Chin. J. Astron. Astrophys. Vol. 6 (2006), No. 3, 372–378
(*http://www.chjaa.org*)

Chinese Journal of
Astronomy and
Astrophysics

On the Station-Keeping and Control of the World Space Observatory/Ultraviolet *

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Received 2005 October 26; accepted 2006 December 2

Abstract Collinear libration points play an important role in deep space exploration because of their special positions and dynamical characteristics. Since motion around them is unstable, we need to control the spacecraft if we wish to keep them around such a libration point for a long time. Here we propose a continuous low-thrust control strategy, illustrated with numerical simulations combined with the orbit design and control of the World Space Observatory/UltraViolet (WSO/UV).

Key words: celestial mechanics — WSO/UV — low thrust

1 INTRODUCTION

The Circular Restricted Three-Body Problem (CRTBP) describes the motion of a small body under the gravitation of two large bodies which revolve each other in circular orbits. The small body does not influence the two large bodies. This system has five libration points, as shown in Figure 1.

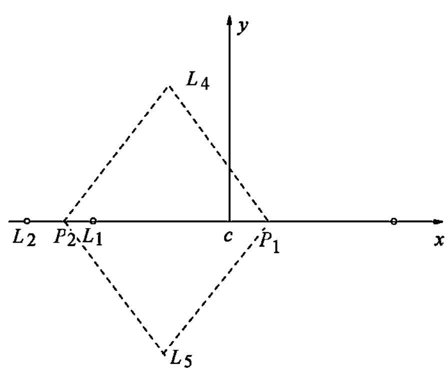

Fig. 1 The synodic frame and the five libration points.

The points L_1, L_2, L_3 are collinear libration points, while the points L_4 and L_5 are equilateral libration points. Libration points are the regions where gravitational forces of the two large bodies are balanced.

* Supported by the National Natural Science Foundation of China.

Spacecraft would need little fuel to stay around them and the geometrical configuration is advantageous for observation, control and communication. Different from the equilateral ones, motion around collinear libration points is unstable (Szebehely 1967), so to keep a spacecraft around a collinear libration point for a long time, control must be applied.

ISEE-3 which was sent to L_1 of the Sun-Earth system is the first successful attempt. After ISEE-3, more spacecrafts have been (such as WIND, SOHO) or will be sent to the collinear libration points. WSO/UV is such a one; it will be sent to L_2 of the Sun-Earth system. Up to now the necessary control is effected by impulsive manoeuvers. In this paper we have studied the practicability of low-thrust control strategy. Although some work has been done (Breakwell et al. 1974; Scheeres et al. 2000), our strategy is simpler and the result is better, in actual practice.

2 NOMINAL ORBITS

The equation of motion of the CRTBP is

$$\begin{cases} \ddot{\boldsymbol{r}} + 2(-\dot{y}, \dot{x}, 0)^T = (\partial\Omega/\partial\boldsymbol{r})^T, \\ \Omega(x, y, z) = [\mu(1-\mu) + x^2 + y^2]/2 + (1-\mu)/r_1 + \mu/r_2, \end{cases} \tag{1}$$

where $\mu = m_2/(m_1+m_2)$, m_1 and m_2 being the masses of the two large bodies with $m_2 \leq m_1$, and r_1 and r_2, the distances of the small body from them. In order to study the motion around the collinear libration point, we transfer the coordinate origin to the collinear libration point, rotate the x and y axes by $180\,^\circ$ and multiply the new coordinates by a factor γ_j (γ_j is the distance between the collinear libration point and the primary nearest to it). Denote the position vector under the new coordinate as $(\xi, \eta, \zeta)^T$, we can write the equation of motion in the new coordinates as

$$\begin{cases} \ddot{\xi} - 2\dot{\eta} - (1 + 2c_2)\xi = \frac{\partial}{\partial\xi}\sum_{n\geq 3}^{\infty} c_n(\mu)\rho^n P_n(\frac{\xi}{\rho}), \\ \ddot{\eta} + 2\dot{\xi} - (1 - c_2)\eta = \frac{\partial}{\partial\eta}\sum_{n\geq 3}^{\infty} c_n(\mu)\rho^n P_n(\frac{\xi}{\rho}), \\ \ddot{\zeta} + c_2\zeta = \frac{\partial}{\partial\zeta}\sum_{n\geq 3}^{\infty} c_n(\mu)\rho^n P_n(\frac{\xi}{\rho}), \end{cases} \tag{2}$$

where $\rho = \sqrt{\xi^2 + \eta^2 + \zeta^2}$, and $c_n (n \geq 2)$ are functions of μ.

The linearized model corresponds to the homogenous form of Equation (2), and the conditionally stable solution can be written as

$$\begin{cases} \xi(t) = \alpha\cos(\omega_0 t + \phi_1), \\ \eta(t) = \kappa\alpha\sin(\omega_0 t + \phi_1), \\ \zeta(t) = \beta\sin(\nu_0 t + \phi_2), \end{cases} \tag{3}$$

where $\omega_0 = \sqrt{(\sqrt{9c_2^2 - 8c_2} - c_2 + 2)/2}$, $\nu_0 = \sqrt{c_2}$, $\kappa = -(\omega_0^2 + 2c_2 + 1)/2\omega_0$, ϕ_1, ϕ_2 are the initial phase angles and can be chosen at will, α, β are the planar and vertical amplitudes, respectively. For the general case of incommensurability of ω_0 and ν_0, Equation (3) descries a quasi-periodic orbit in space, which is called an order-one Lissajou orbit. Now, WSO/UV requires a large halo orbit in order to avoid the influence of the eclipse of the Earth and the Moon. It can be proved that halo orbits exist only when the high order terms on the right side of Equation (2) are considered. The corresponding analytical formulation of order-three periodic orbit (i.e., Halo orbit) is

$$\begin{cases} x = -\alpha\cos\tau + a_{21}\alpha^2 + a_{22}\beta^2 + (a_{23}\alpha^2 - a_{24}\beta^2)\cos 2\tau \\ \quad + (a_{31}\alpha^3 - a_{32}\alpha\beta^2)\cos 3\tau, \\ y = \kappa\alpha\sin\tau + (b_{21}\alpha^2 - b_{22}\beta^2)\sin 2\tau \\ \quad + (b_{31}\alpha^3 - b_{32}\alpha\beta^2)\sin 3\tau + (b_{33}\alpha^3 - (b_{34} - b_{35})\alpha\beta^2)\sin\tau, \\ z = \beta\cos\tau + d_{21}\alpha\beta(\cos 2\tau - 3) + (d_{32}\beta\alpha^2 - d_{31}\beta^3)\cos 3\tau, \end{cases} \tag{4}$$

where the coefficients can be found in references (Richardson 1980), $\tau = \omega t + \phi$ is the phase angle, ω is related to amplitudes α and β. Solutions of orders higher than three under CRTBP are useless in the Sun-Earth system because of the existence of perturbations. In fact the solar system is much more complicated than the CRTBP. Besides the gravitation of the two primaries, a spacecraft is perturbed by other large bodies.

Moreover, the orbit of the two primaries is an ellipse rather than a circle. We can take the order-three halo orbit under CRTBP as a starting point and use the method of iteration to construct conditionally quasi-periodic orbit (quasi-halo orbits) under the actual gravitation model (i.e., the model that takes into account the motion and gravitation of the two primaries as well as other large bodies of the solar system).

Either an order-three halo orbit or an orbit under the real gravitation model can be taken as nominal orbits of the spacecraft. Different nominal orbits require different energies of control. Obviously the consumed energy is less if the model under which the nominal orbit is constructed is closer to the real gravitation model. This point will be verified further in the following numerical simulations.

3 LOW-THRUST CONTROL STRATEGY

To control the spacecraft we use the linear feedback method shown in Figure 2. Here, AB indicates the nominal orbit. $A'B'$ indicates the controlled orbit, $A'B''$ indicates the uncontrolled orbit; δX_0 is the initial deviation from the nominal orbit and δX is the final one. The goal of orbit control is to keep δX small. Different from impulsive manoeuvers, we apply constant low thrust $u = (u_x, u_y, u_z)^T$ to the spacecraft over the whole orbit segment $A'B'$ instead of applying an impulse at point A'. The time of each segment in Figure 2 is ΔT. The nominal orbit AB can be any kind of orbit and the low thrust is obtained from the deviation between B and B''. The equation of motion of the uncontrolled orbit can be written as

$$\dot{X} = F(X, t) \,. \tag{5}$$

Taking $x(t)$ as the deviation between controlled and uncontrolled orbit, considering the low thrust, the equation of motion can be written as

$$\dot{x} = A(t)x + Bu + O(x^2) \,, \tag{6}$$

where

$$A = \partial F/\partial X \,, \quad B = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \,.$$

Suppose the solution of Equation (5) is of the following form:

$$x = a_0 + a_1\Delta t + a_2(\Delta t)^2 + \cdots\cdots \,, \tag{7}$$

Put Equation (7) back into Equation (6) and noting the initial conditions ($t = t_0, x = 0$), we have

$$a_0 = 0, \;\; a_1 = Bu, \;\; a_2 = A_0 Bu/2, \;\; \cdots\cdots . \tag{8}$$

The coefficients of orders higher than 2 are very difficult to obtain; here we stop at order 2. Then we have

$$x = Bu\Delta t + \frac{1}{2} A_0 Bu(\Delta t)^2. \tag{9}$$

If $X = (\boldsymbol{r}, \dot{\boldsymbol{r}})$, then generally matrix A has the form of

$$A = \begin{bmatrix} (0)_{3\times 3} & I_3 \\ A_1 & A_2 \end{bmatrix} . \tag{10}$$

So Equation (9) can be rewritten as

$$x = \begin{bmatrix} \delta \boldsymbol{r} \\ \delta \dot{\boldsymbol{r}} \end{bmatrix} = \begin{bmatrix} (0)_{3\times 3} \\ u \end{bmatrix} \Delta t + \frac{1}{2} \begin{bmatrix} u \\ A_2 u \end{bmatrix} (\Delta t)^2 \,. \tag{11}$$

When the spacecraft is at point B, the deviation from the nominal orbit is

$$\delta X = x + (X''_B - X_B) \,, \tag{12}$$

where $(X''_B - X_B)$ can be found from the state transition matrix (in this situation the nominal orbit should be constructed under the real gravitation model) or by integration. Take the cost function as

$$Q = \frac{1}{2}\delta X^T M \delta X\,, \tag{13}$$

where M is symmetric positive matrix called weight matrix. Suppose that we have the same demands in the three direction of position and velocity, then we take M as

$$M = \begin{bmatrix} kI_3 & 0 \\ 0 & I_3 \end{bmatrix}\,, \tag{14}$$

where k is the weight factor, indicating the different demands on the velocity and position. When $k = \infty$, we control the spacecraft only by position; and when $k = 0$, we control the spacecraft only by velocity.

Our numerical simulations indicate that k has great effects on the results and should be chosen carefully in order to obtain good control results. Our numerical simulations also indicate that we can omit the second order term $A_2 u(\Delta t)^2/2$ in Equation (13) in the presence of the first order term $u\Delta t$, so the results can be expressed as

$$u = -2\left(\frac{kx_1\Delta t + 2x_4}{s},\ \frac{kx_2\Delta t + 2x_5}{s},\ \frac{kx_3\Delta t + 2x_6}{s}\right)^T, \tag{15}$$

where $s = k(\Delta t)^3 + 4\Delta t$.

If we control the spacecraft only by position, the controlled orbit is indeed around the nominal orbit, but the amplitude with respect to the nominal orbit is very large and the energy needed is very large; if we control the spacecraft only by velocity, the controlled orbit is relatively smooth and the energy needed is much less, but the controlled orbit has a drift from the nominal orbit. So we have to control the spacecraft by both position and velocity. Here k is decided by ΔT and generally k increases with decreasing ΔT. Because the velocity changes linearly with the low thrust, "Δv" and "$u\Delta t$" should not appear simultaneously in the cost function Q.

As a result of the linear feedback method, the control result is better if ΔT is smaller. Considering the practicability, ΔT should not be too small, nor should it be too large, or nonlinear effects will destroy the linear feedback. As we have mentioned, higher order coefficients are very difficult to obtain and we have stopped at order 2.

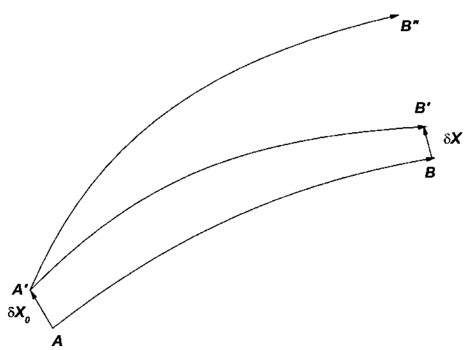

Fig. 2 Illustration of linear feedback.

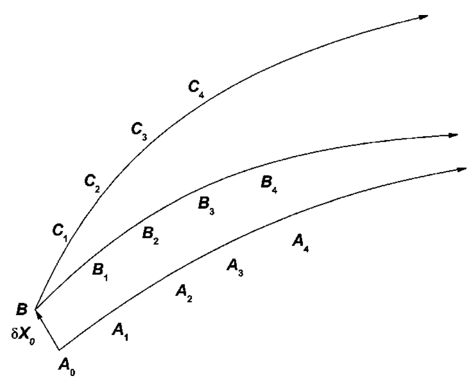

Fig. 3 Illustration of multi-point feedback.

Figure 2 shows the case of the feedback on one single point. In practice we can consider the feedback on more than one point, as shown in Figure 3. Here, A_0A_k indicates the nominal orbit, BB_k the controlled orbit and BC_k the uncontrolled orbit. Denoting by $x = (\boldsymbol{r}, \dot{\boldsymbol{r}})$ the deviation between the controlled orbit and uncontrolled orbit, we have

$$\begin{cases} \boldsymbol{r}_l = A_2 t_l^2 + A_3 t_l^3 + \cdots + A_N t_l^3 + \cdots\cdots\,, \\ \dot{\boldsymbol{r}}_l = 2A_2 t_l + 3A_3 t_l^2 + \cdots + NA_N t_l^{N-1} + \cdots\cdots\,, \end{cases} \tag{16}$$

where $A_l = (a_{lx}, a_{ly}, a_{lz})^T$ is the coefficients of x, y, z, t_l is the time between B_k and B. Comparing with Equation (9), we have

$$A_2 = \frac{1}{2}u = \frac{1}{2}(u_x, u_y, u_z)^T. \tag{17}$$

Take the cost function as

$$J = \sum_{l=1}^{M} \left[(\boldsymbol{r}_l + \bar{\boldsymbol{r}}_l)^T R_l (\boldsymbol{r}_l + \bar{\boldsymbol{r}}_l) + (\dot{\boldsymbol{r}}_l + \dot{\bar{\boldsymbol{r}}}_l)^T S_l (\dot{\boldsymbol{r}}_l + \dot{\bar{\boldsymbol{r}}}_l) \right], \tag{18}$$

where $(\boldsymbol{r}_l, \dot{\boldsymbol{r}}_l)$ is the deviation between the uncontrolled and nominal orbits, M is the number of points considered, R_l, S_l are the half-positive symmetric matrices. If take $R_l = k_l I_3$, $S_l = s_l I_3$, Equation (16) can be rewritten as

$$\begin{aligned} J &= J_x + J_y + J_z \\ &= \textstyle\sum_{l=1}^{M} \left\{ [k_l (x_l + \bar{x}_l)^2 + s_l (\dot{x} + \dot{\bar{x}}_l)^2] \right\} \\ &\quad + \textstyle\sum_{l=1}^{M} \left\{ [k_l (y_l + \bar{y}_l)^2 + s_l (\dot{y} + \dot{\bar{y}}_l)^2] \right\} \\ &\quad + \textstyle\sum_{l=1}^{M} \left\{ [k_l (z_l + \bar{z}_l)^2 + s_l (\dot{z} + \dot{\bar{z}}_l)^2] \right\}. \end{aligned} \tag{19}$$

Then the extreme of J in three directions can be separated. If we wish to obtain the coefficients of the x direction, we have

$$H_x X = B_x, \tag{20}$$

where $X = (a_{2x}, a_{3x}, \cdots, a_{Nx})^T$, N is the order of the polynomial in Equation (16), H_x is a $(N-1) \times (N-1)$ matrix and B_x, an $(N-1)$ vector. Their elements are

$$\begin{cases} h_{ij} = \sum_{l=1}^{M} \left[k_l t_l^{i+j+2} + (i+1)(j+1) s_l t_l^{i+j} \right], \\ b_i = -\sum_{l=1}^{M} \left[k_l t_l^{i+1} \bar{y}_l + (i+1) s_l t_l^{i} \dot{\bar{y}}_l \right]. \end{cases} \tag{21}$$

We can obtain X from Equation (21) and $u_x = 2a_{2x}$. In the same way we can acquire u_y and u_z. When $N = 2$, $M = 1$, $k_1 = k$ and $s_1 = 1$, the results of Equation (21) are the first component of Equation (15).

4 NUMERICAL SIMULATIONS

In order to verify the practicability of the linear feedback control, we carried out numerical simulations with some possible nominal orbits for WSO/UV. All the simulations have included the perturbations of the large bodies in the solar system. The ephemeris is analytical and this does not influence the order of magnitude of the results. The initial epoch is 2008a 1m 1d 0h, the initial deviation is 10^{-6} (here the unit is the distance between the sun and the earth, 10^{-6} is of the order of 10 km) in position and 10^{-6} (of the order of 0.1 m s^{-1}) in velocity. The mass of the spacecraft is 1t. In the numerical simulations we could add the effects of error of observation and control, but as these effects will not influence the quality of the control, these effects were not considered.

WSO/UV should stay around the L_2 point of the Sun-Earth system for 5–10 years. Its nominal orbit should be halo or quasi-halo around L_2 and the size of the orbit should be large.

4.1 Order-Three Halo Orbit

The expression of the nominal orbit is Equation (4), $\alpha = 0.15$ and $\beta = 0.16$ (the amplitude in three directions is about 225 000 km in x, 675 000 km in y, and 240 000 km in z). Figure 4 displays the results of orbit control over 10 years ($k = 10^6$). The low thrust is changed every 1.8 day, and the energy needed for 10 years is 1274.80701 m s^{-1}. The precision of orbit control is of order 1 km. Compared with the results of the order-one Lissajou orbit of the same amplitude, the needed energy is much less and the precision is better.

When higher order nominal orbits are used the energy consumed is less and the precision of orbit control is better. This is quite understandable. The goal of low thrust is two-fold: first, to compensate the deviation of force between the real model and the model under which the nominal orbit is constructed; second, to compensate the deviation between the nominal and controlled orbit. The model for the order-three solution is much closer to the real gravitation model than the model for the order-one solution, so the order-three solution is closer to the orbit under the real gravitational forces.

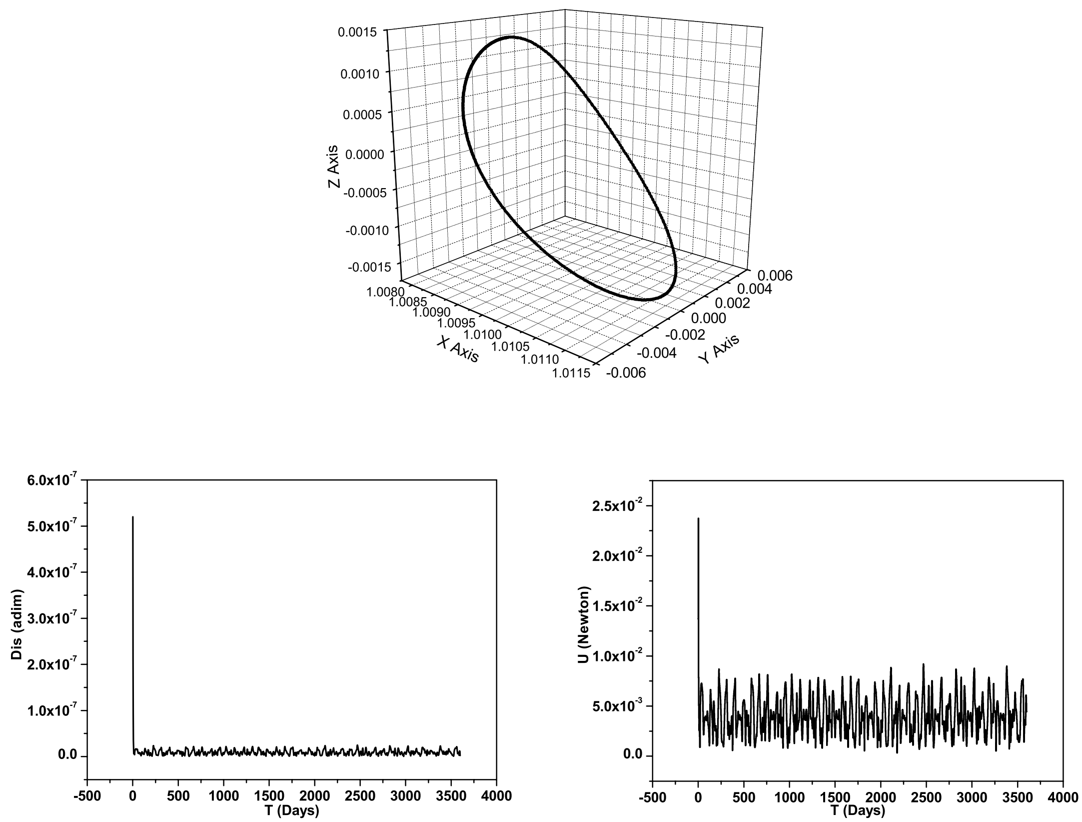

Fig. 4 Controlled orbit for 10 years (upper), distance between the controlled and nominal orbit at nodal points (left), and low thrust (right) for order-three Halo orbits.

4.2 Quasi-Halo Orbit Under The Real Gravitation Model

Figure 5 gives the results of orbit control for 10 years ($k = 15$). The nominal orbit is about the same size as the nominal orbit in Figure 4.

Every 18 days we change the low thrust, and the energy needed for 10 years is 0.67606 m s $^{-1}$. The thrust is large for the first few segments of the orbit and then decreases to zero. The distance between the controlled and nominal orbit behaves in the same way. Because the model for the nominal orbit is the real gravitation model, the goal of the low thrust is simply to pull the orbit back to the nominal orbit. The initial thrust is large because the initial deviation is set to be large. When the controlled orbit is pulled back to the nominal orbit, the thrust naturally vanishes.

The case that the low thrust is close to zero is not practicable. In fact there is a minimum for the low thrust in practice: when the thrust is smaller than this value, it is impossible to control it. Here, we take it as 1mN. That is to say, in the orbit control, if the computed low thrust is smaller than 1mN, we take it as zero, i.e., we do not control the spacecraft on this orbit segment. If we change the low thrust every 18 days, the energy consumed is 203.68161 m s^{-1} and the control precision is 1000 km.

The above numerical simulations show that spacecraft can be kept around the collinear libration point using the low thrust control strategy. The energy needed and the precision of orbit control depend on the nominal orbit and ΔT. Generally, the energy is less if ΔT is smaller or if a higher order orbit is taken as the nominal orbit. The precision of orbit control behaves similarly.

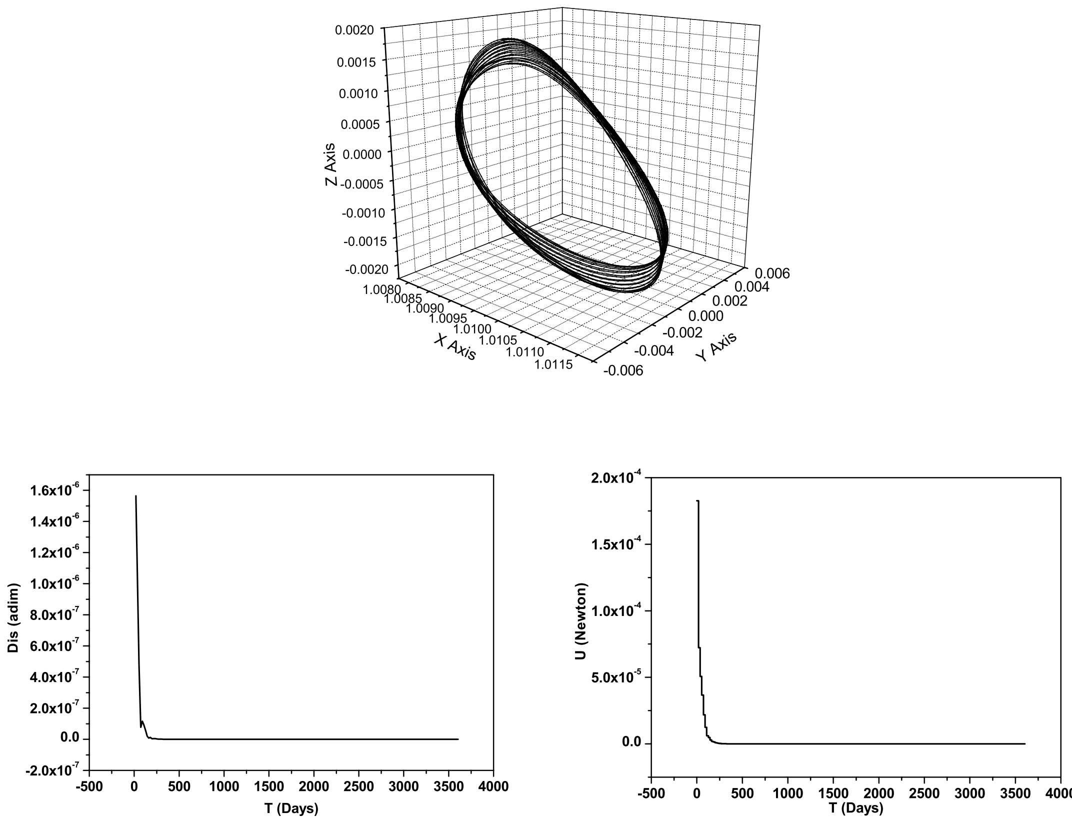

Fig. 5 Controlled orbit for 10 years (upper), distance between the controlled and nominal orbit at nodal points (left), and low thrust (right) for Quasi-Halo Orbits.

5 CONCLUSIONS

We propose a low-thrust control strategy for the orbit control of spacecraft around collinear libration points using the linear feedback method. We also carried out some numerical simulations to support our strategy. In these simulations, we omit the effects of errors in the observation and control. Where there is strong instability of the motion around the collinear libration points, the accumulation of these effects will increase the cost of orbit control. So, in practice we must take all of these effects into account.

Acknowledgements This work was supported by the National Natural Science Foundation of China (NSFC, G10073009).

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