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Gnomon shadow lengths recorded in the *Zhoubi Suanjing*: the earliest meridian observations in China? *

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Abstract The *Zhoubi Suanjing*, one of the most important ancient Chinese books on mathematical astronomy, was compiled about 100 BC in the Western Han dynasty (BC 206 – AD 23). We study the gnomon shadow lengths for the 24 solar terms as recorded in the book. Special attention is paid to the so-called law of 'cun qian li', which says the shadow length of a gnomon of 8 chi (about 1.96 m) high will increase (or decrease) 1 cun (1/10 chi) for every 1000 li (roughly 400 km) the gnomon moves northward (or southward). From these data, one can derive the time and location of the observations. The results, however, do not fit historical facts. We suggest that compilers of the *Zhoubi Suanjing* must have modified the original data according to the law of 'cun qian li'. Through reversing the situation, we recovered the original data, our analysis of which reveals the best possible observation time as 564 BC and the location of observation as 35.78° N latitude. We conclude that this must be the earliest records of solar meridian observations in China. In the meantime, we give the errors of solar altitudes for the 24 solar terms. The average deviation is 5.22°, and the mean absolute deviation is 5.52°, signifying the accuracy of astronomical calculations from that time.

Key words: history and philosophy of astronomy — solar meridian observation — accuracy — solar-terrestrial relations — methods: statistical

1 INTRODUCTION

When looking back into ancient Chinese annals, we find a huge amount of material on astronomy, such as observation records of all kinds of celestial phenomena, astronomical calendars and related issues, astronomical instruments and theories about the origin, structure, shape and movements of the planets (Zhong Hua Books Company Editorial Office 1976). These records were studied by Western missionaries starting in the 17th century. In the 20th century, through the works of Joseph Needham, in particular the well-known series entitled *Science and Civilization in China*, ancient Chinese achievements in science became known to the world. Astronomy was a major part of Chinese science (Needham 1959).

Inspired by Needham's work, Chinese scientists initiated major projects on the history of Chinese science in the second half of the 20th century. As far as astronomy is concerned, Chinese science historians collected a large amount of astronomical data recorded in the historical documents of China and published a compendium (Beijing Astronomical Observatory 1988) which includes more than 1000 solar eclipses with 100 being total eclipses, 2000 lunar eclipses with 400 being total ones, 200 lunar

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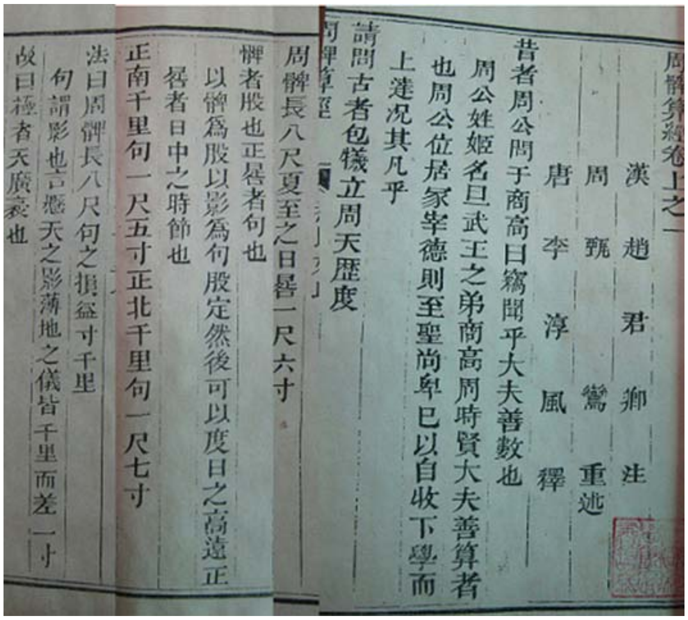

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Fig. 1 Block-printed edition of *Zhoubi Suanjing* from the Tang dynasty (AD 618 to AD 907). This page shows the well-known law of 'cun qian li'.

occultations of the planets, 200 sunspots, 500 comets, 100 novas, 1000 meteors, 100 meteor showers, 100 aerolites and 100 records of northern lights.

Li & Sun (2009) pointed out that in Chinese history there existed two types of data about the gnomon shadow lengths for the 24 solar terms. According to their analysis, the data in the *Zhoubi Suanjing* (hereafter referred to as the *Zhoubi*) were not based on actual measurements, but rather they were derived by means of linear interpolation from the gnomon shadow lengths for the Winter Solstice and Summer Solstice. The second type of data was found in later documents and fit more closely to actual values. Since the values for both solstices in the *Zhoubi* should have existed before linear interpolation, the question is where did they come from?

Qian (1958) and Bo (1989) investigated the astronomical data in the *Zhoubi* in detail. Both of them concluded that the data of the solar shadow lengths recorded in the *Zhoubi* were not from actual observations. It is generally believed that the data are not reliable because the derived observation location (latitude $> 35.3^\circ$ N) does not fit with the capital ($< 34.8^\circ$ N) of the Zhou dynasty (BC 1046 to BC 256), and the derived year of observation (earlier than BC 2500) does not correspond to that period. Such analysis is valuable for understanding the data, but the question still remains: How were these seemingly impossible data obtained?

Zhao (2009) suggested BC 511 as the observational year of the *Zhoubi*, which differs from the previous results. In this paper, we will reinvestigate the records on gnomon shadow lengths at noon for the 24 solar terms. We suggest that these data have been modified before they were passed down to us. To prove this, we need a detailed investigation of the solar meridian observations in ancient China. The reliability and accuracy of observational data are particulary relevant.

2 HISTORICAL DOCUMENTS AND THE OBSERVATIONAL INSTRUMENT

The *Zhoubi* (Fig. 1) is the earliest classic work on mathematics and astronomy, which was probably compiled about BC 100 in the Western Han dynasty (BC 206 – AD 23). The title literally means gnomon and mathematics of the Zhou dynasty.

Fig. 2 *Top*: The Dengfeng ancient astronomical observatory, in Dengfeng county of Henan province. It was designed in 1279 by Guo Shoujing of the Yuan dynasty (AD 1271–AD 1368). The relic shows a huge gui biao, with the biao being 40 chi high (9.7468 m) and the gui being 128 chi long (31.196 m). *Bottom*: This gui biao, in its typical design, was made from bronze with a height of 8 chi (1.96 m). It is on exhibition at the Ancient Astronomical Observatory of Beijing.

The gui biao, which indicates sundial or gnomon, originated from ancient antiquity. It consisted of a vertical biao and a horizontal gui placed in the South-North direction. It had many uses, such as the determination of local time, of directions, and of the length of the tropical year, with its 24 solar terms. Basically, it was an instrument for solar meridian observations. We do not know exactly when the gui biao was invented, but we know that it was first used for meridian observations in very ancient times. At least from the middle of the Chunqiu period (ca. 500 BC), the gui biao shadow length measurements had become a very important method for calendar making. Figure 2 shows two different kinds of gui biaos in ancient China. The one shown in Figure 2(a) was built in the Yuan dynasty (AD 1271 – AD 1368) and the other (Fig. 2(b)) in the Ming dynasty (AD 1368 – AD 1644). Li (2001) studied the structure and historical development of the gui biao. Here, we are only concerned with the data measured by using this instrument.

In the Yuan dynasty, Guo Shoujing enlarged the biao to a height of 40 chi in order to obtain more accurate measurements of the shadow lengths at noon.

3 RECORDED DATA AND THE LAW OF CUN QIAN LI

The data of the shadow length for the 24 solar terms recorded in the *Zhoubi* are gathered here in Table 1. The height of the biao is 8 chi.

Table 1 Data of Shadow Lengths for the 24 Solar Terms at Noon as Recorded in the *Zhoubi*

No	24 Solar terms	Modern solar longitude（°）	Shadow length		Solar altitude at noon（°）
			Length（chi）	In Chinese Characters	
1	Dongzhi (Winter Solstice)	270.0	13.5000	冬至丈三尺五寸	30.65
2	Xiaohan (Lesser Cold)	285.0	12.5083	小寒丈二尺五寸小分五	32.60
3	Dahan (Greater Cold)	300.0	11.5167	大寒丈一尺五寸一分小分四	34.79
4	Lichun (Beginning of Spring)	315.0	10.5250	立春丈五寸二分小分三	37.24
5	Yushui (Rain Water)	330.0	9.5333	雨水九尺五寸三分小分二	40.00
6	Qizhe (Awakening from ibernation)	345.0	8.5417	啓蟄八尺五寸四分小分一	43.12
7	Chunfen (Spring Equinox)	0.0	7.5500	春分七尺五寸五分	46.66
8	Qingming (Fresh Green)	15.0	6.5583	清明六尺五寸五分小分五	50.66
9	Guyu (Grain Rain)	30.0	5.5667	穀雨五尺五寸六分小分四	55.17
10	Lixia (Beginning of Summer)	45.0	4.5750	立夏四尺五寸七分小分三	60.24
11	Xiaoman (Lesser Fullness)	60.0	3.5833	小滿三尺五寸八分小分二	65.87
12	Mangzhong (Grain in Ear)	75.0	2.5917	芒種二尺五寸九分小分一	72.05
13	Xiazhi (Summer Solstice)	90.0	1.6000	夏至一尺六寸	78.69
14	Xiaoshu (Lesser Heat)	105.0	2.5917	小暑二尺五寸九分小分一	72.05
15	Dashu (Greater Heat)	120.0	3.5833	大暑三尺五寸八分小分二	65.87
16	Liqiu (Beginning of Autumn)	135.0	4.5750	立秋四尺五寸七分小分三	60.24
17	Chushu (End of Heat)	150.0	5.5667	處暑五尺五寸六分小分四	55.17
18	Bailu (White Dew)	165.0	6.5583	白露六尺五寸五分小分五	50.66
19	Qiufen (Autumnal Equinox)	180.0	7.5500	秋分七尺五寸五分小分一	46.66
20	Hanlu (Cold Dew)	195.0	8.5417	寒露八尺五寸四分小分一	43.12
21	Shuangjiang (First Frost)	210.0	9.5333	霜降九尺五寸三分小分二	40.00
22	Lidong (Beginning of Winter)	225.0	10.5250	立冬丈五寸二分小分三	37.24
23	Xiaoxue (Light Snow)	240.0	11.5167	小雪丈一尺五寸一分小分四	34.79
24	Daxue (Heavy Snow)	255.0	12.5083	大雪丈二尺五寸小分五	32.60

In the *Zhoubi*, the Chinese terms for the shadow lengths include five length units: zhang, chi, cun, fen and xiao fen (mini fen). Usually zhang, chi, cun and fen have decimal bases, i.e., 1 zhang = 10 chi; 1 chi = 10 cun; and 1 cun = 10 fen, but 1 fen = 6 xiao fen.

In volume 2 of the Zhoubi, lengths of gnomon shadows at noon for all the 24 solar terms are given. In the text, it is stated that shadow lengths for 22 solar terms (Table 1) can be derived from that of the Winter Solstice 13.5 chi (No. 1) and that of the Summer Solstice 1.6 chi (No. 13). In the first place, the average difference in sunyi from Winter Solstice to Summer Solstice was calculated as follows:

$$\mathrm{sunyi} = (13.5 - 1.6)/12\,\mathrm{chi} = 0.9917\,\mathrm{chi} = 9\,\mathrm{cun} + (1/6)\,\mathrm{fen}. \tag{1}$$

Then starting from Winter Solstice, the length for the next solar term equals that for the Winter Solstice minus the value of sunyi; and starting from Summer Solstice, the next value increases by sunyi. For example, the value of Lesser Cold (No. 2) = Winter Solstice - sunyi $= 13.5 - 0.9917 = 12.5083$ chi, and the value of Greater Cold (No. 3) = Lesser Cold (No. 2) - sunyi $= 11.5167$ chi. The value of Lesser Heat (No. 14) = Summer Solstice (No. 13) + sunyi $= 1.6 + 0.9917 = 2.5917$ chi, and so on.

It is clear in the *Zhoubi* that the shadow length decreases linearly from the Winter Solstice to the Summer Solstice and then increases linearly from the Summer Solstice back to the Winter Solstice. This, however, does not correspond to the actual situation. Except for the Winter Solstice and Summer Solstice, the values of the shadow lengths were more likely calculated than observed.

In the meantime, the *Zhoubi* suggests that the shadow lengths of an 8 chi high biao would increase (or decrease) 1 cun if it is moved 1000 li northward (or southward). It is the so-called law of ‘cun qian li’. 1000 li in the Zhou dynasty was equivalent to about 400 km, and 1 cun was about 2.3 cm.

Monk Yixing, a well-known astronomer in the Tang dynasty, doubted the validity of this law. From AD 724 to 725, he carried out a large project on astronomical and geographical surveys that included measuring gnomon shadows at 13 different locations. After analyzing his results, he finally denounced the law of ‘cun qian li’. Now, undoubtedly, this law is incorrect, but by seeing how this incorrect law was used to produce data in the *Zhoubi*, we can have some clues for recovering the original observational data.

4 DATA PRE-PROCESSING

From recorded data, we can retrieve very important information, in this case the location and time of observations. According to spherical astronomy, the solar altitudes at noon for Winter Solstice and Summer Solstice are related to the local latitude. Neglecting corrections for both atmospheric refraction and the radius of the Sun, we have the solar altitude for Summer Solstice:

$$h_{\mathrm{s}} = 90^\circ + \varepsilon - \varphi. \tag{2}$$

And for Winter Solstice:

$$h_{\mathrm{w}} = 90^\circ - \varepsilon - \varphi\,, \tag{3}$$

where φ is the local latitude and ε is the ecliptic obliquity, so we have $h_{\mathrm{s}} - h_{\mathrm{w}} = 2\varepsilon$ and $\varphi = 90^\circ - 1/2(h_{\mathrm{s}} + h_{\mathrm{w}})$. From values of h_{s} and h_{w} in Table 1, using the above functions, we can obtain ε and φ values.

In 1976, the International Astronomical Union (IAU) issued the equation for the ecliptic obliquity:

$$\varepsilon = 23^\circ 26' 21''.448 - 46''.8150T - 0''.00059T^2 + 0''.001813T^3, \tag{4}$$

where T represents the Julian Century from J2000.0. With the ε value known, the T value can be calculated.

Because both shadow lengths and the law of ‘cun qian li’ are found in the *Zhoubi*, we suppose that the shadow lengths as shown in Table 1 might be modified by ancient astronomers according to the law of ‘cun qian li’. Suppose the data were originally measured in one location other than the capital, but for the capital the data must be adjusted. We postulate that the original data for each solar term had been changed δ cun, so

$$\text{Shadow length} = \text{Original value} + \delta. \tag{5}$$

The original values for the 24 solar terms are listed in the column named "shadow length" in Table 1. When the shadow lengths are changed, all the results for solar altitudes at Winter Solstice and Summer Solstice, the ecliptic obliquity (ε), the observation latitude (φ) and the observation year will change accordingly. The results are shown in Table 2.

Table 2 Relations between δ and Shadow Lengths, Solar Altitude, Ecliptic Obliquity, Local Latitude and Observation Year

No.	δ [cun]	L_w [chi]	L_s [chi]	H_w (°)	H_s (°)	ε (°)	φ (°)	Year
1	–3	13.5 – 0.3	1.6 – 0.3	31.22	80.77	24.7759	34.01	–6157
2	–2	13.5 – 0.2	1.6 – 0.2	31.03	80.07	24.5234	34.45	–4998
3	–1	13.5 – 0.1	1.6 – 0.1	30.84	79.38	24.2713	34.89	–3682
4	0	13.5	1.6	30.65	78.69	24.0197	35.33	–2931
5	1	13.5 + 0.1	1.6+0.1	30.47	78.00	23.7688	35.77	–475
6	2	13.5 + 0.2	1.6+0.2	30.28	77.32	23.5186	36.20	1391
7	3	13.5 + 0.3	1.6+0.3	30.10	76.64	23.2693	36.63	3298

Note: L_w stands for shadow length at noon for Winter Solstice, H_w stands for solar altitude, and L_s for shadown length for Summer solstice. H_s is the altitude.

Table 2 gives the results for δ values from -3 cun to 3 cun. For example, with $\delta = -3$ cun, the real location of observation should be 3000 li to the south, and the original shadow length for each solar term should be 3 cun less than that given in the *Zhoubi*. With this change, we can obtain a set of values for the ecliptic obliquity (ε), the observation latitude (φ) and the observation year, as presented in Table 2.

Of the seven sets of data shown in Table 2, only Nos. 5 and 4 can be considered. With No. 5, the observation time would be BC 476, which falls in the period of the Zhou dyansty. This reveals that the location of observation is no more than 1000 li away from the capital towards the north.

5 POSSIBLE OBSERVATIONAL PARAMETERS

If the shadow lengths for Winter Solstice and Summer Solstice were obtained by real observations, we may thus derive the observational year and the latitude of the location for ancient measurements. However, for accurate results we must take into consideration the corrections for atmospheric refraction and for the radius of the Sun. We design one program and use DE 406 ephemeris to process the two possible results of Nos. 5 and 4 in Table 2. In order to recover historical observations, we assume the longitude of the location $L = 111.5°$ E just for the purpose of computation. We use the iteration method to scan the possible year range. When the sum of the differences between the observational data and modern computations reaches its minimum, its related year and location should be the real observation year and location.

For the set of No. 4, we scan the year from BC 3000 to BC 2860 which is centered at BC 2932, and the span of location latitudes is $35.33° \pm 0.40°$ N. For our iterations, we set 0.01 h as the step of time length and $0.02°$ as the step of latitude. For this period, we calculate the solar altitude every day at noon, and compare it with the recorded shadow lengths. Then, we look for the year and latitude when the sum of the difference for calculation and observation of the shadow lengths of Winter Solstice and Summer Solstice becomes minimum. Using DE 406 ephemeris for computation, we obtain the accurate year = BC 3000, the latitude $= 35.33°$ N, and the minimum sum of differences $= 0.0317°$.

Similarly, for No. 5, the years span from BC 576 to BC 376, centered at BC 476; and the latitude span is $35.77° \pm 0.40°$. We have the year = BC 564, latitude $= 35.78°$ and the minimum sum of differences $= 0.0239°$.

Table 3 Calculation Results and Error Analysis for the Shadow Lengths of the Sun at Noon for 24 Solar Terms as Recorded in the *Zhoubi* in both BC 3000 and BC 564

No.	Solar terms	Shadow length (chi)		Transit altitude (°)		Date		Error of altitude (°)	
		BC 3000	BC 564	BC 3000	BC 564	BC 3000	BC 564	BC 3000	BC 564
1	Dongzhi	13.5000	13.6000	30.65	30.47	Jan. 12	Dec. 27	0.03	0.02
2	Xiaohan	12.5083	12.6083	32.60	32.40	Jan. 27	Jan. 10	–1.11	–1.09
3	Dahan	11.5167	11.6167	34.79	34.55	Feb. 12	Jan. 25	–0.68	–0.78
4	Lichun	10.5250	10.6250	37.24	36.98	Feb. 27	Feb. 09	0.68	0.64
5	Yushui	9.5333	9.6333	40.00	39.71	Mar. 15	Feb. 24	3.05	2.78
6	Qizhe	8.5417	8.6417	43.12	42.79	Mar. 30	Mar. 12	5.39	5.59
7	Chunfen	7.5500	7.6500	46.66	46.28	Apr. 15	Mar. 27	8.00	7.92
8	Qingming	6.5583	6.6583	50.66	50.23	May. 01	Apr. 12	10.13	10.12
9	Guyu	5.5667	5.6667	55.17	54.69	May 17	Apr. 27	11.37	11.06
10	Lixia	4.5750	4.6750	60.24	59.70	Jun. 01	May 13	11.05	11.08
11	Xiaoman	3.5833	3.6833	65.87	65.28	Jun 17	May 29	9.42	9.42
12	Mangzhong	2.5917	2.6917	72.05	71.40	Jul. 03	Jun. 14	5.81	5.78
13	Xiazhi	1.6000	1.7000	78.69	78.00	Jul. 18	Jun. 29	0.00	–0.02
14	Xiaoshu	2.5917	2.6917	72.05	71.40	Aug. 02	Jul. 15	5.82	5.71
15	Dashu	3.5833	3.6833	65.87	65.28	Aug. 18	Jul. 30	9.35	9.46
16	Liqiu	4.5750	4.6750	60.24	59.70	Sep. 02	Aug. 15	11.05	11.03
17	Chushu	5.5667	5.6667	55.17	54.69	Sep. 16	Aug. 30	11.44	11.20
18	Bailu	6.5583	6.6583	50.66	50.23	Oct. 01	Sep. 14	10.19	10.08
19	Qiufen	7.5500	7.6500	46.66	46.28	Oct. 16	Sep. 29	8.03	8.05
20	Hanlu	8.5417	8.6417	43.12	42.79	Oct. 31	Oct. 14	5.39	5.51
21	Shuangjiang	9.5333	9.6333	40.00	39.71	Nov. 14	Oct. 29	3.08	2.88
22	Lidong	10.5250	10.6250	37.24	36.98	Nov. 29	Nov. 13	0.74	0.60
23	Xiaoxue	11.5167	11.6167	34.79	34.55	Dec. 14	Nov. 27	–0.77	–0.66
24	Daxue	12.5083	12.6083	32.60	32.40	Dec. 29	Dec. 12	–1.09	–1.05

Notes: 1) Some geographic parameters are needed for calculation. We choose 111.5° E for longitude for both cases, and 35.33° N for latitude for BC 3000 and 35.78° N for latitude for BC 564. 2) The clock error (ET-UT) corresponding to BC 3000 is given as 71940.56∼71968.62 s and to BC 564 is from 17568.86∼17583.06 s. 3) The column "Error of altitude" represents the theoretical value from modern astronomical calculation minus those data converted from the shadow length which are recorded in the *Zhoubi*. 4) The column "Date" gives the modern definition of 24 solar terms.

When the possible year and location of observation are given, we could analyze the accuracy of the shadow lengths, no matter whether they are obtained from ancient calculation or from actual observations. Here, we give the solar altitude of 24 solar terms (calculated according to the modern definition for celestial longitude) at noon for both BC 3000 and BC 564. By comparing them all with the data recorded in the Zhoubi, we obtain the difference between them. Detailed results are shown in Table 3.

In Table 3 according to the modern definition of 24 solar terms, we calculate the celestial longitude of the Sun for each day. Then, we have the dates when the Sun moves to those points. From these, we can also determine the errors of ancient records in the *Zhoubi Suanjing*. For BC 3000, the average error of the solar altitude is 5.27° and the average of the absolute value is 5.57°. For BC 564, they are 5.22° and 5.52° respectively.

From the above analysis, we suggest that either the concept of 24 solar terms in ancient times was quite different from ours – for example, they might be defined by the shadow length at noon directly – or these data were obtained from calculations rather than from measurement by means of the gui biao. We believe the latter case is more likely, for in the *Zhoubi*, the author says that the shadow lengths for 22 solar terms are calculated from those for Winter Solstice and Summer Solstice by means of linear interpolation.

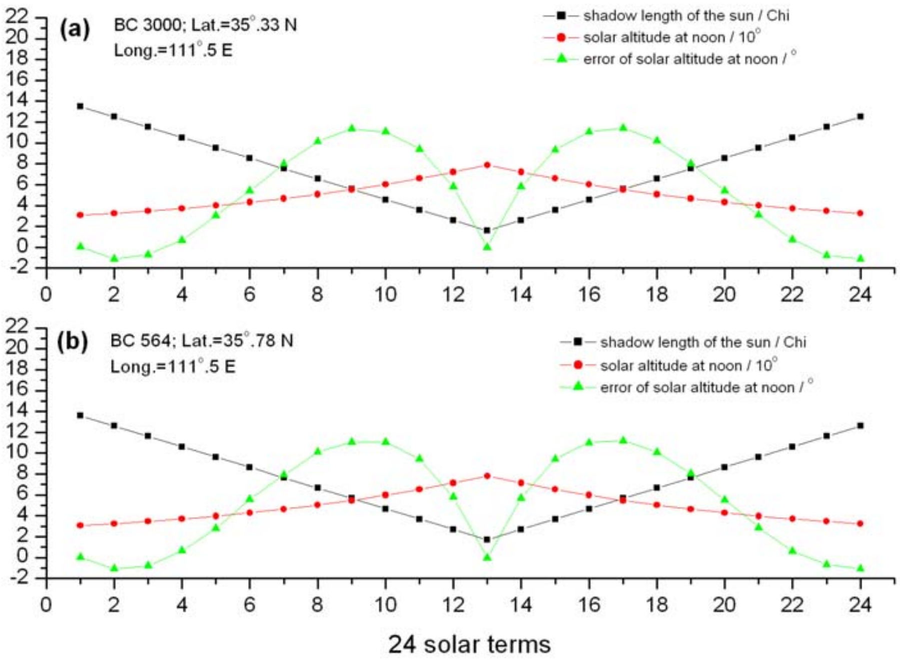

Fig. 3 Shadow lengths for 24 solar terms at noon as recorded in the *Zhoubi Suanjing*, their corresponding solar altitudes and the results of error analysis. (a): In BC 3000, the location of observation is Lat.= $35.33°$ N, Long.= $111.5°$ E. (b): In BC 564, the location of observation is Lat.= $35.78°$ N, Long.= $111.5°$ E.

Figure 3(a) shows the results for BC 3000 and Figure 3(b) for BC 564. The two diagrams have almost the same shape. The errors of solar altitude show large fluctuations. When at Winter Solstice and Summer Solstice, the errors are less than $0.03°$. At Guyu (No. 9 in Table 3) or Chushu (No. 17), the error is bigger than $11.0°$. Li (2005) investigated 98 solar meridian observations in the $13^{\rm th}$ century in China and the derived average error of the altitude of the Sun is $-4.05'$ and the absolute value of average error is $6.78'$. Here, the fluctuation of the errors of solar altitude in the *Zhoubi* is much larger.

6 CONCLUSIONS AND DISCUSSION

6.1 Did Records Come from Calculation or Observation?

For both BC 3000 and BC 564, the average errors of solar altitude are bigger than $5°$, fluctuating between $-1°$ and $11°$. This is unthinkable if they are from direct observations. Two possible solutions can be suggested. First, all shadow lengths except the two for Winter Solstice and Summer Solstice were calculated rather than observed. The text of the *Zhoubi* actually specifies the method of calculation. Here, we suggest that the data have been adjusted by ancient astronomers according to the law of ‘cun qian li’. There have been many studies on the calculation of gnomon shadow lengths in ancient China (Chen 1989; Ji 1994; Qu 1997). They show that different methods were used by Chinese astronomers from different dynasties (Qu et al. 2001). No doubt, the records in the *Zhoubi* were the earliest in China.

The second possibility is that a different definition for 24 solar terms was used. The solar terms might be determined directly according to the shadow length, instead of according to the longitude of the Sun. The 24 solar terms could be indicated on the template of the gnomon, but this definition would cause the intervals between solar terms to vary from 9 to 45 d for BC 564. It seems quite unlikely.

Therefore, we conclude that only the shadow lengths for Winter Solstice and Summer Solstice were obtained from actual observations, and the shadow lengths for the other 22 solar terms were derived from that for Winter Solstice and Summer Solstice by means of linear interpolation. All shadow lengths were changed according to the law of ‘cun qian li’, which was accepted as a sort of dogma.

6.2 Probable Observation Location

The modification of shadow lengths by ancient astronomers according to the incorrect law of ‘cun qian li’ caused a lot of contradictions in the data. No matter how the data for the 22 solar terms were derived, they would affect the results of our analysis. As long as we have data for the two solstices, we can always obtain seven possible results as shown in Table 2, but only two of them (Nos. 4 and 5) are possible solutions. We consider the observation year BC 564 as the best choice, because it falls in the period of the Eastern Zhou dynasty. However, the observation location should be at $35.78\,^\circ$ N latitude. That means the location should be 1000 li to the south of the capital.

In the Western Zhou dynasty (from BC 1100 to BC 771), the capital is Gaojing. It is present-day Xi’an, with long. $= 108.92^\circ$ E and lat. $= 34.25^\circ$ N. The capital of Eastern Zhou (from BC 770 to BC 221) is Luoyi, now called Luoyang, with long. $= 112.43\,^\circ$ E and lat. $= 34.72^\circ$ N. Denfeng was traditionally recognized as the center of the land for astronomical purposes. Its long. $= 113.02\,^\circ$ E and lat. $= 34.27^\circ$ N. All these places do not fit the data in the *Zhoubi Suanjing*. Recently, a prehistoric astronomical site has been discovered in Taosi in Xiangfen County of Shanxi Province (Shanxi Archaeological Team of Institute of Archaeology of Chinese Academy of Social Sciences 2007; Jiang et al. 2006). Its latitude is 35.88° N, which suits the data of the *Zhoubi* very well. It is generally assumed that the observation place of the *Zhoubi* was in Yangcheng (near Luoyang). Taosi is just about 1000 li north of Yangcheng. Li & Sun (2009) suggested that Taosi might be the observation location of the gnomon shadow lengths in the *Zhoubi*. Our analysis confirms this hypothesis. The data were originally observed in Taosi, but later modified according to the law of ‘cun qian li’ to suit the new capitals.

6.3 Origin of the *Zhoubi Suanjing*

So far most studies have stated that the *Zhoubi* and its theory originated from China, with only a few exceptions. For example, Chen Y. K. once suggested that the Theory of Canopy-Heavens in the *Zhoubi* was related to the Buddhist cosmology from India. Jiang (1997) elaborated on this idea. He pointed out that it was very likely that the *Zhoubi* was compiled under the influence of Indian cosmology. However, according to our anaylsis, if the original data were from India, with the latitude being less than $33.5\,^\circ$ in India, we would have to choose the result even earlier than the No. 1 set of data as missing the part listed in Table 2. That means the observation time would be before BC 6000, a quite unlikely date.

Hence, if the data for the Winter Solstice and Summer Solstice were from actual observations and ancient observations were accurate enough, we can ascertain that the *Zhoubi* was originally from China, not from India. This will help end the controversy about the origin of the *Zhoubi*. It is very likely that the original data were observed in the Taosi area, and the observation time was BC 564. The original data were modified later according to the law of ‘cun qian li’.

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