Population Synthesis for the Symbiotic Stars with Main-sequence Accretors *

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Abstract Using a population synthesis code, we have investigated the formation of symbiotic systems in which the hot component is a main-sequence star that is accreting matter from the cool component via Roche lobe overflow (RLOF). The RLOF can be divided into two cases: dynamically unstable and stable. In the first case, the birthrate of symbiotic stars is 0.056 yr$^{-1}$ or 0.045 yr$^{-1}$ depending on different assumptions; in the stable RLOF case, it is 0.002 yr$^{-1}$ or 0.005 yr$^{-1}$. The number of symbiotic stars with main-sequence accretors and unstable RLOF in our galaxy is about 5, that with stable RLOF is about 60 to 280. Comparison between our results with those of Yungelson et al. shows that symbiotic stars with MS accretors make only a small contribution (\(<8\%\)) to the whole population of symbiotic stars in the Galaxy.

Key words: binaries: symbiotic — accretion: accretion disks — stars: evolution — main-sequence star

1 INTRODUCTION

Symbiotic stars (SSs) are a heterogeneous group of variable stars with composite spectra. They typically exhibit both red and blue continua, strong TiO absorption bands, and high-excitation emission lines (Boyarchuk 1970, 1984; Kenyon 1986). The modern model of SSs envisions a three-component system consisting of a binary star with a hot and a cool component and an HII region (Boyarchuk 1970). The cool component is a first giant branch (FGB) or an asymptotic giant ranch (AGB) star. In the majority of SSs the hot component is, most probably, a white dwarf (WD) or subdwarf or an accreting low-mass main-sequence (MS) star (e.g., Kenyon & Webbink 1984; Mürset et al. 1991). The variability of SSs may be due to thermonuclear runaways on the surface of an accreting WD (Tutukov & Yungelson 1976; Yungelson et al. 1995; Iben & Tutukov 1996) or to an accretion disk instability (Paczyński & Rudak 1980; Duschl 1986; Duschl 1989) and/or variations in the mass-loss rate by the cool component (Bath 1977; Bath & Pringle 1982). Recent reviews of the properties of SSs can be found in Mürset & Schmid (1999) and Mikolajewska (2002).

Theoretical studies on the formation and evolution of SSs have been done in a series of studies (e.g., Yungelson et al. 1995; Han et al. 1995a; Iben & Tutukov 1996; Hurley et al. 2002). These investigations

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reproduced successfully many observed properties and provided a wonderful picture for understanding SSs. Those excellent papers are mainly concerned with SSs with WD accretors, while there is a lack of detailed study of SSs with MS accretors. Belczyński et al. (2000) listed 188 SSs, including the highly controversial AX Per and CI Cyg. Kenyon & Webbink (1984), Mikolajewska & Kenyon (1992a) and Mikolajewska & Kenyon (1992b) considered that they are SSs with MS accretors. Bath & Pringle (1982) constructed accretion disk model for CI Cyg, in which the variations of accretion rate produce the eruptions of luminosity. In contrast, Mürset et al. (1991) and Mürset & Schmid (1999) considered that they could be detached systems with hot components that are WDs (also see Skopal 1994) from the statistical relation \( R \leq \frac{1}{2} l_1 \) between the radius of a giant \( R \) and the distance from the center of the giant to the inter Lagrangian point, \( l_1 \).

It is necessary to do detailed study for SSs with MS accretors. In the present paper, we discuss such SSs by a population synthesis code. We attempt to find the formation of SSs which contains MS accretors accreting hydrogen-rich materials and FGB and AGB stars transferring mass via RLOF (Roche lobe overflow). In this model, the birthrate, lifetime and number of SSs in the Galaxy and some potentially observable parameters are given, such as the orbital periods and the masses of the components.

In Section 2 we list the assumptions and describe some details of the modelling algorithm. In Section 3 we discuss the main results and the effects of different parameters. In Section 4 the discussion and conclusions are given.

2 MODEL

Our goal is to construct a model of the subpopulation of SSs which contains main-sequence accretors as the hot sources and FGB or AGB stars as the cool sources. They can be pictured as consisting of a red giant transferring matter into an accretion disk around an MS star. Below we describe our algorithm and give some computational details that are essential for an understanding of our model.

2.1 Simplicity on Accretion Disk

Formation of accretion disk is a basic condition for SSs with MS accretors. The disc formation is possible if the radius of the star is less than 0.1–0.2 times its Roche lobe radius \( R \) (Lubow & Shu 1975; Bath & Pringle 1982). This condition is generally satisfied by the MS components. We assume that accretion disk forms when the primary in FGB or AGB stage begins to fill its Roche lobe.

SSs have long orbital periods of a few hundred days. We can estimate a typical outer radius of the accretion disk in an SS. Paczynski & Rudak (1980) determined the outer radii of geometrically thin accretion disks by the restricted three-body problem. According to their results, we fit the outer radius of the accretion disk by the numerical formula,

\[
\frac{R_{\text{max}}}{a} = 0.17\mu^{2} - 0.7\mu + 0.62 ,
\]

where \( \mu = \frac{M_{\text{MS}}}{M_{\text{MS}} + M_{\text{cool}}} \) and \( a \) is the separation of the binary system.

Duschl (1986) showed that an accretion disk around an MS star with dimensions (\( \sim 10^{12} - 10^{12.6} \) cm) and a constant mass transfer rate from the companion red giant into the disk of \( \sim 10^{-5} - 10^{-4} M_{\odot} \) yr\(^{-1}\) can produce the symbiotic phenomenon. Bath & Pringle (1982) modelled the symbiotic binary CI Cyg by an accretion disk with outer radius \( \sim 10^{12.9} \) cm and varying mass transfer rates between \( \sim 2.38 \times 10^{-5} \) and \( 6.34 \times 10^{-4} M_{\odot} \) yr\(^{-1}\). In the present paper, we write \( R_{\text{max}} \) for the outer radius of the accretion disk. If \( R_{\text{max}} \) is between \( \sim 10^{11.9} \) (\( \sim 11.4R_{\odot} \)) (see fig. 6 in Duschl 1986) and \( 10^{13} \) cm (\( \sim 144R_{\odot} \)), then the symbiotic phenomenon may be produced.

2.2 Mass Transfer Rate

If at the onset of RLOF the mass ratio of the components of the binary system \( q = M_{\text{donor}}/M_{\text{gainer}} \) is larger than certain critical value \( q_c \), the mass transfer rate is dynamically unstable and the system forms common envelope. Otherwise, the system will undergo a stable RLOF. On different assumptions \( q_c \) takes different value. In models of condensed polytropes, Hjellming & Webbink (1987) gave an alternative con-
dition for dynamical mass transfer from a giant donor:

\[ q_c = 0.362 + \left[3 (1 - M_{c1}/M_1)\right]^{-1}, \tag{2} \]

where \( M_{c1} \) and \( M_1 \) are the core mass and mass of the giant donor, respectively. Hurley et al. (2002) fit \( q_c \) by the formula

\[ q_c = \left[1.67 - x + 2 \left(\frac{M_{c1}}{M_1}\right)\right]/2.13, \tag{3} \]

where \( x \) is the exponent in the mass-radius relation at constant luminosity for giant stars (Hurley et al. 2000) and is taken to be 0.3 in this work. The range above \( q_c \) is suitable for conservative RLOF. However, the RLOF at FGB/AGB is not conservative as a matter of fact. Han et al. (2000) showed that \( q_c \) heavily depends on the mass transfer efficiency. According to Han et al. (2002), we take \( q_c \) to be around 1.25 for non-conservative RLOF in this work. For checking the effects of \( q_c \), we use the \( q_c \) given by Equations (2) & (3) and \( q_c = 1.25 \) in different models.

Dynamically unstable mass transfer includes two phases (Hjellming & Webbink 1987): the mass transfer proceeds first on a thermal time scale at the onset of RLOF, then on a dynamical time scale. We assume that the symbiotic phenomenon can only be produced in the first phase. In the second phase, the binary system very rapidly evolves to the common envelope stage. In the first phase, the mass transfer rate is roughly given by

\[ \dot{M} = \frac{M}{\tau_{KH}}, \tag{4} \]

where the Kelvin-Helmholtz time-scale \( \tau_{KH} \) is given by

\[ \tau_{KH} = \frac{10^7 M}{RL} (M - M_c) \text{yr}, \tag{5} \]

and \( M, R, L \) and \( M_c \) are the mass, radius, luminosity and core mass of the cool giant in solar units. In the first phase, the transferred mass depends on the entropy profile of the donor (Hjellming & Webbink 1987; Hjellming 1989). Figure 1 displays, for different binary systems, the entropy profiles at the onset and the end of the first phase. According to Figure 1, if we assume that roughly one percent of the donor’s mass is transferred in the first phase the duration of the first phase or the lifetime of the SS is about 0.01 \( \tau_{KH} \).

For stable RLOF, we accept the prescription of Hurley et al. (2002) for the mass transfer rate given by

\[ \dot{M} = 3 \times 10^{-6} \left[\min(M_1, 5.0)\right]^{2/3} \left[\ln(R_1/R_{L1})\right]^{3/2} M_\odot \text{yr}^{-1}, \tag{6} \]

where \( M_1, R_1 \) and \( R_{L1} \) are, respectively, the mass, radius and Roche lobe radius of the donor.

According to Duschl (1986) and Bath & Pringle (1982), the symbiotic phenomenon will be produced if the mass transfer rate from the red giant to the accretion disk is between \( \sim 10^{-5} \) and \( 10^{-3} M_\odot \text{yr}^{-1} \).

### 2.3 Basic Parameters in the Monte Carlo Simulation

For the population synthesis of a binary stellar population, the main input parameters are: (i) the initial mass function (IMF) of the primaries; (ii) the lower and upper cut-offs of the IMF, \( M_1 \) and \( M_8 \); (iii) the mass-ratio distribution of the binaries; (iv) the distribution of orbital separations; (v) the eccentricity distribution; and (vi) the metallicity \( Z \) of the binary systems.

A simple approximation to the IMF of Miller & Scalo (1979) is used. The primary mass is generated with the formula of Eggleton, Tout & Fitchett (1989),

\[ M_1 = \frac{0.19 X}{(1 - X)^{0.75} + 0.032(1 - X)^{0.25}}, \tag{7} \]

where \( X \) is the uniform random variable in the range \([0, 1]\), and \( M_1 \) is the primary mass from \( 0.8 - 8 M_\odot \).

The mass ratio of the two components, \( q = M_2/M_1 \), is a very important parameter for the evolution of the binary system but its distribution is quite controversial. We take a uniform mass-ratio distribution (Mazeh et al. 1992; Goldberg & Mazeh 1994):

\[ n(q) = 1, 0 < q \leq 1. \tag{8} \]
Fig. 1 Entropy profiles of donors when the donor fills up its Roche lobe (solid lines) and when the dynamical regime begins (dashed lines). Four cases are shown with respective initial masses and orbital periods: (a) $1.0 \ M\odot + 0.8 \ M\odot$, 23.0 d; (b) $1.4 \ M\odot + 1.1 \ M\odot$, 30.5 d; (c) $2.0 \ M\odot + 1.8 \ M\odot$, 32.6 d; (d) $3.0 \ M\odot + 2.6 \ M\odot$, 36.0 d

We assume that all stars are members of binary systems, and that the distribution of separations is constant in log $a$ at the large end and falls off smoothly at the small end:

$$a n(a) = \begin{cases} \alpha_{\text{sep}}(\frac{a}{a_0})^m & a \leq a_0; \\ \alpha_{\text{sep}} & a_0 < a < a_1, \end{cases}$$

where $\alpha_{\text{sep}} \approx 0.070$, $a_0 = 10 R\odot$, $a_1 = 5.75 \times 10^6 R\odot = 0.13$ pc and $m \approx 1.2$. This distribution implies that there is an equal number of wide binary systems per unit logarithmic interval, and that approximately 50 percent of the systems have orbital periods less than 100 yr (Han et al. 1995b).

As the calculating model of the population of binary stars we use the rapid binary code (Hurley et al. 2002) in which the important input parameters are: the metallicity $Z$ set at 0.02, and the initial eccentricity for the binary orbit at $e = 0.0$.

2.4 Selection of Models

We take a binary systems as an SS with MS accretor if the following conditions are all satisfied: (1) the primaries are FGB or AGB stars with their Roche lobe filled; (2) The secondaries are still MSs whose initial masses are greater than $0.08 M\odot$, the minimum mass for hydrogen core burning; (3) The maximum outer radius of the accretion disk is in the range $11.4 \ R\odot \sim 144 \ R\odot$; (4) The mass transfer rate is between $10^{-3}$ and $10^{-8} M\odot$ yr$^{-1}$. 
According to $q_c$ in Section 2, we construct three models: In model 1, $q_c=\text{Eq.}(3)$; in model 2, $q_c=\text{Eq.}(2)$; in model 3, $q_c=1.25$. To calculate the birthrate of SSs, we assume that one binary with $M_1 \geq 0.8M_\odot$ is formed every year in the Galaxy (Iben & Tutukov 1984; Yungelson et al. 1993; Han et al. 1995a; Yungelson et al. 1995; Zhu & Zha 2005).

3 RESULTS

We take $5 \times 10^5$ binary systems for every model, with mass of primaries between $0.8M_\odot$ and $8.0M_\odot$.

3.1 Birthrate and Number of SSs with MS Accretors

For model 1, the birthrate of SSs with MS accretors in the Galaxy is $\sim 0.058\text{ yr}^{-1}$, in RLOF accounts for $\sim 0.002\text{ yr}^{-1}$. For model 2, they are $\sim 0.058\text{ yr}^{-1}$, $\sim 0.003\text{ yr}^{-1}$, $\sim 0.050\text{ yr}^{-1}$ and $\sim 0.055\text{ yr}^{-1}$.

As mentioned in Section 2.2, the lifetime of SSs with dynamically unstable RLOF is usually very short. In our models their average value is about 100 yr. For SSs with stable RLOF, however, their average lifetimes are 30000, 33000 and 56000 yr in models 1, 2 and 3, respectively. If we assume that the number of SSs is determined by their birthrate and lifetime, then the number of SSs with dynamically unstable RLOF in the Galaxy is about $\sim 5$ and, for stable RLOF, the number is $\sim 60$ for model 1 and 280 for model 2.

Observational estimates of the total number of SSs range from several thousands (Boyarchuk 1970) to about 30000 (Kenyon 1994) or up to $\sim 300000$ (Munari & Renzini 1992), depending on the assumptions on the distance to typical SSs and on observational selection. In Yungelson et al. (1995), the birthrate of SSs with WD accretors is $0.073\text{ yr}^{-1}$ and their number is 3300–30000 in their standard model.

Comparing with observed values the theoretical results of Yungelson et al. (1995), we find that SSs with MS accretors are negligible, being ($< 8\%$) of the total number of SSs in the Galaxy. SSs with dynamically unstable RLOF have a higher birthrate but their lifetime is too short, and SSs with stable RLOF have a longer lifetime but their birthrate is too low. Whether the binary system undergoes dynamically unstable RLOF or stable RLOF is determined by the criterion for dynamically unstable RLOF, $q_c$. SSs with dynamically unstable RLOF have different properties from SSs with stable RLOF (see next section), and $q_c$ has a great effect on SSs with MS accretors.

3.2 Properties of SSs with MS Accretors

In this section, we describe the properties of potentially observable physical quantities of SSs with MS accretors. All the analyses are for model 1.

Figure 2 displays the distributions of the initial primary masses, separations and mass ratios of the SS progenitors in model 1. There are great differences between SSs with dynamically unstable and stable RLOF: those with stable RLOF have larger initial primary masses. In order to stably transfer mass via the Roche lobe, the mass ratio of the system must be smaller than $q_c$. This condition can be satisfied in two ways: by reducing the mass ratio or by increasing $q_c$. In the progenitor systems of SSs with stable RLOF, the massive primary fills its Roche Lobe before becoming an RG and losing matter which may be transferred to the secondary or lost from the system. Then, when the primary evolves to FGB or AGB, the mass ratio will be smaller. For massive primaries, the core mass is larger, then $q_c$ can be larger. For the progenitor systems of SSs with stable RLOF, the initial primary masses and mass ratios are larger, as shown in Figure 2.

**Orbital Period:** Figure 3 displays the distribution of the birthrates on orbital periods of SSs with MS accretors. The ranges for SSs with dynamically unstable RLOF and stable RLOF are between $\sim 7$ days and 60 days and between $\sim 10$ days and 120 days, respectively. For AX Per (orbital period 680.8 d) and CI Cyg (855.3 d) (Mikołajewska 2002), their orbital periods are outside our calculated range.

**Mass of MS:** In Figure 4 the distribution of the birthrates of the MS masses in SSs with MS accretors. For SSs with dynamically unstable RLOF the peak is between $\sim 0.08M_\odot$ and $2.0M_\odot$; for those with stable RLOF, between $\sim 6.0M_\odot$ and $11.0M_\odot$. For the latter, mass transfer may cause the mass of secondary to exceed that of the primary. The measured masses of the hot components of AX Per and CI Cyg are $0.37 \pm 0.06$ and $0.43 \pm 0.04M_\odot$, respectively (Mikołajewska 2002). Obviously, they are in the range of SSs with dynamically unstable RLOF.
Fig. 2 Distributions of initial primary masses, initial separations and initial mass ratios of the progenitor of SSs with MS accretors in model 1. Solid line (scale on the left) for SSs with dynamically unstable RLOF and dashed line (scale on the right) for those with stable RLOF.

Fig. 3 Distribution of the birthrate of SSs with MS accretors based on the orbital periods in model 1. Same conventions regarding solid line and dashed line as in the previous figure.

**Mass of Cool Giant:** Figure 5 displays the distribution of the birthrates of the mass of the cool giant component in SSs with MS accretors. For SSs with dynamically unstable RLOF the peak is between \( \sim 0.08 \) \( M_\odot \) and \( 2.0 \) \( M_\odot \); for those with stable RLOF, between \( \sim 1.7 \) \( M_\odot \) and \( 2.5 \) \( M_\odot \). The measured masses of the hot components of AX Per and CI Cyg are \( 0.9 \pm 0.2 \) and \( 1.3 \pm 0.3 M_\odot \), respectively (Mikołajewska 2002).
Fig. 4 Distribution of the birthrate of the mass of the MS for SSs with MS accretors. Same conventions on the solid and dashed lines as in the previous figures.

Fig. 5 Distribution of the birthrate of the MS accretor mass in SSs with MS accretors. Same conventions regarding the solid and dashed lines.

According to Figures 4 and 5, if AX Per and CI Cyg are SSs with MS accretors, they should belong to SSs with dynamically unstable RLOF.

4 DISCUSSION AND CONCLUSIONS

As mentioned Section 3, there are fewer SSs with MS accretors in the Galaxy and their contribution to the whole SS population is less than 8%. AX Per and CI Cyg may not be SSs with MS accretors for two reasons: First, their orbital periods (680.8 d and 855.3 d) are much longer than in our results (shorter than 200 d); Secondly, based on their masses, they may be SSs with dynamically unstable RLOF, but the number (~ 5) of SSs with dynamically unstable RLOF in the Galaxy are too low. However, thin accretion disk are considered in our models. Mikolajewska & Kenyon (1992b) have argued that AX Per and CI Cyg are best explained by the presence of an unstable thick disk around a low-mass MS accretor. Thus, before a more detailed study, we can not exclude them from being SSs with MS accretors.

In this paper, we constructed rough models of SSs with MS accretors, estimate their birthrate (about $0.058 \sim 0.050\, \text{yr}^{-1}$) and their number in the Galaxy (about 60 \sim 280). There are two main areas of uncertainty in our model: the accretion disc model and the evolution from semidetached to contacted binary.
systems. Future theoretical work in these two areas will hopefully give more detailed and reliable results for SSs with MS accretors.

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References

Hjellming M. S., 1989, Doctor Thesis
Kenyon S. J., 1994, MmSAI, 65, 135
Mikalajewska J., astro-ph/0210489
Tutukov A.V., Yungelson L. R., 1976, Astrophysics, 12, 321
Zhu C. H., Zha C. Z., 2005, ChJAA, 5, 419