1$. We discuss the zero order and first order Einstein equations and YM field kinetic energy equations of the free YM field models. From the zero order equations, we find that $\omega + 1 \propto a^{-2}$, from which it follows that the equation of state of the YM field always goes to -1 , independent of the initial conditions. By solving the first order Einstein equations and the YM field equations, we find that in the YM field inflationary models, the scale-invariant primordial perturbation power spectrum cannot be generated. Therefore, only using this kind of YM field is not enough to account for inflationary sources. However, as a kind of candidate for dark energy, the YM field has the ‘sound speed’ $c_s^2 = -1/3 < 0$, which makes the perturbation ϕ have a damping behavior at large scales. This provides a way to distinguish the YM field dark energy models from other kinds of models.

Key words: cosmology: theory — gravitation — methods: analytical

1 INTRODUCTION

It is well known that the physics of inflationary fields and dark energy are two of the most important problems in modeling cosmological. They all need a kind of matter with negative pressure $p \simeq -\rho$ (Kolb & Turner 1990; Bennett et al. 2003; Riess et al. 1998; Tegmark et al. 2004; Feng et al. 2005), where p and ρ are the pressure and energy density of matter, respectively. People always describe them with a scalar field, which can naturally give an equation of state of $-1 \leq \omega \leq 0$ (Kolb & Turner 1990; Wetterich 1995; Bharat & Peebles 1988). In particular, when the potential of the scalar field is dominant, ω will go to -1 . Thus, the expansion of the Universe is close to the de Sitter expansion. Recently, observations of the cosmic microwave background radiation (CMB) temperature and polarization anisotropies by the Wilkinson Microwave Anisotropy Probe (WMAP) show that the spectral index of the primordial density perturbation $n_s = 1.20_{-0.11}^{+0.12}$ at wavenumber $k = 0.002 \text{ Mpc}^{-1}$ (Bennett et al. 2003). Observations of Type Ia Supernovae (SNeIa), CMB and large scale structure (LSS) (Bennett et al. 2003; Riess et al. 1998; Tegmark et al. 2004) also suggest that the equation of state of the dark energy may be $\omega < -1$ (Corasaniti et al. 2004). These are all very difficult to obtain from single scalar field models. So, it is necessary to look for a new candidate for the inflationary field and dark energy.

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Recently, a number of authors have considered using a vector field as the candidate of the inflationary field or dark energy (Ratra 1992). We have advocated using the effective YM condensate (Zhang 1994, 2003; Zhao & Zhang 2006a,b; Zhang et al. 2007; Zhao & Xu 2007; Zhao 2008) as a kind of candidate, which can be used to describe the inflation at high energy scales and dark energy at very low energy scales. In our models, a quantum effective YM condensate is used as the source of inflation or dark energy, instead of a scalar field. This model has a desired interesting feature: the YM field is an indispensable cornerstone to any particle physics model with interactions mediated by gauge bosons, so it can be incorporated into a sensible unified theory of particle physics. Besides, the equation of state of this field is different from that of general matter as well as scalar fields, and the state $\omega < -1$ can be naturally realized.

In this paper, we shall discuss the evolution of the equation of state of the YM field and cosmic perturbations by solving the zero and first order Einstein equations and kinetic energy equations. From the zero order Einstein equations, we find that the YM field can easily give a state of homogeneity and isotropy, and it can naturally give an equation of state $\omega < -1$ or $\omega > -1$. We also find that $\omega + 1 \propto a^{-2}$ from the zero order equations. It follows that ω naturally goes to -1 with the expansion of the Universe, independent of the initial condition. By considering the evolution of cosmic perturbations, we investigate the first order Einstein equations and kinetic energy equations. In the simplest condition with only an electric field, we find that this YM field has the sound speed $c_s^2 = -1/3 < 0$, which is very different from scalar field models. We also find that a scale-invariant primordial perturbation power spectrum cannot be generated, which shows that an electric YM field alone cannot be a candidate for the inflationary field. However, as a candidate for dark energy, the YM field makes the cosmic fluctuation ϕ damped at large scales. This is helpful for answering large scale damping of the CMB anisotropy power spectrum.

2 THE QUANTUM EFFECTIVE YANG-MILLS FIELD

In the quantum effective YM field dominated Universe, the effective YM field Lagrangian is given by (Adler 1981)

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}bF \ln |F/e\kappa^2|, \quad (1)$$

where κ is the renormalization scale with dimension of squared mass, and $F \equiv -\frac{1}{2}F_{\mu\nu}^a F^{a\mu\nu}$ plays the role of the order parameter of the YM field. The Callan-Symanzik coefficient $b = 11N/24\pi^2$ for $SU(N)$ when the fermion's contribution is neglected. For the gauge group $SU(2)$ considered in this paper, one has $b = 2 \times 11/24\pi^2$. For the case of $SU(3)$, the effective Lagrangian in Equation (1) leads to a phenomenological description of the asymptotic freedom for the quarks inside hadrons (Adler 1981). It should be noted that the $SU(2)$ YM field is introduced here as a model for a cosmic inflationary field or dark energy, and it may not be directly identified as a QCD gluon field, and also not the weak-electromagnetic unification gauge field.

An explanation can be given for the form in Equation (1) as an effective Lagrangian up to a 1-loop quantum correction (Pagels & Tomboulis 1987; Adler 1981). A classical $SU(N)$ YM field Lagrangian is $\mathcal{L} = \frac{1}{2g_0^2}F$, where g_0 is the bare coupling constant. As is known, when the 1-loop quantum corrections are included, the bare coupling g_0 will be replaced by a running g as used in the following (Gross & Wilczek 1973; Pagels & Tomboulis 1987; Adler 1981), $g_0^2 \rightarrow g^2 = \frac{4 \times 12\pi^2}{11N \ln(k^2/k_0^2)} = \frac{2}{b \ln(k^2/k_0^2)}$, where k is the momentum transfer and k_0 is the energy scale. To build up an effective theory (Pagels & Tomboulis 1987; Adler 1981), one may just replace the momentum transfer k^2 by the field strength F in the following manner, $\ln(k^2/k_0^2) \rightarrow 2 \ln |F/e\kappa^2|$, yielding Equation (1). The expression of $1/g^2$ is based on the renormalization group estimates. As emphasized in the references (Adler 1981; Adler & Piran 1982), this estimate is formally valid whenever the running coupling g^2 is small in magnitude, which is true both when $F/\kappa^2 \gg 1$ (given g^2 is small and positive) and when $F/\kappa^2 \ll 1$ (given g^2 is small and negative).

The attractive features of this effective YM model include the gauge invariance, the Lorentz invariance, the correct trace anomaly, and the asymptotic freedom (Pagels & Tomboulis 1987). With the logarithmic dependence on field strength, \mathcal{L}_{eff} has a form similar to the Coleman-Weinberg scalar effective potential (Coleman & Weinberg 1973), and the Parker-Raval effective gravity Lagrangian (Parker & Raval 1999). The dielectric constant is defined by $\epsilon = 2\partial\mathcal{L}_{\text{eff}}/\partial F$, and, in the 1-loop order, it is given by

$$\epsilon = b \ln |F/\kappa^2|. \quad (2)$$

As analyzed in (Adler 1981), the 1-loop model is a universal, leading to a semi-classical approximation. Thus, depending on whether the field strength $|F| \geq \kappa^2$ or $|F| \leq \kappa^2$, the YM condensate belongs to the family of matter whose dielectric constant ϵ can be positive or negative. The properties mentioned above are still true even if 2-loop order corrections are taken into account, an essential feature of the effective model (Adler 1981; Adler & Piran 1982).

It is straightforward to extend the model to the expanding Robertson-Walker (R-W) spacetime. For simplicity, we shall work in a spatially flat R-W spacetime with a metric $ds^2 = a^2(\tau)(d\tau^2 - \gamma_{ij}dx^i dx^j)$, where we have set the speed of light $c = 1$ and $\gamma_{ij} = \delta_j^i$, denoting that the background space is flat, and $\tau = \int (a_0/a)dt$ is the conformal time. The dominant matter is assumed to be the quantum YM condensate, whose effective action is $S = \int \mathcal{L}_{\text{eff}} a^4(\tau) d^4x$, and the Lagrangian \mathcal{L}_{eff} is defined in Equation (1). By variation of S with respect to $g^{\mu\nu}$, one obtains the energy-momentum tensor,

$$T_{\mu\nu} = -g_{\mu\nu} \frac{b}{2} F \ln |F/e\kappa^2| + \epsilon F_{\mu\sigma}^a F_{\nu}^{a\sigma}, \quad (3)$$

where the energy-momentum tensor is the sum of three energy-momentum tensors of vectors, $T_{\mu\nu} = \sum_a T_{\mu\nu}^a$. In order to keep the total energy-momentum tensor homogeneous and isotropic, we assume that the gauge fields are only functions of time t , and $A_\mu = \frac{i}{2}\sigma_a A_\mu^a(t)$ (Zhao & Zhang 2006a). YM field tensors are defined as usual:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c. \quad (4)$$

This tensor can be written in the form with electric and magnetic fields as

$$F_{\nu}^{a\mu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}. \quad (5)$$

From the definition in Equation (4), we can find that $E_1^2 = E_2^2 = E_3^2$ and $B_1^2 = B_2^2 = B_3^2$. Thus, F has a simple form $F = E^2 - B^2$, where $E^2 = \sum_{i=1}^3 E_i^2$ and $B^2 = \sum_{i=1}^3 B_i^2$. Inserting Equation (4) in Equation (3), we can obtain the energy density and pressure of the YM field given by (Zhao & Zhang 2006a),

$$\rho = \frac{1}{2}\epsilon(E^2 + B^2) + \frac{1}{2}b(E^2 - B^2), \quad (6)$$

$$p = \frac{1}{6}\epsilon(E^2 + B^2) + \frac{1}{2}b(B^2 - E^2), \quad (7)$$

and

$$\rho + p = \frac{2}{3}\epsilon(E^2 + B^2). \quad (8)$$

Equation (8) follows a conclusion of this study: the weak energy condition can be violated so that $\rho + p < 0$, by the effective YM condensate matter in a family of quantum states with the negative dielectric constant $\epsilon < 0$. The whole range of allowed values of F is divided into two domains with $|F| > \kappa^2$ and $|F| < \kappa^2$, respectively. In the domain where $|F| > \kappa^2$, one always has $\epsilon > 0$, so the WEC is still satisfied. The other domain is $|F| < \kappa^2$ in which $\epsilon < 0$, so that the WEC is now violated.

3 THE ZERO ORDER EQUATIONS

3.1 The Zero Order Einstein Equations

Let us firstly investigate the Friedmann equations, which can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$

where “dot” denotes d/dt . If we consider the simplest case with only an “electric” field, as used in the previous works (Zhang et al. 2007; Zhao & Zhang 2006a), the Friedmann equations can be reduced to

$$\frac{d\rho}{da^3} = -\frac{4}{3}\frac{\beta\rho}{a^3}, \quad (9)$$

where we have defined $\beta \equiv \epsilon/b$. If $|\beta| \ll 1$, one gets

$$a^2\beta = \text{constant}, \quad (10)$$

which follows the relation of the equation of state:

$$\omega + 1 \propto a^{-2}. \quad (11)$$

From this relation, we find that ω will run to the critical condition with $\omega = -1$ with the expansion of the Universe. In the next subsection, we will find that from the zero order YM field kinetic equation, we can get the same result, which means that the YM field equation and the Einstein equations are self-rational.

3.2 The Zero Order Yang-Mills Field Kinetic Energy Equations

By variation of S with respect to A_μ^a , one obtains the effective YM equations (Zhao & Zhang 2006a)

$$\partial_\mu(a^4\epsilon F^{a\mu\nu}) - f^{abc}A_\mu^b(a^4\epsilon F^{c\mu\nu}) = 0, \quad (12)$$

which can be simplified as (Zhao & Zhang 2006a),

$$a^2E\epsilon = \text{constant}. \quad (13)$$

If the Universe is dominated by an “electric” YM field, the kinetic energy equation follows as

$$a^2\beta e^{\beta/2} = \text{constant}, \quad (14)$$

when $|\beta| \ll 1$, then this equation becomes

$$a^2\beta(1 + \beta/2) \simeq a^2\beta = \text{constant}.$$

which also follows as

$$\omega + 1 \propto a^{-2}. \quad (15)$$

This is exactly the same as that of the zero order Einstein equations.

4 THE FIRST ORDER EQUATIONS

4.1 The First Order Einstein Equations

As a kind of candidate of inflationary field or dark energy, it is very important to study the evolution of perturbations in the YM field and cosmic fluctuations. In this section, let us consider the flat R-W metric with a scalar perturbation in the conformal Newtonian gauge

$$ds^2 = a^2(\tau)[(1 + 2\phi)d\tau^2 - (1 - 2\psi)\gamma_{ij}dx^i dx^j]. \quad (16)$$

The gauge-invariant metric perturbation ψ is the Newtonian potential and ϕ is the perturbation of the intrinsic spatial curvature. The first order Einstein equations become (Mukhanov et al. 1992):

$$-3\mathcal{H}(\mathcal{H}\phi + \psi') + \nabla^2\psi = 4\pi G a^2 \delta T_0^0, \quad (17)$$

$$(\mathcal{H}\phi + \psi')_{,i} = 4\pi G a^2 \delta T_i^0, \quad (18)$$

$$[(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \psi'' + 2\mathcal{H}\psi' + \frac{1}{2}\nabla^2 D]\delta_j^i - \frac{1}{2}\gamma^{ik}D_{|kj} = -4\pi G a^2 \delta T_j^i, \quad (19)$$

where $\mathcal{H} = a'/a$, $D = \phi - \psi$ and the prime denotes $d/d\tau$. From the definition of the energy-momentum tensor (3) and the metric of (16), one can get the first order energy-momentum tensor:

$$\delta T_0^0 = (\epsilon - b)(B\delta B - E\delta E) + 2\epsilon E\delta E + 2\phi(\epsilon - b)B^2 + \frac{B^2 + E^2}{2}\delta\epsilon, \quad (20)$$

$$\begin{aligned} -\delta T_i^i &= (\epsilon - b)(-B\delta B + E\delta E) - 2\phi(\epsilon - b)B^2 + \epsilon[(2B_2\delta B_2 + 2B_3\delta B_3 \\ &\quad - 2E_1\delta E_1) + 4(B_2^2 + B_3^2)\phi] + \frac{1}{2}(E^2 - B^2)\delta\epsilon + \frac{2B^2 - E^2}{3}\delta\epsilon, \end{aligned} \quad (21)$$

and others are all zero, where $E\delta E = E_1\delta E_1 + E_2\delta E_2 + E_3\delta E_3$, and similar for $B\delta B$.

To obtain the gauge-invariant equations of motion for cosmological perturbations in a Universe dominated by this kind of YM field, we insert the general equations of the energy-momentum tensor into δT_ν^μ . First of all, from the $i - j$ ($i \neq j$) equation, it follows that we can set $\phi = \psi$, since $\delta T_j^i = 0$ ($i \neq j$). Substituting the energy-momentum tensor δT_ν^μ into the general equations and setting $\psi = \phi$, we find:

$$-3\mathcal{H}(\mathcal{H}\phi + \phi') + \nabla^2\phi = 4\pi G a^2 \{(\epsilon - b)(B\delta B - E\delta E) + 2\epsilon E\delta E + 2\phi(\epsilon - b)B^2 + \frac{B^2 + E^2}{2}\delta\epsilon\}, \quad (22)$$

$$(\mathcal{H}\phi + \phi')_{,i} = 0, \quad (23)$$

$$\begin{aligned} [(2\mathcal{H}' + \mathcal{H}^2)\phi + \mathcal{H}\phi' + \phi'' + 2\mathcal{H}\phi'] &= 4\pi G a^2 \{(-B\delta B + E\delta E)(\epsilon - b) - 2\phi B^2(\epsilon - b) \\ &\quad + \frac{\epsilon}{3}[4B\delta B - 2E\delta E + 8\phi B^2] + \frac{E^2 + B^2}{6}\delta\epsilon\}, \end{aligned} \quad (24)$$

where $\delta\epsilon = (2E\delta E - 2B\delta B - 4B^2\phi)/(E^2 - B^2)$, the relation between δE and δB depends on the YM field equations as below.

4.2 The First Order Kinetic Energy Equations of the YM Field

The metric as before with $\psi = \phi$ is

$$ds^2 = a^2(\tau)[(1 + 2\phi)dt^2 - (1 - 2\phi)\gamma_{ij}dx^i dx^j], \quad (25)$$

then $\sqrt{-g} = a^4(1 - 2\phi)$. The kinetic energy equation is

$$\partial_\mu(a^4(1 - 2\phi)\epsilon F^{a\mu\nu}) - f^{abc}A_\mu^b(a^4(1 - 2\phi)\epsilon F^{c\mu\nu}) = 0, \quad (26)$$

from which, one can get the first order perturbation equations:

$$\partial_\mu [a^2(\epsilon\partial_\tau\delta A + \delta\epsilon\partial_\tau A - 2\epsilon\phi\partial_\tau A)] = 0, \quad (\mu = 0, 1, 2, 3) \quad (27)$$

$$\partial_i[\delta\epsilon A^2 + 2\epsilon\phi A^2 + 2\epsilon A\delta A] = 0, \quad (i = 1, 2, 3) \quad (28)$$

which can also be written as

$$\partial_i(B\delta\epsilon + 2\epsilon\phi B + 2\epsilon\delta B) = 0, \quad (29)$$

$$\partial_i(\epsilon\delta E + \delta\epsilon E - 2\epsilon\phi E) = 0, \quad (30)$$

$$\partial_\tau[a^2(\epsilon\delta E + \delta\epsilon E - 2\epsilon\phi E)] = 0. \quad (31)$$

From these equations, we can immediately get the simple relation between δB and δE :

$$a^2(\epsilon\delta E + \delta\epsilon E - 2\epsilon\phi E) = \text{constant}, \quad (32)$$

which is useful when solving the first order Einstein Equations (22)–(24).

4.3 The Solution of the Perturbations

Here, we only discuss the simplest case with $B \equiv 0$, so the first order energy-momentum tensor becomes very simply

$$\delta T_0^0 = (\epsilon + 2b)E\delta E, \quad (33)$$

$$-\delta T_i^i = (\epsilon - 2b)E\delta E/3, \quad (34)$$

$$\delta T_i^0 = -2\epsilon[E_1(\delta B_3 - \delta B_2)] = 0, \quad (35)$$

where we have used $\delta\epsilon = 2bE\delta E/E^2$ when $B \equiv 0$. From Equations (34)–(35), we find that δT_ν^μ is independent of the metric perturbation of ϕ , which is different from the scalar field models (Weller & Lewis 2003; Armendariz-Picon et al. 2000; DeDeo et al. 2003). The first order Einstein equations become

$$-3\mathcal{H}(\mathcal{H}\phi + \phi') + \nabla^2\phi = 4\pi G a^2 E\delta E(\epsilon + 2b), \quad (36)$$

$$(\mathcal{H}\phi + \phi')_{,i} = 0, \quad (37)$$

$$[(2\mathcal{H}' + \mathcal{H}^2)\phi + \phi'' + 3\mathcal{H}\phi'] = 4\pi G a^2 E\delta E(\epsilon - 2b)/3. \quad (38)$$

From these, we obtain the main equation, which describes the evolution of the metric perturbation ϕ with time:

$$\phi'' + 3\mathcal{H}(1 - \gamma)\phi' + \gamma\nabla^2\phi + (2\mathcal{H}' + \mathcal{H}^2 - 3\mathcal{H}^2\gamma)\phi = 0, \quad (39)$$

where $\gamma \equiv \frac{2b-\epsilon}{6b+3\epsilon}$. From this equation, one can easily find that the evolution of ϕ only depends on γ and \mathcal{H} , but not on the first order YM field kinetic energy equations. This is because we have only considered the “electric” YM field. If we also consider the B components, the YM Equation (38) will be used to relate δE and δB .

For the inflationary field, the most important prediction is that the inflation can generate a scale-invariant primordial scalar perturbation power spectrum, which has been found in the CMB power spectrum and large scale structure. Now, we shall firstly consider whether a Universe dominated by this YM field can also generate a scale-invariant spectrum using the general scalar inflationary models.

If we consider the YM field as a kind of candidate for an inflationary field, the YM field should satisfy the following constraints:

- (1) Firstly, we should require that the inflation can exist, which means that the YM field has a state with $\omega < -1/3$. Using the p and ρ in Equation (12), one can easily get a constraint on the YM field:

$$\epsilon < b. \quad (40)$$

- (2) The energy density of the YM field should be positive, which induces the second constraint on the YM field:

$$\epsilon > -b. \quad (41)$$

- (3) Due to Equation (39), if we can define an adiabatic vacuum state at very high frequency ($k \rightarrow \infty$), this would require that $\gamma < 0$. This induces the third constraint on the YM field:

$$\epsilon > 2b, \text{ or } \epsilon < -2b. \quad (42)$$

We find that these three simple constraints on the YM field cannot be satisfied at the same time, so this model cannot generate a scale-invariant primordial power spectrum as a general scalar field inflationary model. However, it is necessary to notice that this does not mean that the YM field cannot be the source of the inflation. Recently, some authors have discussed a kind of curvaton reheating mechanism in non-oscillatory inflationary models (Feng & Li 2003). In this kind of model, the primordial spectrum and the reheating can be generated by the other curvaton field. So, although the YM field cannot generate a scale-invariant primordial spectrum, the YM field can also be the background field in the curvaton field inflationary models, which will be discussed in a future work.

However, the YM field can be a good candidate for dark energy (Zhao & Zhang 2006a; Zhang et al. 2007). Now let us discuss the evolution of the cosmic scalar fluctuations ϕ in the YM field dark energy models.

Neglecting anisotropic stress, the potential ϕ evolves as (Weller & Lewis 2003; Ma & Bertschinger 1995; Gordon & Hu 2004)

$$\phi'' + 3\mathcal{H} \left(1 + \frac{p'}{\rho'}\right) \phi' - \frac{p'}{\rho'} \nabla^2 \phi + \left[\left(1 + 3\frac{p'}{\rho'}\right) \mathcal{H}^2 + 2\mathcal{H}' \right] \phi = 4\pi G a^2 \left(\delta p - \frac{p'}{\rho'} \delta \rho \right), \quad (43)$$

where $p = \sum_i p_i$ and $\rho = \sum_i \rho_i$, which should include the contributions of the baryon, photon, neutron, cold dark matter, and the dark energy. Here, we consider the simplest case with only the YM field dark energy, and it has the equation of state $\omega_{de} = -1$. The ‘sound speed’ is defined by $c_s^2 = \delta p / \delta \rho$. From Equations (39) and (40), one finds that $c_s^2 = -1/3$. So then, the Equation (43) becomes

$$\phi'' + 2\mathcal{H}\phi' + \frac{1}{3}\nabla^2\phi + 2\mathcal{H}'\phi = 0, \quad (44)$$

which is the same as Equation (39) with $\gamma = 1/3$. Defining $u \equiv a\phi$, one gets

$$u_k'' - \frac{k^2}{3}u_k = 0, \quad (45)$$

where u_k is the Fourier component with wavenumber k , and we have used $a \propto 1/\tau$, which has the solution of

$$u_k \propto e^{\pm \frac{k\tau}{\sqrt{3}}}. \quad (46)$$

For the large wavelength case ($k\tau \ll 1$), we have

$$\phi \propto a^{-1}, \quad (47)$$

and when $k\tau \gg 1$, one has the growing solution

$$u \propto e^{-\frac{k\tau}{\sqrt{3}}}, \text{ and } \phi \sim a^{-1} e^{-\frac{k\tau}{\sqrt{3}}}. \quad (48)$$

We find that in this universe, the evolution of the cosmic fluctuation ϕ is very different from that in the scalar field dark energy models. The fluctuations with large wavelengths $k\tau \ll 1$ are always damped with the expansion of the Universe. However, fluctuations with small wavelengths show rapid growth with time, which is because of the negative sound speed $c_s^2 = -1/3 < 0$. This should have an important effect on the large scale integrated Sachs-Wolfe effect of CMB. We should mention that the result in this subsection is only qualitative since we have not considered other effects, such as baryons, photons, neutrons, dark matter and so on, which are also important for the evolution of the cosmic fluctuation ϕ , especially at small scales.

5 CONCLUSIONS AND DISCUSSION

In this paper, we have investigated the effective YM field as a kind of candidate for an inflationary field or dark energy. From the zero order Einstein equations and YM field kinetic energy equations, we find that this field can naturally give an equation of state $-1 < \omega < 0$ and $\omega < -1$. This is one of the most important features of this model. Also, we get $\omega + 1 \propto a^{-2}$, which suggests that ω will go to -1 with the expansion of the Universe. This makes the Universe dominated by this field have a de Sitter expansion.

When considering the evolution of the perturbations, we solved the first order Einstein equations and YM field kinetic energy equations. We find that the model with only an electric field cannot generate a scale-invariant primordial scalar perturbation power spectrum. This means that a model that only uses the YM field is not a good candidate for describing the inflationary field. However, as a kind of candidate for dark energy, we obtained the equation of the YM perturbation, and found that this field is very different from the general scalar field dark energy models. This YM field has a negative sound speed, which dampens cosmic perturbations at large scales. This is a very important source of the integrated Sachs-Wolfe effect of the CMB power spectrum. This damping of ϕ at large scales may be helpful for answering the very small quadrupole problem of the CMB temperature anisotropy power spectrum.

We should mention that, in this paper, we have not considered the possible interaction between the YM field dark energy and the other components, especially the dark matter (Zhang et al. 2007). We leave this topic to future work.

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