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Corrections to Solar Thermal Structure when a Turbulent Magnetic Field is Included *

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Abstract Correction of non-ideal effect due to a magnetic fluctuating tensor is derived from the ideal MHD equations. The inclusion of a magnetic turbulent field leads to modifications of the hydrostatic equilibrium equation and thermodynamical variables such as the temperature T, the adiabatic exponent γ, the adiabatic temperature gradient $\nabla_{\rm ad}$ and the temperature gradient ∇. In particular, the modifications in the adiabatic and radiative temperature gradients will result in a change in the Schwarzchild criterion, hence in the location of the base of the convective zone. Incorporating the modifications, we construct a modified thermodynamical equilibrium structure of the Sun.

Key words: solar convection — MHD: thermodynamics variables — Sun: hydrostatic equilibrium equation

1 INTRODUCTION

With the increasing quality of observational methods, more and more observed data have revealed magnetic activities on the surface of the Sun and sun-like stars (Meunier 2003; Vaquero & Gallego 2002; Chaplin et al. 2001; Neugebauer et al. 2000). However, the generation of the magnetic activities is still not clearly understood, and still awaits further investigations of the physical processes in the solar interior.

Previously, the physics of pure turbulence was applied to improve the condition of solar convection. The properties of turbulent convection and the important effect of pure turbulent pressure on solar structure and solar p-mode oscillations have been well studied (Christensen-Dalsgaar & Frandsen 1983; Kosovichev 1995; Gabriel 1995; Böhmer & Rüdiger 1998, 1999; Bi & Xu 2000, 2002). Recently, the effects of magnetic fields on the solar structure and solar p-mode oscillations are examined by some authors (Bi et al. 2003; Li et al. 2002, 2001; Zhukov 2001; Antia et al. 2000). Although much work has been dedicated to describe the effects of the mean magnetic field on solar structure, the effects by the magnetic turbulent field have not been so thoroughly examined.

In this paper, we examine how a turbulent magnetic field influences the solar thermal structure. The non-ideal effect of MHD turbulence on the solar thermal structure depends not only

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on how it is modeled, but also on how it is incorporated into the solar model. Our study will proceed in three steps: (1) Since the radial component of the magnetic fluctuating tensor contributes to the pressure and also influences the internal energy density, we modify the equation of state through changes in the total pressure and internal energy; (2) We write down the first law of thermodynamics with the magnetic energy included and correct the thermodynamical variables due to the turbulent magnetic field. (3) We accordingly improve one of the equations of stellar structure, namely, the equation of hydrostatic equilibrium.

This paper is organized as follows. Section 2 presents the basic physical formulae including a mathematical description of the MHD turbulence pressure, the improved equation of state containing the ratio of turbulent magnetic pressure to gas pressure $\beta_{\rm m}$, and the improved hydrostatic equilibrium equation. Section 3 shows the expression of turbulent magnetic field used in the calculation of the MHD turbulent pressure. Sections 4 and 5 present, respectively, the numerical results obtained and our conclusion.

2 BASIC PHYSICAL FORMULAS

2.1 MHD Turbulent Pressure

We consider an ideal MHD turbulence in a homogeneous, isotropic and stationary medium. The motion of the fluid obeys the MHD equations (Unno et al. 1989):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{1}$$

$$\rho \left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla \right) \boldsymbol{v} = -\nabla P + \frac{1}{4\pi} (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \rho \boldsymbol{g}, \tag{2}$$

where ρ is the total density, P is the pressure, $\boldsymbol{v}$ is the velocity vector, and the magnetic field $\boldsymbol{B}$ is described by the induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}), \tag{3}$$

$$\nabla \cdot \boldsymbol{B} = \boldsymbol{0}, \tag{4}$$

where the gravitational acceleration $\boldsymbol{g}$ can be written as the gradient of a gravitational potential $\boldsymbol{\Phi}$:

$$\boldsymbol{g} = \nabla \boldsymbol{\Phi}, \tag{5}$$

and $\boldsymbol{\Phi}$ is determined by Posson's equation

$$\nabla^2 \boldsymbol{\Phi} = -4\pi G \rho, \tag{6}$$

G being the gravitational constant.

In the MHD equations, each variable can be considered as consisting of a mean part, indicated by the overbar, and a fluctuating part, indicated by the prime (the fluctuating part of the velocity $\boldsymbol{v}$ being the turbulent velocity $\boldsymbol{u}$):

$$\rho = \overline{\rho} + \rho', \quad \boldsymbol{v} = \overline{\boldsymbol{v}} + \boldsymbol{u}, \quad P = \overline{P} + P', \quad \boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{B}', \tag{7}$$

Taking into account that the fluctuating components have zero average,

$$\overline{\rho'} = \overline{\boldsymbol{B}'} = \overline{P'} = \overline{\boldsymbol{u}} = 0, \tag{8}$$

and, assuming the velocity field to be solenoidal,

$$\left(\overline{\boldsymbol{B}}\cdot\nabla\right)\overline{\boldsymbol{v}}-\left(\overline{\boldsymbol{v}}\cdot\nabla\right)\overline{\boldsymbol{B}}=0. \tag{9}$$

We obtain, after some averaging, the mean field MHD equations:

$$\frac{\partial\overline{\rho}}{\partial t}+\nabla\cdot\left(\overline{\rho}\,\overline{\boldsymbol{v}}\right)=0\,, \tag{10}$$

$$\begin{aligned}\overline{\rho}\frac{d}{dt}\overline{\boldsymbol{v}} &= -\nabla\left(\overline{P}+\overline{P_B}\right)-\nabla\cdot\overline{\rho}\left(\langle\boldsymbol{u}\boldsymbol{u}\rangle-\frac{1}{4\pi\overline{\rho}}\left\langle\boldsymbol{B}'\boldsymbol{B}'\right\rangle\right) \\ &\quad+\frac{1}{4\pi}\left[\left(\nabla\times\boldsymbol{B}'\right)\times\overline{\boldsymbol{B}}+\left(\nabla\times\overline{\boldsymbol{B}}\right)\times\boldsymbol{B}'\right]+\overline{\rho}\boldsymbol{g},\end{aligned} \tag{11}$$

$$\frac{\partial\overline{\boldsymbol{B}}}{\partial t}=\nabla\times\left(\overline{\boldsymbol{v}}\times\overline{\boldsymbol{B}}+\left\langle\boldsymbol{u}\times\boldsymbol{B}'\right\rangle\right), \tag{12}$$

$$\nabla\cdot\boldsymbol{B}=0, \tag{13}$$

where the mean magnetic pressure is defined as:

$$\overline{P}_B=\frac{\overline{B}^2}{8\pi}. \tag{14}$$

It should be noted that we have assumed that the variation of the mean field $\overline{\boldsymbol{B}}$ along the magnetic field lines can be neglected, i.e., $\overline{\boldsymbol{B}}\cdot\nabla\overline{\boldsymbol{B}}=0$. Also, in the averaging procedure, we adopted the algorithm, $\overline{\rho'\boldsymbol{f}}=\overline{\rho'}\quad\overline{\boldsymbol{f}}$, $\boldsymbol{f}$ denoting any one of the variables except the magnetic field $\boldsymbol{B}$.

We introduce the MHD turbulence pressure P_t. It can be written as:

$$\overline{\rho}\left(\langle\boldsymbol{u}\boldsymbol{u}\rangle-\frac{1}{4\pi\overline{\rho}}\left\langle\boldsymbol{B}'\boldsymbol{B}'\right\rangle\right)=P_t\boldsymbol{I}+\boldsymbol{II}, \tag{15}$$

where $\boldsymbol{I}$ is the unit tensor, $\boldsymbol{II}$ is the turbulent viscosity stress tensor due to anisotropic properties in the real situation. For an ideal MHD turbulence in a homogeneous, isotropic and stationary medium, the MHD turbulent pressure (including the dynamical and the magnetic pressure) has a simple expression (Landau & Lifshitz 1975, 1984):

$$P_t=\frac{1}{3}P_{\rm m}^{(0)}+\frac{2}{3}P_{\rm k}^{(0)}, \tag{16}$$

where $P_{\rm m}^{(0)}=\left\langle\boldsymbol{B}'\cdot\boldsymbol{B}'\right\rangle/8\pi$ is the energy density of the magnetic fluctuations and $P_{\rm k}^{(0)}=\overline{\rho}\left\langle\boldsymbol{u}\cdot\boldsymbol{u}\right\rangle/2$ is the energy density of the turbulent hydrodynamic motion. Combining Eqs. (15) and (11), we obtain

$$\overline{\rho}\frac{d}{dt}\overline{\boldsymbol{v}}=-\nabla P_{\rm tot}+\overline{\rho}\boldsymbol{g}+\frac{1}{4\pi}\left[\left(\nabla\times\boldsymbol{B}'\right)\times\overline{\boldsymbol{B}}+\left(\nabla\times\overline{\boldsymbol{B}}\right)\times\boldsymbol{B}'\right]. \tag{17}$$

It is easily found that the total pressure has three parts: (1) a mean pressure $\overline{P}$ which consists of the thermal ideal gas pressure and the radiation pressure, (2) a mean magnetic pressure $\overline{P}_B$ and (3) an MHD turbulent pressure $P_{\rm t}$ which consists of a magnetic fluctuating pressure $P_{\rm m}^{(0)}$ and an isotropic turbulent hydrodynamic pressure $P_{\rm k}^{(0)}$. Thus, the total pressure is

$$P_{\rm tot}\equiv\overline{P}+P_{\rm m}, \tag{18}$$

where $P_{\rm m}=\overline{P}_B+P_{\rm t}$.

2.2 Corrections to the Thermodynamical Variables

Here we derive the corrections to the various thermodynamic variables due to the presence of a turbulent magnetic field. These corrections will be indicated by the symbol Δ. The magnetic turbulent pressure can be written as the sum of a mean magnetic pressure and a part due to the fluctuating magnetic field (Kleeorin 1994),

$$P_{\mathrm{m}} = \frac{1}{3}\frac{\langle \boldsymbol{B}' \cdot \boldsymbol{B}' \rangle}{8\pi} + Q_{\mathrm{p}}\frac{\overline{B}^2}{8\pi}, \tag{19}$$

and

$$Q_{\mathrm{p}} = 1 - \frac{1}{6}\ln\left(R_{\mathrm{m}}\right) , \tag{20}$$

R_{m} being the magnetic Reynolds number.

In terms of the ratio of radiation pressure to gas pressure $\beta = P_{\mathrm{R}}/P_{\mathrm{g}}$ and the ratio of magnetic pressure to gas pressure $\beta_{\mathrm{m}} = P_{\mathrm{m}}/P_{\mathrm{g}}$, the total pressure can be written as (Esin 1997):

$$P_{\mathrm{tot}} = \overline{P} + \left(\beta_{\mathrm{m}}\frac{\rho k T}{\mu m_u}\right) . \tag{21}$$

The correction to the pressure is, therefore,

$$\Delta P = \left(\beta_{\mathrm{m}}\frac{\rho k T}{\mu m_u}\right) . \tag{22}$$

The part of the internal energy due to the magnetic field is,

$$\Delta u = \frac{1}{\rho}P_{\mathrm{m}} = \beta_{\mathrm{m}}\frac{kT}{\mu m_u} . \tag{23}$$

The first law of thermodynamics states that

$$T\mathrm{d}s = \mathrm{d}u - \frac{P}{\rho^2}\mathrm{d}\rho, \tag{24}$$

We note that the magnetic pressure is much smaller than the gas pressure and that β_{m} can be taken to be a constant when we derive the corrections to the other variables. The correction to ds is then

$$\Delta(\mathrm{d}s) = \beta_{\mathrm{m}}\frac{k}{\mu m_u}\frac{\mathrm{d}T}{T} - \beta_{\mathrm{m}}\frac{k}{\mu m_u}\frac{\mathrm{d}\rho}{\rho}, \tag{25}$$

and the correction to the specific heat per unit mass at constant density is

$$\Delta c_v = \beta_m \frac{k}{\mu m_u}, \tag{26}$$

and the correction to the specific heat per unit mass at constant pressure is

$$\Delta c_{\mathrm{p}} = 2\beta_{\mathrm{m}}\frac{k}{\mu m_u} . \tag{27}$$

The corresponding correction to the adiabatic exponent is then

$$\Delta\gamma = \frac{1}{c_v}\Delta c_{\mathrm{p}} - \frac{c_{\mathrm{p}}}{c_v^2}\Delta c_v . \tag{28}$$

Hence, according to the expression of the adiabatic temperature gradient,

$$\nabla_{\rm ad} = \frac{\gamma - 1}{\gamma}, \tag{29}$$

the correction to the adiabatic temperature gradient is

$$\Delta(\nabla_{\rm ad}) = \frac{\Delta\gamma}{\gamma^2}. \tag{30}$$

Meanwhile the turbulent magnetic field makes a contribution to the total pressure and internal energy, so leading to changes in the thermodynamical variables. It also modifies the superadiabatic temperature gradient, resulting in changes in the solar convection. It should be noted that the corrections derived here apply only to the solar convective zone and the derivations represent rough modifications of the equation of state.

2.3 Improving the Equation of Hydrostatic Equilibrium

We assume spherical symmetry of the solar structure and that all the physical quantities vary only along the radial direction and, being equilibrium values, are time independent. Using Eq. (17) in the stationary limit, the equation of hydrostatic equilibrium can be written as:

$$\frac{\rm d}{{\rm d}r}(\overline{P} + P_{\rm m}) = -g\overline{\rho}\,. \tag{31}$$

Note that it includes a turbulent magnetic pressure from the turbulent magnetic field.

3 TURBULENT MAGNETIC FIELD

The turbulent magnetic field is generated by turbulence, so we assume that its energy spectrum is similar to the turbulence spectrum. Assuming energy equipartition for the local spectra (Kleeorin et al. 1996; Bi et al. 2003), we have

$$\frac{\rho\left\langle \boldsymbol{uu}\right\rangle}{2} = \frac{\left\langle \boldsymbol{B}'\boldsymbol{B}'\right\rangle}{8\pi}\,. \tag{32}$$

For a simple case, consider an ideal incompressible MHD turbulence in a homogeneous, isotropic and stationary medium, with a power-law velocity correlation function, $\langle \boldsymbol{uu}\rangle$ along the radial direction (Boldyrev et al. 2004). From the condition of energy equipartition, the correlation function of the magnetic field has the form

$$\langle \boldsymbol{B}'\boldsymbol{B}'\rangle \propto r^{\mu}\,. \tag{33}$$

In this paper, we only calculate the effects of the mean and fluctuating magnetic fields on the thermodynamical variables and the equilibrium structure. As Eq. (19) and Eq. (32) show, $P_{\rm m}$ has the form:

$$P_{\rm m} = -\frac{1}{3}\rho\Lambda r^{\mu} + Q_{\rm p}\frac{\overline{B}^2}{8\pi}\,, \tag{34}$$

with Λ is a coefficient measuring the rate of transfer of turbulent energy to magnetic energy.

4 THE NUMERICAL RESULTS

We consider the effects of turbulent magnetic fields with the help of an improved time-dependent mixing-length theory. We construct a solar model with the Eggleton's code (Eggleton et al. 1973; Pols et al. 1995). Our numerical integration was carried out for three cases:

1. We incorporate the tables of radiative opacity derived from OPAL, and from Alexander (Alexander & Ferguson 1994a, 1994b) for low temperatures where molecular opacity is important. The initial composition is $X = 0.70$, $Y = 0.28$, $Z = 0.02$. As in usual solar models, $P_{\rm m}$ is not included;
2. The influence of a magnetic field is considered: we take, for the part of turbulent magnetic pressure, $\mu = 0.85632$ and $\Lambda = 1/3 \times 1.68521 \times 10^{-8}$; for the part of mean magnetic pressure, $\overline{B} = 30\,{\rm G}$ and $R_{\rm m} = 1 \times 10^5$. At the surface, the total pressure is modified in the boundary condition of the pressure equation.
3. We take a different set of the parameter values. For the turbulent magnetic pressure: $\mu = 0.85632$, and $\Lambda = 1/2 \times 1.68521 \times 10^{-8}$; for the mean magnetic pressure: $\overline{B} = 50\,{\rm G}$ and $R_{\rm m} = 1 \times 10^6$.

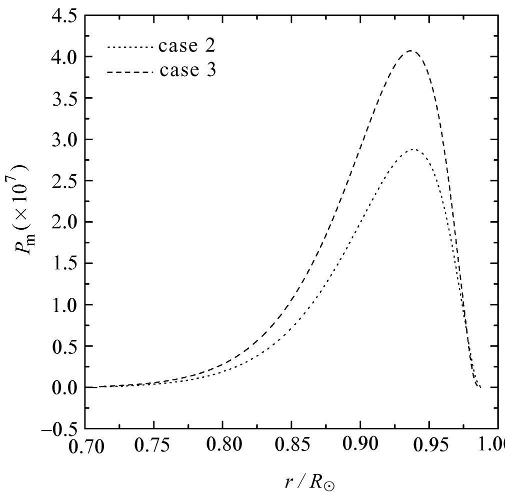

Fig. 1 Radial profile of the turbulent magnetic pressure for case 2 (dotted line) and case 3 (dashed line).

The variation of the turbulent magnetic pressure along the solar radius is shown in Fig. 1. In the solar convective zone, the contribution of the turbulent magnetic pressure to the total pressure is directly related to the degree of the turbulence. Near the surface, the turbulent magnetic pressure has the highest value, so the influence on the thermal variables will be more apparent there. As the gas density decreases at the solar surface, energy transportation becomes less efficient and the turbulent magnetic pressure decreases.

The position of the base of convective zone can be determined very precisely from solar oscillation frequencies (Chrestensen-Dalsgaard 1996, 2000). For its location at $0.7166\,R_\odot$, we calculated the corresponding ages of the Sun in the three cases. For case one, the age is about 4.60×10^9 yr; for case two, about 4.38×10^9 yr; for case three, about 4.23×10^9 yr. These results tell us that the Sun is younger than what we thought previously. It shows that a magnetic field speeds up the evolution of the star, the stronger the magnetic field the clearer the evidence for the speed-up.

Our calculations refer to ages from the ZAMS to the present age of the Sun, 4.6×10^9 yr. The base of the convective zone was found to move from $0.7166\,R_\odot$ to $0.7249\,R_\odot$ (case 2) or to $0.7373\,R_\odot$ (case 3). Our calculation clearly indicates that a magnetic field inhibits the generation of convection.

The position of the base of the convective zone is determined by the Schwarzchild criterion involving the temperature gradient. The radial profile of the superadiabatic temperature gradient is shown in Fig. 2 for case 1 (no magnetic field, solid line) and case 3 (with magnetic field, dashed line). At the base of the convective zone, the superadiabatic temperature gradient decreases and moves outwards. It influences the zero position of the Schwarzchild criterion. So the base of the convective zone moves outwards.

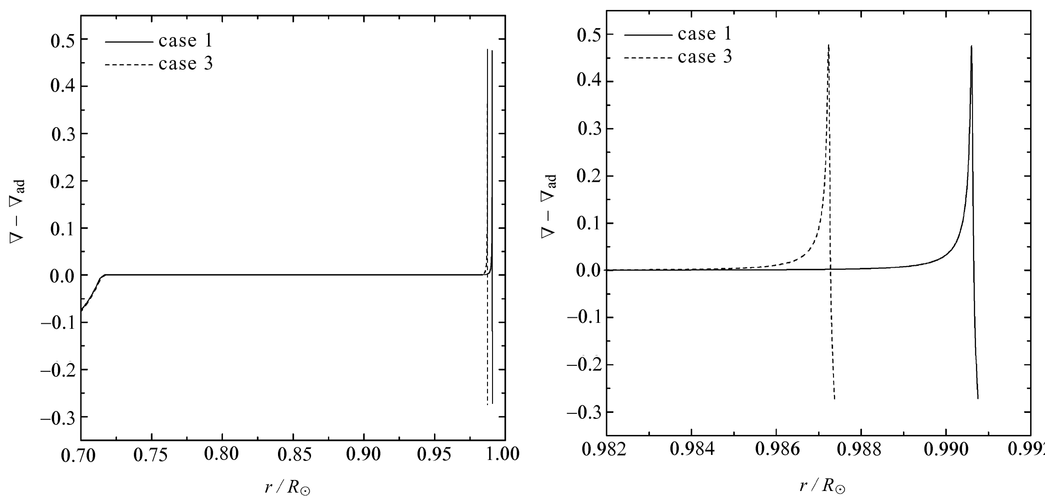

Fig. 2 Radial profile of the superadiabatic temperature gradient. Solid line and dashed line refer to the models with no magnetic field (case 1) and with magnetic field (case 3), respectively. The right panel is a blow-up of the range $0.982 - 0.992\,R_\odot$.

To understand these results, we note that the energy which is stored in the turbulent magnetic field accouts for part of the total energy flux at the bottom of the convective zone. To the radiative flux $F_{\rm rad}$ and the convective flux $F_{\rm conv}$, the turbulent magnetic energy flux $F_{\rm m}$ will be added in the total energy flux. So the sum of $F_{\rm rad}$ and $F_{\rm conv}$ decreases. The convective temperature gradient $\nabla_{\rm conv}$ is determined by $F_{\rm rad}$ and $F_{\rm conv}$, so the profile of the superadiabatic temperature gradient moves outwards and the effective temperature of the Sun decreases at the surface when a turbulent magnetic field is included.

Figure 3 illustrates the changes in the thermodynamical variables when a tubulent magnetic field is included: their values are decreased in the convective zone (compare the dotted and dashed "with-field" curves with the solid "no-field" curve). The reason is that the internal energy of the gas decreases, which then causes the other variables and the thermal structure to change. It shows that, when a turbulent magnetic fields is included, energy is redistributed, leading to changes in the thermodynamical properties of the solar material.

5 CONCLUSIONS

The purpose of this paper is to show how the inclusion of a turbulent magnetic field affects the equilibrium structure of the Sun.

1. The main conclusion is that the thermodynamics equilibrium structure of the Sun is modified when the turbulent magnetic field is considered. The modification is especially evident in the convective zone.

2. The magnetic field inhibits the generation of convection and it also speeds up the evolution of the star.

3. Energy is redistributed by the inclusion of a turbulent magnetic field.

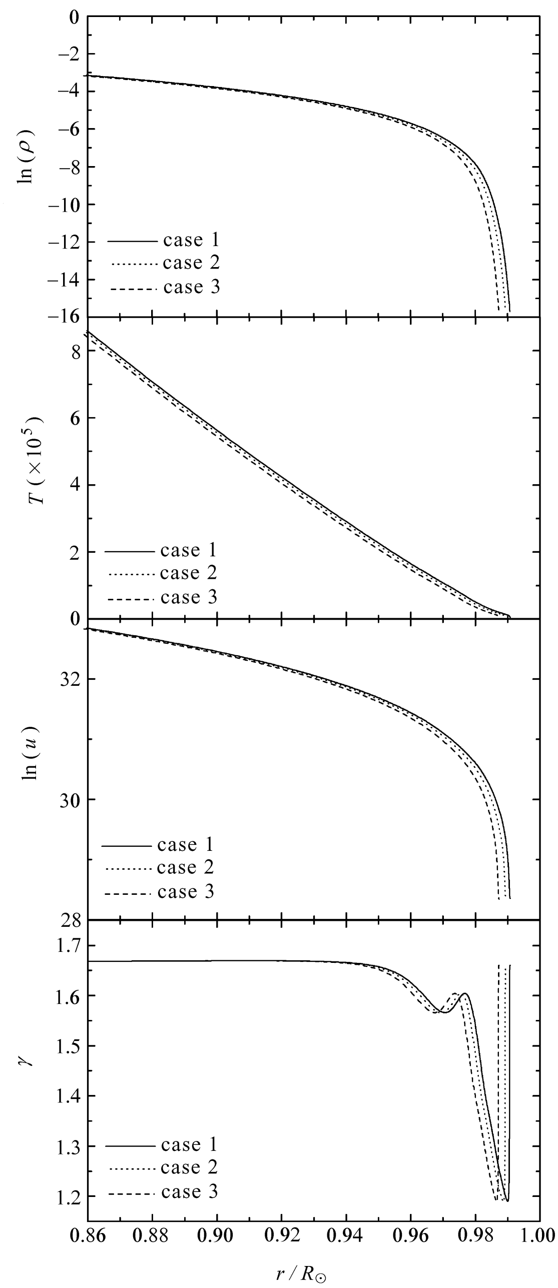

Fig. 3 Thermodynamical variables as a function of depth in the convective zone for case 1 (no magnetic field, solid line) and case 3 (with magnetic field, dotted/dashed line).

Although our formulation can be applied to improve the solar model, further investigation about the geometry of the magnetic field and more precision calculation of the field strength should be considered in future work.

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