The Black Hole Mass and Magnetic Field Correlation in Active Galactic Nuclei

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Abstract The observed $\nu L_{\nu}(5100\text{Å})$-$M_{\text{BH}}$ correlation is used to probe the magnetic field of black holes harbored in active galactic nuclei (AGNs). The model is based on the assumption that the disk is heated by energy injection due to the magnetic coupling (MC) process and the gravitational dissipation due to accretion. The MC process can transfer energy and angular momentum from a rotating Kerr black hole to its surrounding disk. The relation of $\nu L_{\nu}(5100\text{Å})$-$M_{\text{BH}}$ as functions of the spin ($a_*$) and magnetic field ($B_{\text{BH}}$) of the black hole (BH) is modelled. The model predicts that $\nu L_{\nu}(5100\text{Å})$ emitted from the disk is sensitive to the strength of the poloidal component of the magnetic field on the BH horizon. Based on the observations of $\nu L_{\nu}(5100\text{Å})$ for 143 AGN sources, we obtain the correlation between $M_{\text{BH}}$ and $B_{\text{BH}}$, which shows completely different results from the situation without considering the magnetic field coupling process.

Key words: Magnetic field — Black Hole Physics — AGN

1 INTRODUCTION

Recently, as a possible mechanism for transferring energy and angular momentum from a fast-rotating BH to its surrounding disk, the magnetic coupling (MC) process through the closed field lines connecting the Kerr BH with its exterior disk, has been widely studied (Li 2002a; Wang et al. 2002, 2003), which can be regarded as one of the variants of the Blandford-Znajek (BZ) process proposed two decades ago (Blandford & Znajek 1977). This mechanism has been applied to account for the recent XMM-Newton observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15 (Li 2002b) which has a very steep emissivity. It is assumed that the disk is stable, perfectly conducting, thin and Keplerian. The magnetic field is assumed to be constant on the black hole horizon and to vary as a power law with the radial coordinate of the disk.

The model involving the magnetic field configurations with both poloidal and toroidal components still remains an open question. Since the magnetic field $B_{\text{BH}}$ on the BH horizon is brought and held by its surrounding magnetized disk, there must exist some relations between the magnetic field and the accretion rate. However, studying the structure of $B_{\text{BH}}$ from its relation with the accretion rate is difficult because estimation of the accretion rate from observations is not easy (Wang 2003). Motivated by recent studies on the BH mass in AGNs and

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inactive galaxies, the relation of $\nu L_{\nu}(5100\text{Å})$-$M_{\text{BH}}$ is used to investigate the magnetic field of a BH.

2 MODEL PREDICTIONS OF THE $\nu L_{\nu}(5100\text{Å})$-$M_{\text{BH}}$ CORRELATION

It is known that the energy output from an accretion disk strongly depends on its structure, particularly, the amounts of thermal excess in the spectrum of the escaped radiation strongly depends on the disk temperature. Assuming that the disk is heated by considering the MC effects and the gravitational energy dissipation, the disk temperature can be estimated. Because thermodynamic equilibrium is postulated inside the accretion disk based on the equation of energy balance, consequently, the temperature of the disk can be approximated by balancing absorptions with emissions (Wang et al. 2002, 2003)

$$\sigma T^4 = F_{\text{MC}} + F_{\text{DA}},$$

where $F_{\text{MC}}$ and $F_{\text{DA}}$ are the emission fluxes due to the MC effects and the disk accretion in the situation of no-torque boundary condition (Page & Thorne 1974), respectively. $F_{\text{DA}}$ and $F_{\text{MC}}$ are formulated by

$$F_{\text{DA}}(x) = \frac{3M c^6}{8\pi M^2 G^2 x^4 (x^3 - 3x + 2a_s)} \times \left[ x - x_{\text{ms}} - \frac{3}{2} a_s \ln \left( \frac{x}{x_{\text{ms}}} \right) - \frac{3(x_1 - a_s)^2}{x_1(x_1 - x_2)(x_1 - x_3)} \ln \left( \frac{x - x_1}{x_{\text{ms}} - x_1} \right) - \frac{3(x_2 - a_s)^2}{x_2(x_2 - x_1)(x_2 - x_3)} \ln \left( \frac{x - x_2}{x_{\text{ms}} - x_2} \right) - \frac{3(x_3 - a_s)^2}{x_3(x_3 - x_1)(x_3 - x_2)} \ln \left( \frac{x - x_3}{x_{\text{ms}} - x_3} \right) \right],$$

$$F_{\text{MC}}(x) = \frac{3G z^3}{2 M c^2 x(x^3 + a_s)^2} \left[ E^+ - \Omega D L^+ \right]^{-2} \times \int_1^{\xi_{\text{out}}} (E^+ - \Omega D L^+) H \xi d\xi,$$

where $\xi_{\text{out}}$ is the outer boundary of the disk defined in the later formula of Eq.(7).

$$A_1 = 1 + (1 - a_s^2)^{1/3} [(1 + a_s)^{1/3} + (1 - a_s)^{1/3}] , \quad A_2 = (3a_s^2 + 3) / 2 ,$$

$$x_{\text{ms}} = \{ 3 + A_2 + [(3 - A_1)(3 + A_1 + 2A_2)]^{1/2} \}, \quad x_1 = 2 \cos \left[ \frac{1}{3} \arccos(a_s) - \frac{\pi}{3} \right] ,$$

$$x_2 = 2 \cos \left[ \frac{1}{3} \arccos(a_s) + \frac{\pi}{3} \right] , \quad x_3 = -2 \cos \left[ \frac{1}{3} \arccos(a_s) \right] , \quad \Omega_D = \frac{\Omega}{M(x^3 + a_s)} ,$$

$$E^+ = \frac{(1 - 2x^{-3} + a_s x^{-3})}{(1 - 3x^{-3} + 2a_s x^{-3})^{1/2}} , \quad L^+ = \frac{M x (1 - 2a_s x^{-3} + a_s^2 x^{-4})}{(1 - 3x^{-3} + 2a_s x^{-3})^{1/2}} .$$

The definitions of the basic parameters of a BH are:

$$a = J / M c , \quad r_g = GM / c^2 , \quad a_s = a / r_g , \quad r_H = r_g (1 + (1 - a_s^2)^{1/2}) , \quad x_{\text{ms}}^2 = r_{\text{ms}} / r_g ,$$

$$x^2 = r / r_g , \quad \xi = x^2 / x_{\text{ms}}^2 , \quad q = (1 - a_s^2)^{1/2} , \quad \beta = \frac{\Omega_D}{\Omega_H} = \frac{2(1 + q)}{a_s} [x^3 + a_s]^{-1} ,$$

where $r_H$ is the radius of BHs on the horizon. The mapping of the polar angle with the radius $\xi$ is defined as

$$\cos \theta = \int_1^{\xi} G(a_s, n, \xi) d\xi , \quad \cos \theta_1 = \int_1^{\xi_{\text{out}}} G(a_s, n, \xi) d\xi ,$$

where $\theta_1 = \pi / 6$, and

$$G(a_s, n, \xi) = - \frac{\xi^{1-n} x_{\text{ms}}^2}{2\sqrt{(1 + a_s^2 x_{\text{ms}}^4 + 2a_s^2 x_{\text{ms}}^6)(1 - 2x_{\text{ms}}^2 \xi^{-3} + a_s^2 x_{\text{ms}}^4 \xi^{-2})}} .$$
Since the magnetic field on the horizon of a BH is brought and held by its surrounding magnetized disk, for a cylindrical magnetic field with both poloidal and toroidal components, the poloidal magnetic field on the disk is assumed to vary with \( \xi \) as a power law (Blandford 1976; Wang et al. 2002, 2003)

\[
B_{\text{disk}} \propto \xi^{-n},
\]

where \( n \) is the power law index. The distribution of the angular momentum flux \( H \) transferred between the BH and the disk is (Wang et al. 2002, 2003)

\[
H = \begin{cases} H_0 A(a_*, \xi) \xi^{-n} & 1 < \xi < \xi_{\text{out}} \\ 0 & \xi > \xi_{\text{out}} \end{cases}
\]

where \( H_0 = (B_{\text{BH}})^2 M = 1.48 \times 10^{21} B_4^2 M_8 \text{ g s}^{-2} \), \( B_4 = B/10^4 \text{ G} \), \( M_8 = M/10^8 \text{M}_\odot \), and \( B_{\text{BH}} \) is the magnetic field on the BH horizon.

Substituting \( F_{\text{DA}} \) and \( F_{\text{MC}} \) into Eq.(1), we can obtain \( T(x) \). Then we can estimate the energy flux at frequency \( \nu \) from the disk, which is

\[
\nu L_\nu = \nu \int_{r_{\text{in}}}^{r_{\text{out}}} 2\pi I_\nu r dr,
\]

where \( I_\nu \) is the flux emitted by each unit disk element, which is assumed, very crudely, to satisfy

\[
I_\nu = B_\nu [T(x)] = \frac{2\hbar \nu^3 / c^2}{e^{\hbar \nu / kT(x)} - 1}.
\]

Setting the boundary condition and assuming that the disk extends down to the last stable orbit of the BH, it is thus possible to determine the relation of \( \nu L_\nu(5100\text{Å})-M_{\text{BH}} \) from Eq.(1) to Eq.(12). The \( \nu L_\nu(5100\text{Å}) \) versus \( M_{\text{BH}} \) relationships as functions of \( a_*, n \) and \( B_{\text{BH}} \) are plotted in Fig. 1, Figure 2 and Figure 3, respectively. The figures show that for the Kerr black hole the \( \nu L_\nu(5100\text{Å})-M_{\text{BH}} \) relation is not very sensitive to \( a_* \) and \( n \), but sensitive to the magnetic field strength \( B_{\text{BH}} \). Therefore the magnetic field strength is the only sensitive parameter for \( \nu L_\nu(5100\text{Å})-M_{\text{BH}} \) relation.

### 3 THE CORRELATION BETWEEN \( B_{\text{BH}} \) AND \( M_{\text{BH}} \)

Comparing Eq.(11) with the observational relation of \( \nu L_\nu(5100\text{Å})-M_{\text{BH}} \), we can therefore estimate the magnetic field strength \( B_{\text{BH}} \) for each source. The data used to get the \( \nu L_\nu(5100\text{Å}) \) relation here include 29 objects of Kaspi et al.’s (2000) samples, 30 objects of McLure et al.’s (2001) samples, and 84 quasars of Shields et al.’s (2003) samples. We omitted 5 objects of Kaspi et al.’s (2000) since their lower uncertainties are too large. A correlation between the black hole masses and the magnetic field strength is obtained for the 143 AGN sources discussed above, which is

\[
\log B_{\text{BH}} = 9.26 - 0.81 \log (M/M_\odot).
\]

The derived \( B \) versus \( M_{\text{BH}} \) correlation is plotted as dashed-line in Figure 4, together with data used.

Finally as comparison, we also consider the correlation between \( B_{\text{BH}} \) and \( M_{\text{BH}} \) derived in the situations with and without MC process. In the former case, combining with the consideration of MC process and the standard accretion disk, Eq.(13) shows that

\[
B_{\text{BH}} \propto M_{\text{BH}}^{-0.81}.
\]
Fig. 1 $\nu L_\nu$ versus $M_{\text{BH}}$ relationship for different values of black hole spin parameter $a_*$ with $n = 3$ and $B = 10^4$ G. The dotted, dashed-dotted, dashed and solid lines correspond to $a_* = 0.2$, $0.5$, $0.8$ and $0.995$, respectively.

Fig. 2 $\nu L_\nu$ versus $M_{\text{BH}}$ relationship for different values of the power-law index $n$ of magnetic field distribution (Eq. (9)) with $a_* = 0.5$, and $B = 10^4$ G. The dotted, dashed-dotted, dashed and solid lines correspond to $n = 1.5$, $3.0$, $5.0$ and $7.0$, respectively.

However in the latter case, if $F_{MC} \ll F_{DA}$, e.g., for the standard accretion disk (Shakura & Sunyaev 1973; Novikov & Throne 1973), the relationship between $B_{\text{BH}}$ and $M_{\text{BH}}$ is (Cheng & Lu 2001; Cao 2002),

$$B_{\text{BH}} \propto M_{\text{BH}}^{-0.5}. \quad (15)$$

It is clear that Eq.(14) and Eq.(15) show significantly different relationship between $M_{\text{BH}}$ and $B_{\text{BH}}$. 
\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{$\nu L_\nu$ versus $M_{BH}$ relationship for different values of black hole magnetic field $B_{BH}$ in the case of $\alpha = 0.5$ and $n = 3.0$. The dotted, dashed, dashed-dotted, solid and upper dotted lines correspond to $B_{BH} = 3 \times 10^3$ G, $1 \times 10^4$ G, $3 \times 10^4$ G, $1 \times 10^5$ G and $3 \times 10^5$ G, respectively.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4.png}
\caption{The derived $B$ versus $M_{BH}$ correlation. The sources are from Kaspi et al. 2000, McLure et al. 2001 and Shields et al. 2003, corresponding to dots, squares and crosses, respectively. The dashed-line is the best-fit result from the data points, as shown in Eq. (13).}
\end{figure}

4 SUMMARY

Based on the MC process which can transfer energy and angular momentum from a fast-rotating Kerr BH to its surrounding disk and then radiated in the way of thermal emission, we
investigate the relation of the BH mass with its magnetic field by comparing with observation data, provided that the magnetic field on the black hole horizon is constant and the poloidal magnetic field on the disk varies with the radius as power law. The model shows that the relation of \( \nu L_\nu(5100\text{Å}) - M_{\text{BH}} \) is not very sensitive on the values of \( a_* \) and \( n \). The value of \( B_{\text{BH}} \) on the horizon of Kerr BH is the only sensitive parameter for the emission of \( \nu L_\nu(5100\text{Å}) \) from the disk. Consequently a correlation \( \log B = 9.26 - 0.81 \log M/M_\odot \) is obtained for the samples of 143 AGN sources. This relation is different with the result with respond to the standard accretion disc situation. The model is limited by the available data \( \nu L_\nu(5100\text{Å}) \) from the samples of Kaspi et al. (2000), McLure et al. (2001), and Shields et al. (2003).

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**References**