

## Observational Constraints on Quark Matter in Neutron Stars \*

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Received 2006 December 29; accepted 2007 March 12

**Abstract** We study the observational constraints of mass and redshift on the properties of the equation of state (EOS) for quark matter in compact stars based on the quasi-particle description. We discuss two scenarios: strange stars and hybrid stars. We construct the equations of state utilizing an extended MIT bag model taking the medium effect into account for quark matter and the relativistic mean field theory for hadron matter. We show that quark matter may exist in strange stars and in the interior of neutron stars. The bag constant is a key parameter that affects strongly the mass of strange stars. The medium effect can lead to the stiffer hybrid-star EOS approaching the pure hadronic EOS, due to the reduction of quark matter, and hence the existence of heavy hybrid stars. We find that a middle range coupling constant may be the best choice for the hybrid stars being compatible with the observational constraints.

**Key words:** dense matter — gravitation — stars: neutron — stars: rotation — stars: oscillations

### 1 INTRODUCTION

The interiors of neutron stars contain matter at very high density that is a few times and even up to more than ten times the density of ordinary atomic nuclei. This provides a high-pressure environment in which numerous subatomic particle processes compete with each other. Therefore, the composition and properties of the interiors of neutron stars have attracted much attention (Pandharipande 1971; Glendenning 1985; Glendenning et al. 1992; Sahu et al. 1993; Kutschera & Koltroz 1993; Thorsson et al. 1994; Glendenning 1997; Alford & Reddy 2003), but actually, the equation of state for the neutron star is still an unsolved problem after many years of investigation because of uncertain nuclear physics. Observational constraints on theoretical predictions of the equation of state of high density matter have been regularly proposed (Glendenning 1997; Glendenning & Moszkowski 1991; Zhang et al. 2007). Based on X-ray observations with the XMM Newton observatory, Cottam et al. (2002) reported that neutron stars did not contain strange matter, but Xu (2003) argued that this conclusion was incorrect, and that we still could not rule out strange star models for the X-ray burster EXO 0748–676 from the mass-radius relations for bare and crusted strange stars. Recently an accumulation of neutron-star cooling observations also favors the presence of exotic particles such as hyperons or quark matter in the cores of some neutron stars (Yakovlev & Pethick 2004) and Lackey et al. (2006) have given a particular discussion concerning the existence of hyperons by reference to observables at maximum such as the stellar mass and gravitational redshift. Alford et al. (2005) and Klähn et al. (2006a,b) studied the effect of quark matter inside compact stars on the mass-radius relationship. Klähn et al. (2006a,b) used NJL (Nambu-Jona-Lasinio) model for quark matter. Alford et al. (2005) modelled the quark matter equation of state through a phenomenological parameterization, but their parameterization formalism was based on a consideration of the perturbative QCD corrections. Moreover, they

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\* Supported by the National Natural Science Foundation of China.

mainly focused on the maximum mass of hybrid stars in accordance with the construction of a sharp transition in their work. We shall also estimate the circumstance with the inclusion of quark matter, but the first transition in hybrid stars is based on the Gibbs construction. We expect to uncover how the change of the region embodying mixed phase and quark matter increases the maximum masses of the hybrid stars. We apply the GPS (Ghosh-Patak-Sahu) model (Ghosh et al. 1995) together with an extended MIT bag model for hybrid stars. The extended MIT bag model is called the “effective mass bag model” by Schertler et al. (1997). In this model, medium effects are taken into account in the framework of the MIT bag model by introducing density-dependent effective quark masses. Such quark matter systems in quasi-particle description may involve partial nonperturbative contributions since the coupling constant  $g$  can not be small, as will be shown below. According to the Tolman-Oppenheimer-Volkoff theory (1939), different equations of state would lead to different mass-radius relations and maximum masses. From the mass and radius of a neutron star, its gravitational redshift can immediately be determined, and then the observed mass of the neutron star and the gravitational redshift would set a limit on the equation of state. Our investigation focuses on the influence of the equation of state of quark matter on the limiting mass and gravitational redshift of the star under the consideration whether or not quark matter exist in compact stars.

The most precise observations of neutron-star masses come from radio pulsars in binaries, which are all measured with 95% confidence to be less than  $1.5 M_{\odot}$  (Thorsett & Chakrabarty 1999). The most accurately measured mass up to now of PSR 1913+16 is  $M = 1.442 \pm 0.003 M_{\odot}$  (Taylor & Weisberg 1989), but is not necessarily the maximum possible mass. X-ray measurements have long suggested that accreting neutron stars are more massive, but contamination by the oscillations of the high-mass main sequence companion has been known (van Kerkwijk et al. 1995). So the record of  $1.5 M_{\odot}$  has remained the constraint for many years until Nice et al. (2003, 2004, 2005) obtained a neutron-star mass greater than  $1.6 M_{\odot}$  at the 95% confidence level through recent radio observation of PSR J0751+1807. Also Ransom et al. (2005) discovered that the relativistic periastron advance of the two eccentric systems in the globular cluster Terzan 5 indicates that at least one of the pulsars has a mass  $> 1.68 M_{\odot}$  at 95% confidence. Lackey et al. (2006) suggested a bound of  $\sim 1.68 M_{\odot}$  on the maximum neutron-star mass. However, Özel (2006), in an analysis of EXO 0748–676 observational data found that the neutron-star mass could only be low to  $1.82 M_{\odot}$  within  $1-\sigma$  bar. Although the result has been recently disproved in an alternative analysis by Hynes et al. (2006), we shall still adopt this upper limit of mass because the equations of state of pure hadron matter have predicted such heavy stars. Another constraint is the measurement of a gravitational redshift by Cottam et al. (2002). They analysed the absorption lines in the spectra of 28 bursts of the Low-mass X-ray binary EXO 0748–676 and discovered that several absorption lines are consistent with a redshift  $z = 0.35$ , although with small uncertainties of no more than 5% for the respective transitions.

The organization of the rest of this paper is as follows. In Section 2, we provide details of the equations of state of strange stars and hybrid stars. In Sections 3 and 4, we estimate the constraints on the equations of state given by the observed mass and gravitational redshift. Finally, we give our conclusions and a discussion in Section 5.

## 2 EQUATION OF STATE

### 2.1 Strange Stars

A strange star is composed of pure strange quark matter, the up, down, strange quarks and leptons. For quark matter, it is natural that its equation of state is calculated by lattice quantum chromodynamics, but this could not be carried through at finite density, so researchers often adopt some phenomenological models in the calculation (Baym & Chin 1976; Freedman & McLerran 1978; Chakrabarty 1991; Peng et al. 2000). Here we mainly take the effective mass bag model that takes medium effect into account (Schertler et al. 1997). This model is based on quasi-particle approximation. We could consider  $u$ ,  $d$  and  $s$  quarks to be quasi-particles by utilizing the Debye screen effect in plasma. In this scenario, a quark acquires an effective mass generated by the interaction with other quarks in a dense system. The effective masses are derived from the zero momentum limit of the dispersion relations following from the quark self energy. In the hard dense loop (HDL) approximation, the generic form of the quark self energy is

$$\Sigma = -a P_{\mu} \gamma^{\mu} - b \gamma_0 - c, \quad (1)$$

where

$$a = \frac{1}{4p^2} [tr(P_\mu \gamma^\mu \Sigma) - p_0 tr(\gamma_0 \Sigma)], \quad (2)$$

$$b = \frac{1}{4p^2} [P^2 tr(\gamma_0 \Sigma) - p_0 tr(P_\mu \gamma^\mu \Sigma)], \quad (3)$$

$$c = -\frac{1}{4} tr \Sigma. \quad (4)$$

Here,  $P^2 = p_0^2 - p^2$ . We can then obtain the effective quark mass,

$$m_i^* = \frac{m_i}{2} + \sqrt{\frac{m_i^2}{4} + \frac{g^2 \mu_i^2}{6\pi^2}}. \quad (5)$$

This formula contains the coupling constant  $g$ , the quark chemical potential  $\mu_i$ , and the current quark mass  $m_i$  ( $i = u, d, s$ ). In the HDL approach, one expects that  $g$  to be small as the effective mass is calculated perturbatively, but Schertler et al. (1997) extrapolated it to large values. Here, we take  $g$  from 0 to 5 as was done in their works (Schertler et al. 1997; Schertler et al. 2000). The pressure and energy density for quark matter can be constructed from the statistical mechanics of quasi-particles system,

$$\epsilon = \sum_i \left\{ \frac{d}{16\pi^2} \left[ \mu_i k_i (2\mu_i^2 - m_i^{*2}) - m_i^{*4} \ln \left( \frac{k_i + \mu_i}{m_i^*} \right) \right] + B^*(\mu_i) \right\} + \epsilon_e + B, \quad (6)$$

$$p = \sum_i \left\{ \frac{d}{48\pi^2} \left[ \mu_i k_i (2\mu_i^2 - 5m_i^{*2}) + 3m_i^{*4} \ln \left( \frac{k_i + \mu_i}{m_i^*} \right) \right] - B^*(\mu_i) \right\} + p_e - B, \quad (7)$$

where  $d$  is the degree of degeneracy and  $B^*$  is a function to maintain thermodynamic self-consistency,

$$\frac{dB^*(\mu_i)}{dm_i^*} = -\frac{d}{4\pi^2} \left[ m_i^* \mu_i k_i - m_i^{*3} \ln \left( \frac{k_i + \mu_i}{m_i^*} \right) \right]. \quad (8)$$

Up to now, it has been difficult to estimate the bag constant  $B$  and the strange quark mass  $m_s$  from available data. They are subject to systematic uncertainties, so we follow previous researchers and treat them as free parameters, with for  $B$  ranging from  $140^4$  to  $200^4$  (MeV)<sup>4</sup> and  $m_s$  from 80 to 150 MeV.

## 2.2 Hybrid Stars

A hybrid star mainly consists of a quark matter core, a mixed quark-hadron phase and hadron matter if the surface tension at the boundary between the quark core and hadron phase is low enough. To construct the equation of state for a hybrid star, we need first the equations of state for hadron matter and quark matter. The equation of state of quark matter has been discussed in the previous section. Generally, we shall use the Baym-Pethick-Sutherland (BPS) (Baym et al. 1971) equation of state for subnuclear densities corresponding to the crust of the star, which is matched with the equation of state for nuclear densities at  $\epsilon \approx 10^{14}$  g cm<sup>-3</sup>. The equation of state of hadron matter for nuclear densities could be established in the relativistic mean field theory (Glendenning 1997), it is one of effective field theories describing hadron matter, where nucleons interact through nuclear force mediated by the exchange of isoscalar and isovector mesons ( $\sigma$ ,  $\omega$  and  $\rho$ ). Here we consider the hadron phase to include only four kinds of particles,  $n$ ,  $p$ ,  $e$  and  $\mu$ . The relativistic Lagrangian reads

$$\begin{aligned} \mathcal{L} = & \sum_{B=n,p} \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B} \gamma_\mu \tau \cdot \rho^\mu) \psi_B \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) + \sum_{\lambda=e,\mu} \bar{\psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda) \psi_\lambda \\ & - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \rho_{\mu\nu} \cdot \rho^{\mu\nu} \\ & + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu. \end{aligned} \quad (9)$$

We then solve the Euler-Lagrange equations by replacing the fields by their mean values under the assumption that the bulk matter is static and homogeneous and then calculate the kinetic Dirac equations for baryons and also the meson fields equations. By imposing  $\beta$ -equilibrium, local electric charge neutrality and conservation of baryon number, we obtain the equation of state for pure hadron matter,

$$\begin{aligned} \epsilon = & \frac{1}{3}bm_n(g_\sigma\sigma)^3 + \frac{1}{4}c(g_\sigma\sigma)^4 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ & + \sum_{B=n,p} \frac{2J_B + 1}{2\pi^2} \int_0^{k_B} \sqrt{k^2 + (m_B - g_{\sigma B}\sigma)^2} k^2 dk \\ & + \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \sqrt{k^2 + m_\lambda^2} k^2 dk, \end{aligned} \quad (10)$$

$$\begin{aligned} p = & -\frac{1}{3}bm_n(g_\sigma\sigma)^3 - \frac{1}{4}c(g_\sigma\sigma)^4 - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ & + \frac{1}{3} \sum_{B=n,p} \frac{2J_B + 1}{2\pi^2} \int_0^{k_B} \frac{k^4}{\sqrt{k^2 + (m_B - g_{\sigma B}\sigma)^2}} dk \\ & + \frac{1}{3} \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \frac{k^4}{\sqrt{k^2 + m_\lambda^2}} dk. \end{aligned} \quad (11)$$

Actually, the Euler-Lagrange equations feature five free parameters, which under certain assumptions are fit algebraically to numbers distilled from laboratory measurements of many finite nuclei, i.e., the saturation density, binding energy per nucleon and symmetry energy coefficient at saturated nuclear matter, and the overall compressibility  $K$  and the effective mass of nucleons  $m^*$ . As it is not our motive here, we numerically adopt five fiducial equations of state using the Glendenning (GL) (Glendenning 1997) and the Ghosh-Patak-Sahu (GPS) (Ghosh et al. 1995) parameters for demonstration purpose. These are arranged in Table 1 in order of increasing stiffness.

**Table 1** Parameters of five fiducial hadronic equations of state in the relativistic mean-field theory. Here  $\rho_0$  is the saturation density,  $B/A$  is the binding energy, the incompressibility is denoted by  $K$ , the effective mass by  $m^*/m_N$  and the symmetry energy by  $a_{\text{sym}}$ .

Name	$\rho_0$ (fm $^{-3}$ )	$B/A$ (MeV)	$a_{\text{sym}}$ (MeV)	$K$ (MeV)	$m^*/m_N$	EOS
GL1	0.153	-16.3	32.5	240	0.78	softest
GPS1	0.150	-16.0	32.5	250	0.83	soft
GPS2	0.150	-16.0	32.5	300	0.83	intermediate
GPS3	0.150	-16.0	32.5	350	0.83	stiff
GL2	0.153	-16.3	32.5	300	0.7	stiffest

Considering models of compact stars in which the deconfinement transition occurs at high density (Glendenning 1992, 1997), we allow the hadron phase to undergo a first order phase transition (Schertler et al. 1998; Schertler et al. 2000) to a deconfined quark matter phase above the saturation density of nucleon. This phase transition makes possible the occurrence of mixed hadron-quark phase in a finite density range inside the compact star. From the quark and hadron matter equations of state, the  $\beta$ -equilibrium, global electric charge neutrality and the Gibbs condition between quark and hadron phases, we easily obtain the equation of state for the mixed phase, and hence the equation of state for the hybrid star.

### 3 MAXIMUM MASS

The structure of a neutron star is determined by the local balance between the attractive gravitational force and the pressure force of the neutron star matter. For a neutron star in equilibrium under the condition that the effect of rotation can be overlooked the gravitational field is taken to be static and spherically

symmetric. Considering the effect of general relativity, Tolman, Oppenheimer & Volkoff (1939) established a set of equations (the TOV equations) to determine the structures of such stars:

$$\frac{dp(r)}{dr} = -\frac{[\epsilon(r) + p(r)][m(r) + 4\pi r^3 p(r)]}{r[r - 2m(r)]}, \quad (12)$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r). \quad (13)$$

Here  $G = c = 1$ ,  $p(r)$  and  $\epsilon(r)$  are the pressure and energy density of the matter at radius  $r$ , and  $m(r)$  the total stellar mass within a sphere of radius  $r$ :

$$m(r) = 4\pi \int_0^r \epsilon(r') r'^2 dr'. \quad (14)$$

After the equation of state of the star  $\epsilon(r) = \epsilon(p(r))$  is given, the TOV equations could be finally solved as an initial value problem. Starting with the values at the center of the pressure,  $p(r = 0) = p_c$ , and of the mass  $m(r = 0) = 0$ , we can integrate the TOV equations outward until the surface  $p(r = R) = 0$  reaches, then we have the  $M$ - $R$  relation of the star, which has a maximum mass according to general relativity. Using this property we can rule out those equations of state that are too soft to reproduce the observed mass. Actually, Ter 5 I rotates at 104 Hz (Ransom 2005) and EXO 0748–676 at 45 Hz (Villarreal & Strohmayer 2004), which may increase the TOV maximum mass by less than 1%. When analysing the theoretical results with the observational data, we should also take this small correction into account.

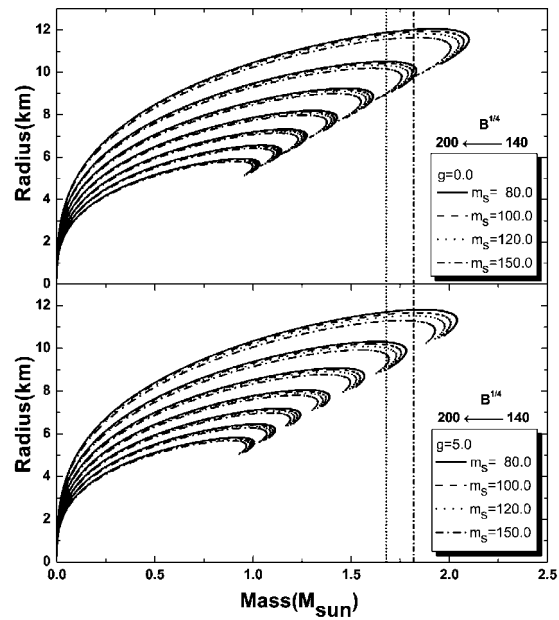
In Figure 1 we plot the  $R$ - $M$  relation for strange stars based on our fiducial effective mass bag model considering medium effect. We find that the consideration of the medium effect and the increase of the current mass of s quark  $m_s$  could only slightly soften the equation of state, but the change of bag constant  $B$  can markedly affect the stiffness of equation of state, and only the ones with smaller bag constants are necessary for strange stars to be consistent with the higher mass limits.

We now investigate the influence of the hadronic equations of state on those of hybrid stars in Figure 2. It displays little effect on the stiffness of the equations of state of hybrid stars for all the maximum masses. See also below for the gravitational redshift. Our choice of the intermediate model, GPS2, for estimating the equations of state of hybrid stars suffices present needs. In Figure 3 we show the  $R$ - $M$  relation for hybrid stars. The crosses on each curve represent the dividing point of the hadron and mixed phases. Only the compact stars lying between the cross and the maximum mass point could have quark matter in their interior. The coupling constant for strong action  $g$  as well as the bag constant  $B$  significantly influences the stiffness of the equation of state of the hybrid stars. For  $g = 0.0$ , almost all the equations of state with quark matter are ruled out by both the mass limits, except the stiffest one with parameters  $B^{1/4} = 200.0$  MeV,  $m_s = 150.0$  MeV, which can be brought to within the 95% confidence limit of  $1.68 M_\odot$  from Ter 5 I after allowing for the rotation correction. Under  $g = 3.0$ , both curves are consistent with the limit from Ter 5 I but only the ones with  $B^{1/4} = 200.0$  MeV could fit  $1.82 M_\odot$ , and when  $g = 4.0$ , all equations of state reach the larger constraint  $1.82 M_\odot$ . For  $g = 5.0$  quark matter disappears in the neutron stars. The equations of state of pure nucleonic matter are consistent with  $1.68 M_\odot$  and  $1.82 M_\odot$ . Obviously, the hybrid stars with the medium effect of quark matter are very different from the ones without the medium effect. The reason for the existence of heavy hybrid stars is that increasing  $g$  and  $B$  softens the EOS of quark matter, which raises the transition density and causes difficulty to occur from hadrons to quark matter and in all hardens the EOS of hybrid star. This effect brings about a hybrid star dominated by hadronic matter, i.e., the extent of quark matter is reduced but the region of hadron matter is increased and this in fact yields “hybrid” stars that are actually hadronic stars with a tiny core of mixed quark-hadron matter inside, and the most of the mass and radius contributions come from the hadron matter, so the Mass-radius relation of the hybrid star tends towards the pure neutron star and the heavier stars.

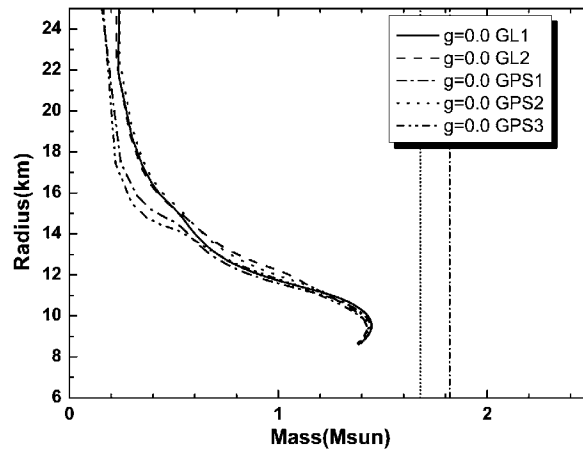
#### 4 GRAVITATIONAL REDSHIFT

Generally speaking, the scale of the gravitational redshift is small, except around a black hole or a neutron star which has a strong gravitational field. For a nonrotating star the redshift  $z$  obeys the relation

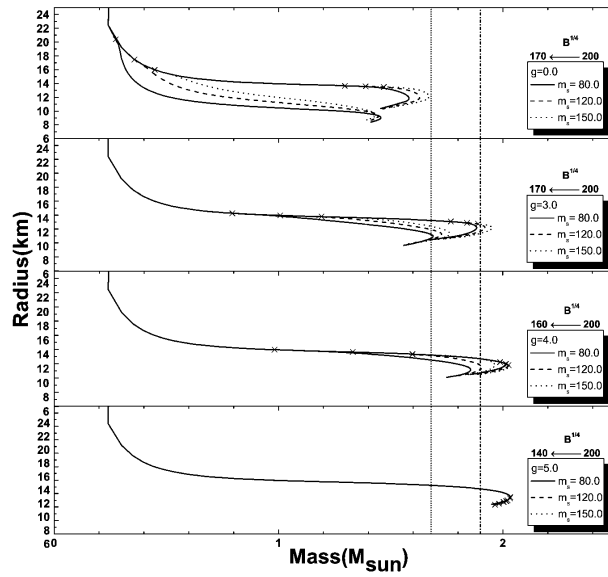
$$z = \left(1 - \frac{2M}{R}\right)^{-1/2} - 1, \quad (15)$$



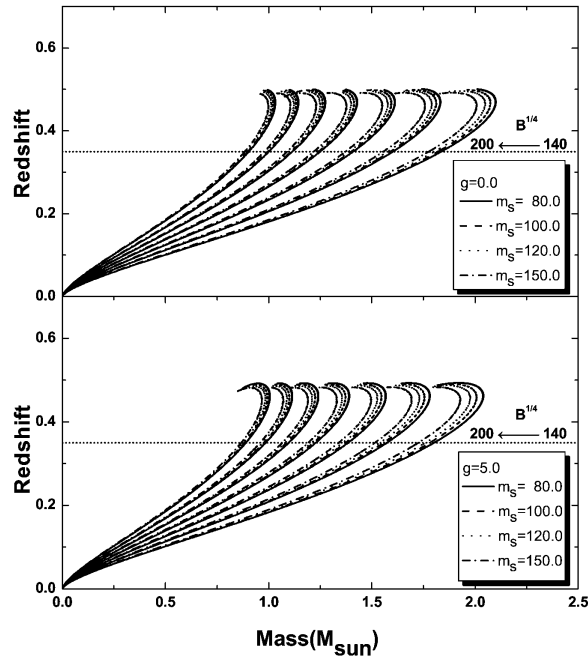
**Fig. 1** Oppenheimer-Volkoff radius-mass curves for strange stars using effective mass bag model including the medium effect (bottom panel) and not including the medium effect (top panel). The two vertical lines represent the two observational 95% confidence limit of  $1.68 M_{\odot}$  from Ter 5 I and  $1.82 M_{\odot}$  within  $1 - \sigma$  bar from EXO 0748-676. The seven sets of four curves in each panel correspond to seven different values of  $B^{1/4}$ , decreasing from 200 MeV (the leftmost set) to 140 MeV (the rightmost) at steps of 10 MeV.



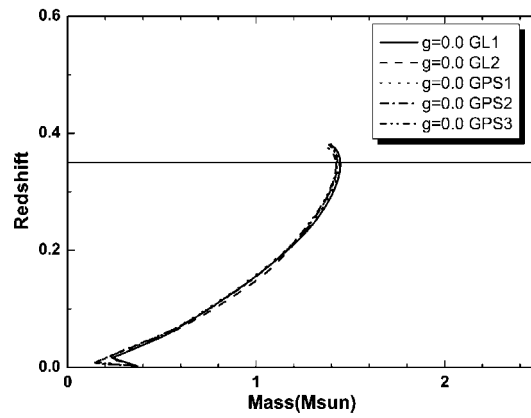
**Fig. 2** Oppenheimer-Volkoff radius-mass curves for hybrid stars with the same equation of state for quark matter and different equations of state for the hadrons (GL1, GPS1, GPS2, GPS3 and GL2). These two vertical lines represent the observational 95% confidence limit,  $1.68 M_{\odot}$  from Ter 5 I and  $1.82 M_{\odot}$  within  $1 - \sigma$  bar from EXO 0748-676. For quark matter, we chose the parameters  $B^{1/4} = 170.0$  MeV,  $g = 0.0$  and  $m_s = 150.0$  MeV in the calculation.



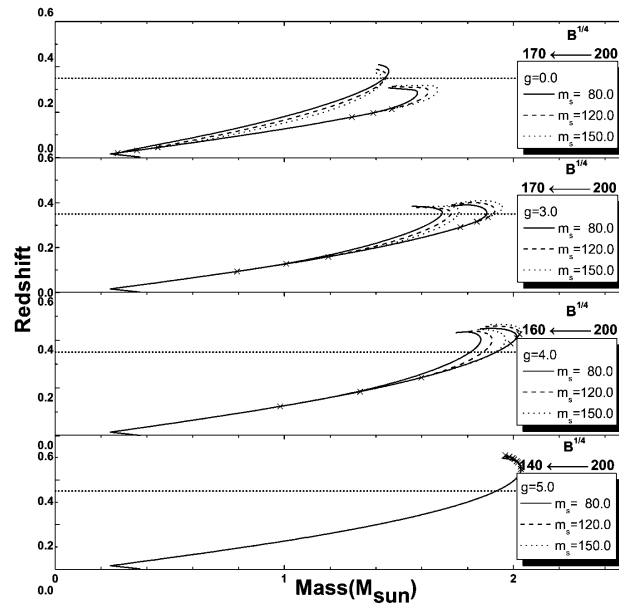
**Fig. 3** Oppenheimer-Volkoff radius-mass curves for hybrid stars with the same equation of state for hadrons (GPS2). The cross on each curve marks the dividing point between the hadron phase and the mixed phase. The two vertical lines have the same meaning as in Fig. 2. Each of the four panels show two sets of three curves. The different sets correspond to different pairs of values of  $B^{1/4}$  in MeV: (from top down) (170, 200), (170, 200), (160, 200) and (140, 200). The different curves within each set correspond to the different values of  $g$  and  $m_s$  shown in the legend.



**Fig. 4** Gravitational redshift vs mass for strange stars using the effective mass bag model including the medium effect (bottom panel) and not including the medium effect (top panel). The horizontal line,  $z = 0.35$ , is the measured redshift of EXO0748-676. The seven sets of four curves in each panel correspond to seven different values of  $B^{1/4}$ , decreasing from 200 MeV (the leftmost set) to 140 MeV (the rightmost) at steps of 10 MeV.



**Fig. 5** Gravitational redshift vs mass for hybrid stars with the same equation of state for quark matter and different equation of state for hadrons (GL1, GPS1, GPS2, GPS3 and GL2). The horizontal line is  $z = 0.35$  measured for EXO 0748–676. For quark matter, we used the parameters  $B^{1/4} = 170.0$  MeV,  $g = 0.0$  and  $m_s = 150.0$  MeV in the calculation.



**Fig. 6** Gravitational redshift vs mass for hybrid stars with the same equation of state for hadrons (GPS2). The cross on each curve marks the dividing point between the hadron phase and the mixed phase. The horizontal line is  $z = 0.35$  measured for EXO0748-676. The pairs of sets of curves refer to the same pairs of values of  $B^{1/4}$  as in Fig. 3.

which is evidently related to the value of  $M/R$  and this fact could be possibly used in a determination of the mass-to-radius ratio in compact stars. Now, we have already known from the  $M-R$  relation that  $R$  decreases with increasing  $M$  until  $M$  approaches its maximum value for a stable star. The redshift reaches a maximum at the point of the maximum mass, this could be also used to rule out those equations of state



that can not produce the observed redshift. Cottam et al. (2002) analysed the absorption lines in the spectra of 28 bursts of the Low-mass X-ray binary EXO 0748–676. They identified the most significant features in the Fe XXVI and XXV  $n = 2 - 3$  and O VIII  $n = 1 - 2$  transitions and discovered all the transitions had a redshift of  $z = 0.35$  with a small scatter. Recent observation suggests that the neutron star rotates at 45 Hz (Villarreal & Strohmayer 2004), but we shall still use the nonrotating approximation as the rotation correction to the redshift should be small.

We plot in Figure 4 the redshift as a function of mass for strange stars based on our fiducial effective mass bag model that includes the medium effect. All the equations of state are consistent with  $z = 0.35$ . Together with the maximum mass constraint discussed in Section 3, we conclude that the observation mass constraint operates strongly on the equations of state for quark matter, as well as the free parameters especially the bag constant  $B$  in strange stars.

Analogically, we investigate the influence of the hadronic equations of state in the frame of relativistic mean-field theory on the redshift (Mass) relations of hybrid stars. See Figure 5. The maximum redshift is not sensitive to the equation of state for hadrons but is sensitive to that of quark matter at several times the nuclear density. See Figure 6. In contrast to the maximum masses limits, those with  $B^{1/4} = 170.0$  MeV under  $g = 0.0$  are both consistent with the redshift constraint 0.35, and the situation is opposite for  $B^{1/4} = 200.0$  MeV. This means that both equations of state not including the medium effect are excluded by the observational constraints. For  $g = 3.0$  and  $g = 4.0$ , all the equations of state for quark matter are consistent with  $z = 0.35$ . The equations of state in the case of  $g = 5.0$  are also compatible with the constraint, but quark matter is absent.

## 5 CONCLUSIONS

We have compared the equations of state for quark matter in quasi-particle description in strange stars and hybrid stars with astronomical observations of mass and gravitational redshift. We studied the effect of the coupling strength among quarks on the strange and hybrid stars. In the strange stars the effect of the coupling constant is small in regard to the observational limits. In the hybrid stars, however, the coupling constant plays an important role in the maximum mass and gravitational redshift. We think that the hybrid star is a more realistic model, where intermediate dense strange quark matter is possible. The hybrid stars have masses much lower than the observed masses when the quark matter exists as a free fermion gas, but can produce the observed mass and redshift when the medium effect of strange quark matter is taken into account. Sections 3 and 4 show that an intermediate coupling constant may be the best choice for making the equations of state of hybrid stars consistent with both the mass and redshift observational constraints, for a wide range of the parameter  $B$ .

Qualitatively, we can obtain similar maximum masses to those given by Alford et al. (2005) and that the presence of quark matter in the neutron star is constrained but not ruled out by the observational mass and gravitational redshift. However, there are quite evident differences between the investigation of Alford et al. (2005) and ours. First, our model contains large coupling constant  $g$  (or  $\alpha_c = g^2/4\pi^2 > 1$ ), which is slightly different from the parameterized EOS based on the perturbative QCD theory. Secondly, the heavier hybrid star is dominated by quark matter in the case of sharp transition, while in our configuration by hadronic matter. The reason is that the softening of EOS of quark matter leads to the existence of an almost pure hadron star in our model. For the sharp transition, Alford et al. (2005) owe it to the hardening of the quark matter EOS by an increased  $c$  and to the existence of a much more larger quark matter core. Our work also indicates that there is a critical value of  $g \sim 5.0$  over which quark matter cannot exist inside stable neutron stars.

It is well-known that the cooling measurement is an additional constraint on the equation of state of compact stars. Current measured temperatures of compact objects are compatible with strange stars and hybrid stars (Yu & Zheng 2006; Kang & Zheng 2007). Existence of hyperons and quark matter in compact objects indicates that these equations of state tend to result in excessively fast cooling due to the onset of direct Urca processes, and so are the stiff nucleonic equations of state in heavier stars (Page et al. 2006). Thus, normal neutron stars or hyperon stars may have to be much cooler to be ruled out by the X-ray data. However, the hybrid stars do not have this problem because a deconfinement heating mechanism is triggered due to the spin-down of the star. We discovered that the deconfinement latent heat can effectively cancel the enhanced neutrino emission (Kang & Zheng 2007).

**Acknowledgements** This work is supported by the NSFC under grant Nos. 10373007 and 10603002.

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